Explanations in CP

Satisfaction problems

- “Why has variable x value a in a/the solution?”
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Optimization problems

- “Why is x = a in a/the optimal solution?”
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Counterfactual explanations

Given a constraint optimization problem with optimal solution x^*

<u>User:</u> “Why not the solution $\hat{x}$ I thought of instead of optimal x^*?”

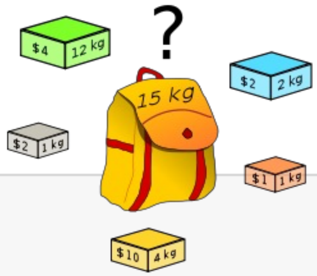
<u>Program:</u> “For $\hat{x}$ to be optimal the objective coefficients must change to d^*”

Ideally: changes to objective are as small as possible to ensure interpretability

Counterfactual explanations

Example

Knapsack:




```
model = Model()

gr,bl,og,ye,gy = boolvar(shape=5)

model += (12*gr + 2*bl + 1*og + 4*ye + 1*gy <= 15)

model.maximize(4*gr + 2*bl + 1*og + 10*ye + 2*gy)


model.solve()
```

```
print(gr.value(), bl.value(), og.value(), ye.value(), gy.value())
```

```
0 1 1 1 1
```

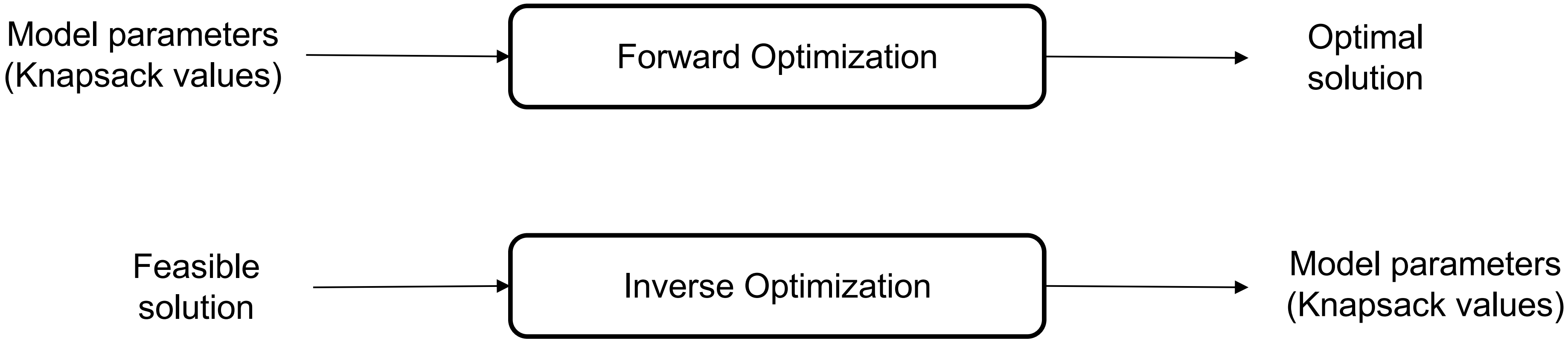
<u>User:</u>

“Why does the optimal not include gr?”

<u>Program:</u>

“Because to include gr into the knapsack, its value should change to at least 11 instead of 4”

(Inverse) Optimization



Inverse optimization and Counterfactual expl

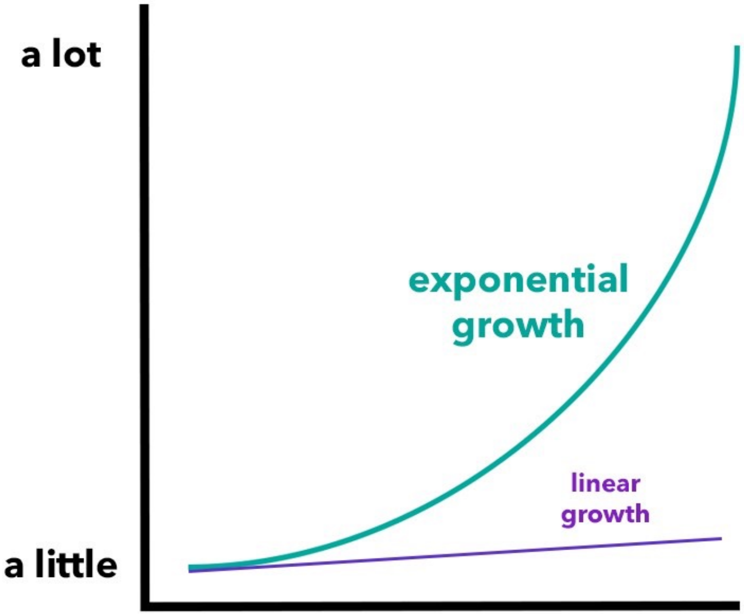
- User provides *part of* new solution $\hat{x}$
  - We want an explanation in terms of variables in this partial assignment
  - First extend partial solution to full solution [1]
- Find optimal objective for $\hat{x}$ and return as explanation

[1] Korikov, Anton, and J. Christopher Beck. "Counterfactual explanations via inverse constraint programming." *27th International Conference on Principles and Practice of Constraint Programming (CP 2021)*. Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2021.

Inverse optimization

$$\begin{aligned}\mathbf{IO}(\mathbf{c}^0, \hat{\mathbf{x}}, \mathcal{X}): \ \underset{\mathbf{c}\in\mathcal{P}}{\text{minimize}} \quad & \left\|\mathbf{c} - \mathbf{c}^0\right\|_1 \\ \text{subject to} \quad & \mathbf{c}^\top \hat{\mathbf{x}} \le \mathbf{c}^\top \mathbf{x}_j, \quad \forall \mathbf{x}_j \in \mathcal{E}(\mathcal{X})\end{aligned}$$

Inverse optimization



$$\forall \mathbf{x}_j \in \mathcal{E}(\mathcal{X})$$

Finding cutting planes

- Master problem:
  - Add cut to feasible region and find new optimal cost vector

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- Sub problem:
  - Find extreme optimal point on convex hull given a cost vector c
  - I.e., solve the original forward problem with a new cost vector

$$\begin{aligned} \mathbf{FP}(\mathbf{c}, \mathcal{X}): \ \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{c}^\top \mathbf{x} \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{X} := \{\mathbf{A}\mathbf{x} \ge \mathbf{b}, \ \mathbf{x} \in \mathbb{Z}^{n-q} \times \mathbb{R}^q\}. \end{aligned}$$

Finding cutting planes

Iteratively add cuts → Incremental MIP solver

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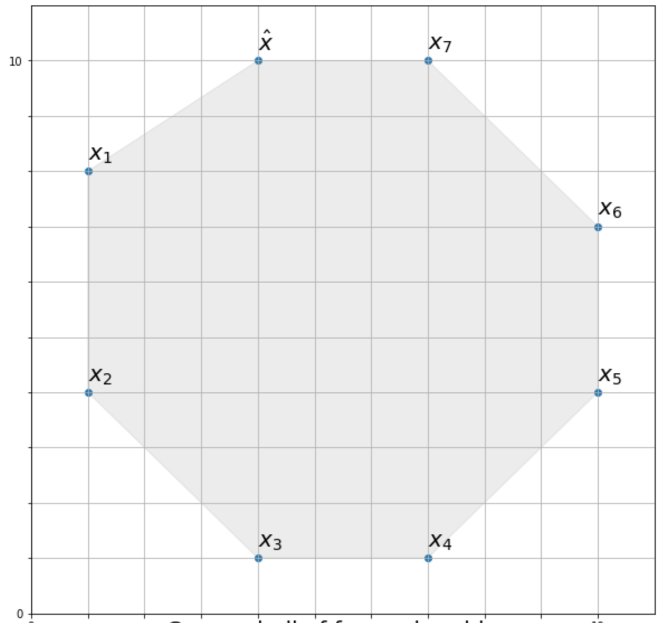
Original problem → problem specific solver

Cutting planes for Inverse Optimization

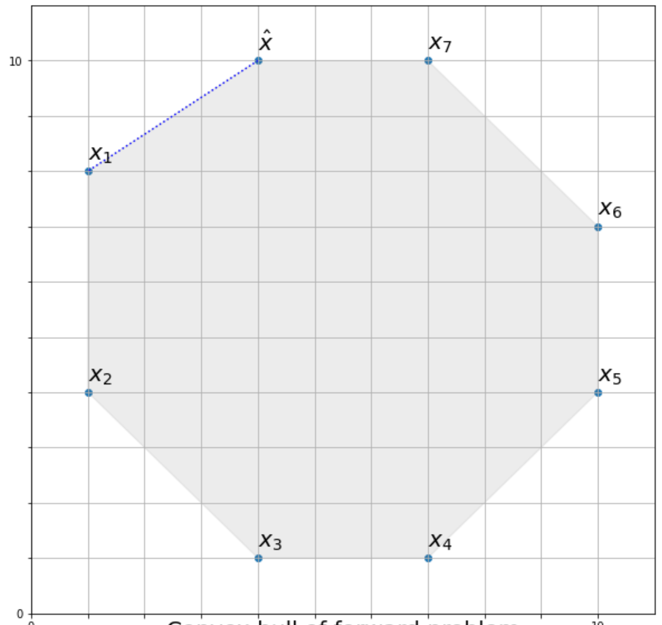


Convex hull of forward problem

Every extreme point on the convex hull corresponds to an optimal solution for <u>*some*</u> cost vector c

Task: find the cost vector <u>making $\hat{x}$ optimal</u>
while minimizing changes to origninal cost vector

Cutting planes for Inverse Optimization



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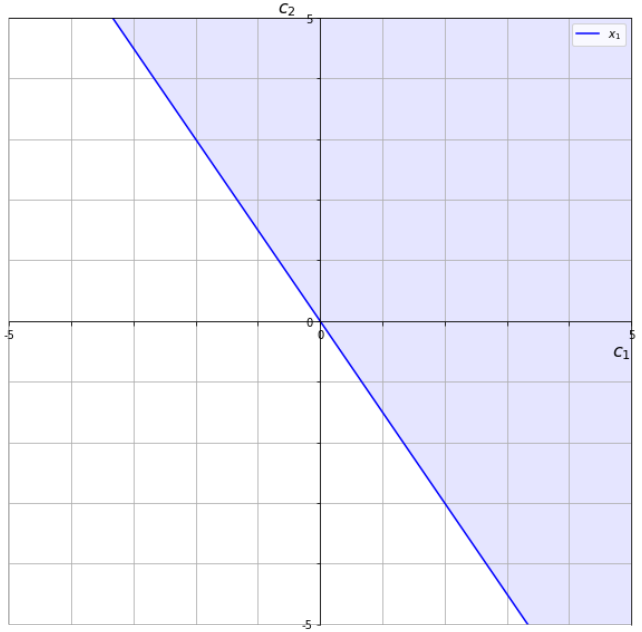

Half spaces induced by cutting planes

Bodur, Merve, Timothy CY Chan, and Ian Yihang Zhu. "Inverse Mixed Integer Optimization: Polyhedral Insights and Trust Region Methods." INFORMS Journal on Computing (2022).

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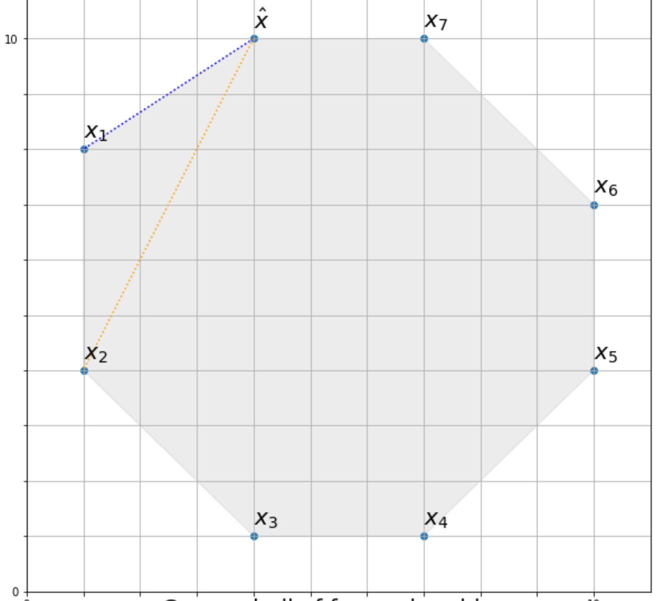

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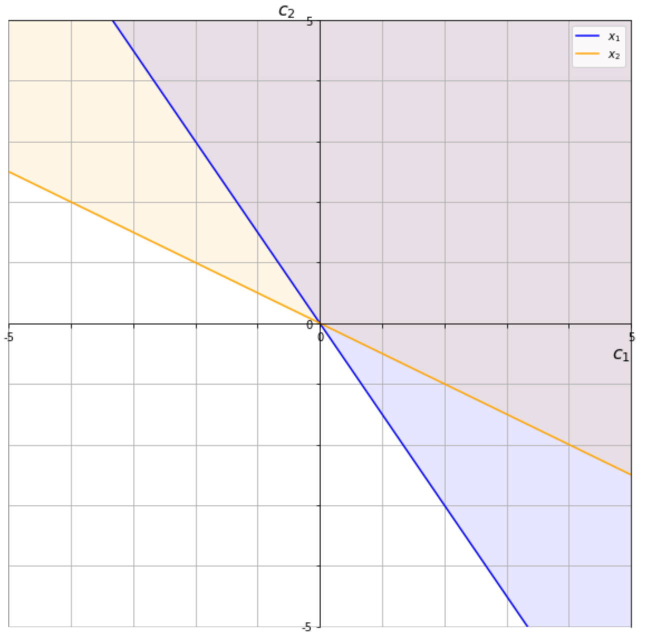

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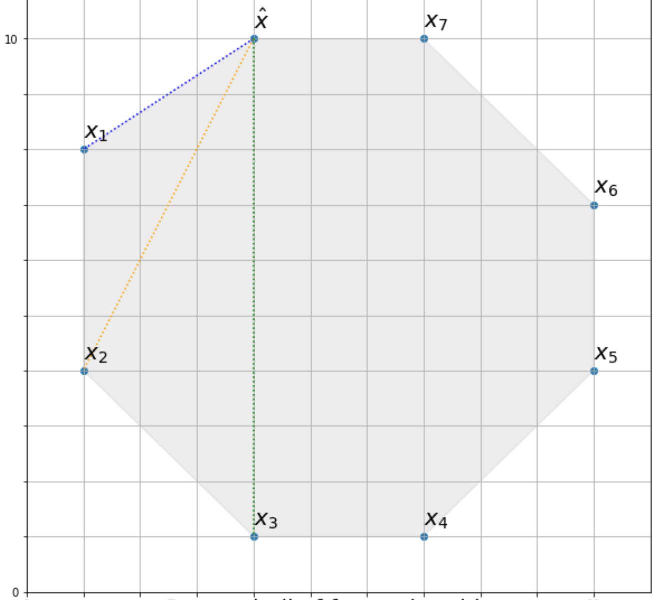

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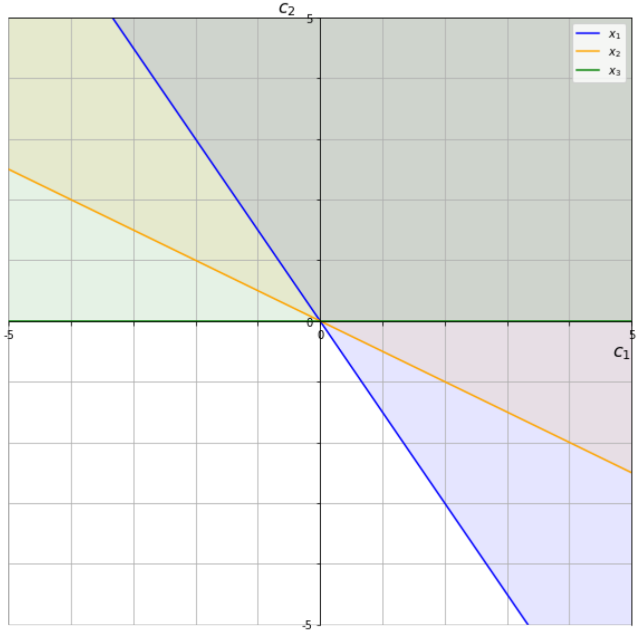

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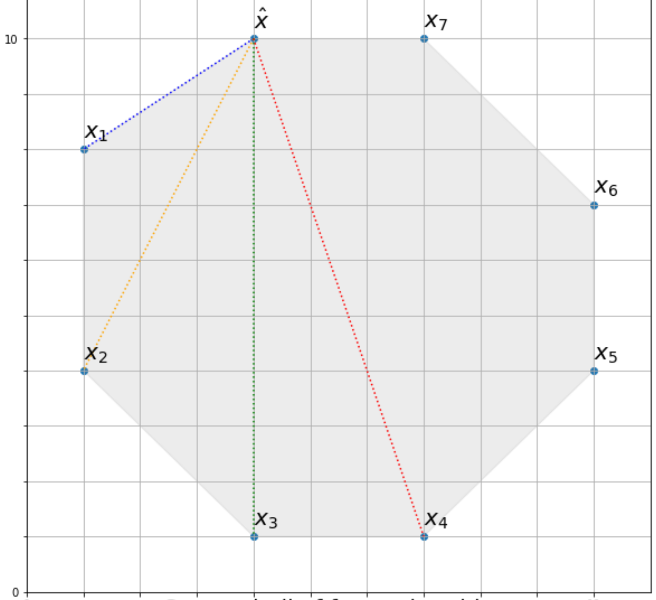

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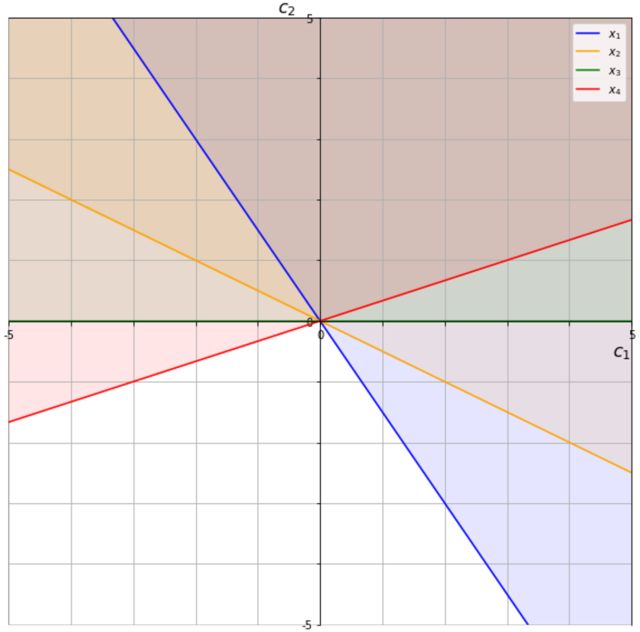

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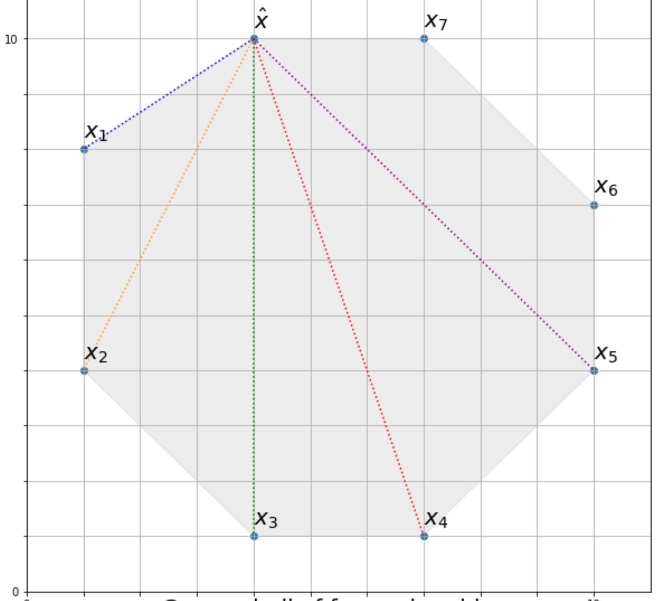

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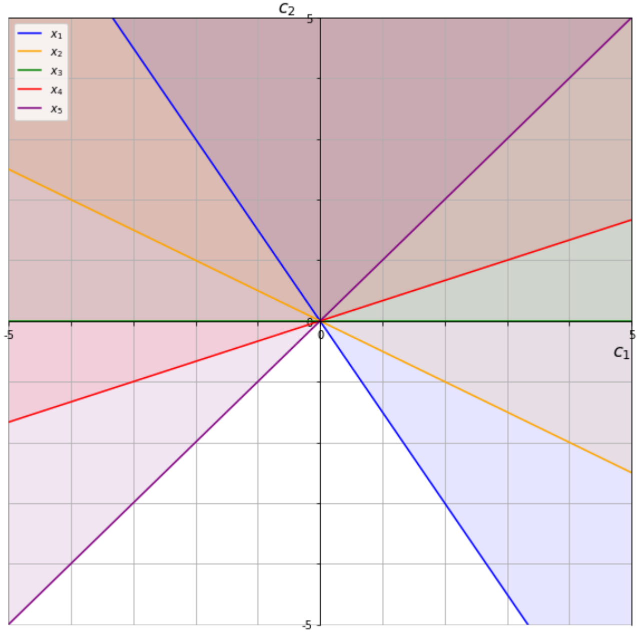

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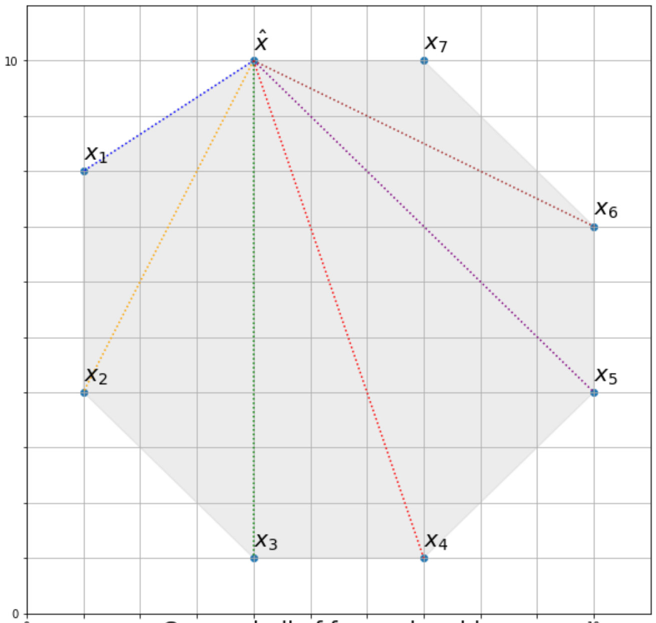

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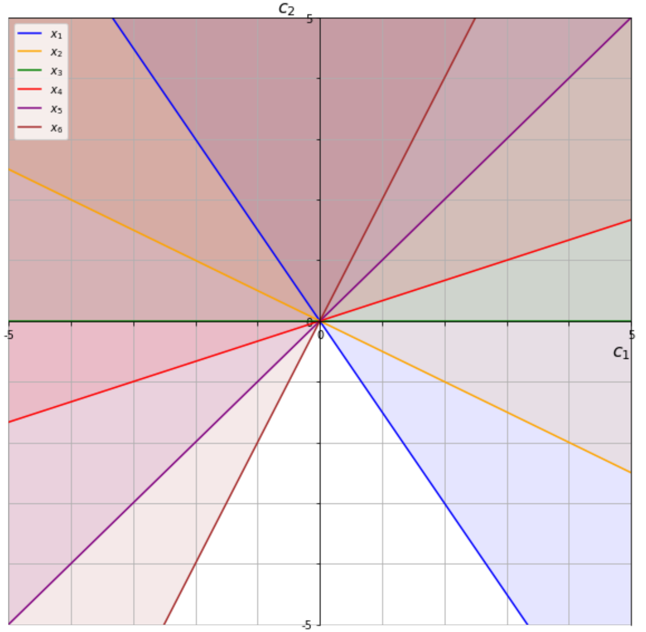

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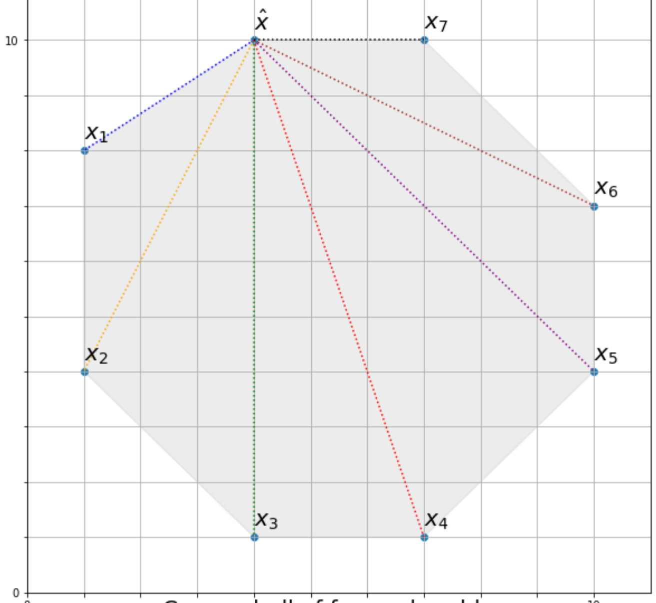

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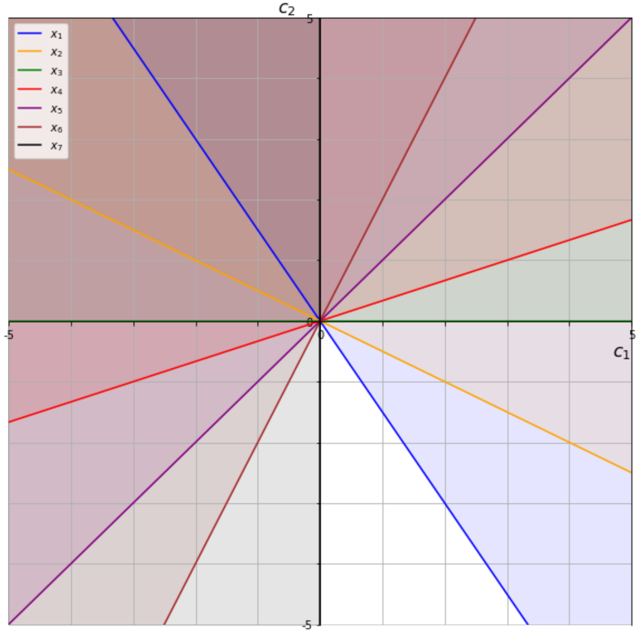

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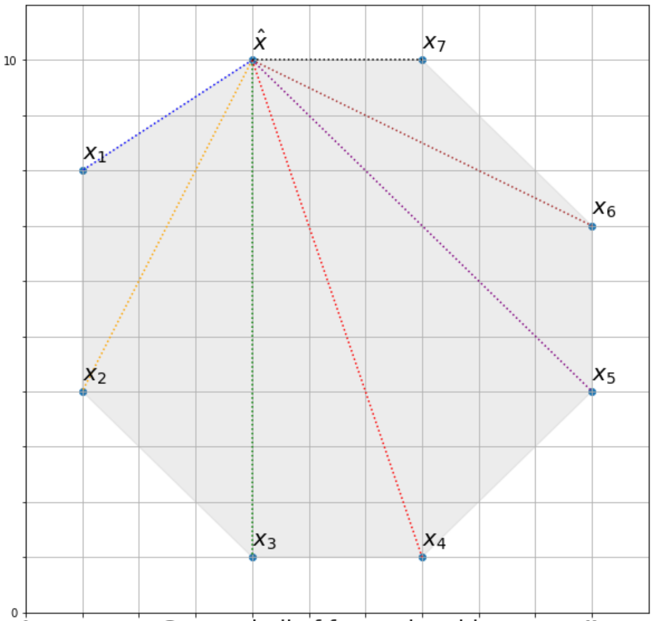

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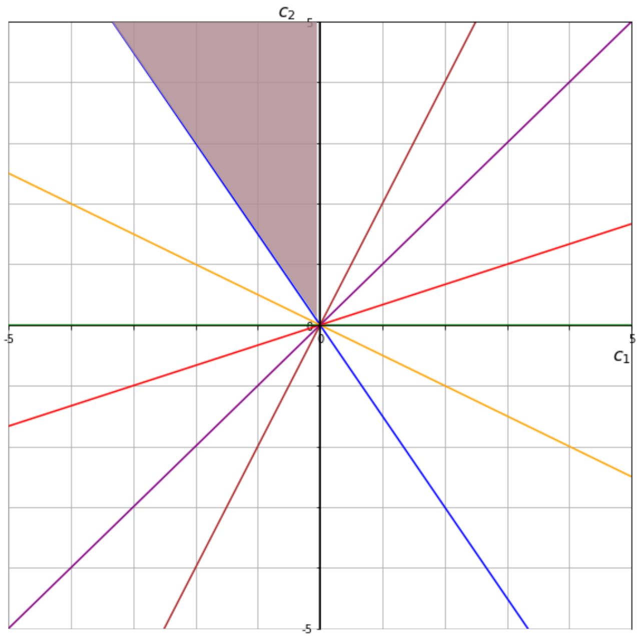

Intersection of all half planes

Bodur, Merve, Timothy CY Chan, and Ian Yihang Zhu. "Inverse Mixed Integer Optimization: Polyhedral Insights and Trust Region Methods." INFORMS Journal on Computing (2022).

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Inverse optimization in CPMpy

```
def inverse_optimize(SP, c, x, x_d, keep_static=None):

    # Decision variable for new parameter vector
    d = intvar(0,INFTY, shape=len(x_d), name="d")

    # create the master problem
    MP = SolverLookup.get("gurobi")
    MP.minimize(norm(c-d,1))
    MP += SP.constraints
    while MP.solve():
        # find new cost vector
        new_d = d.value()
        print(f"New costvector = {new_d}")

        # find point on convex hull corresponding to new_d
        SP.maximize(sum(new_d * x))
        SP.solve()

        if sum(new_d * x_d) >= sum(new_d * x.value()):
            # solution is optimal
            break

        # add new cut to MP
        MP += sum(d * x_d) >= sum(d * x.value())

    return new_d, x.value()
```

Minimizing L1 norm

Solving the forward problem

Iteratively building cut constraints

Example: Knapsack problem

- Solver finds optimal solution to given problem:

```
model, (items, values, weights, capacity) = get_knapsack_problem()
assert model.solve()
print("Objective value:",model.objective_value())
print("Used capacity:", sum(items.value() * weights))

print(f"{values = }")
print(f"{weights = }")
print(f"{capacity = }")

print(f"items = {items.value()}")
```

```
Objective value: 32
Used capacity: 31
values = array([5, 0, 3, 3, 7, 9, 3, 5])
weights = array([2, 4, 7, 6, 8, 8, 1, 6])
capacity = 35
items = [ True False False  True  True  True  True  True]
```

Example: knapsack problem

```
items = [ True False False  True  True  True  True  True]
```

```
# User query
# "I want my solution to really contain item 1 and 2"
model += all(items[[1,2]])
assert model.solve()

x_hat = items.value()
print("Objective value:",model.objective_value())
print("Used capacity:", sum(x_hat * weights))

print(f"Items: {x_hat}")
```

```
Objective value: 29
Used capacity: 35
Items: [ True  True  True False  True  True False  True]
```

Find new solution satisfying extra constraints

Korikov, Anton, and J. Christopher Beck. "Counterfactual explanations via inverse constraint programming." *27th International Conference on Principles and Practice of Constraint Programming (CP 2021)*. Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2021.

Example: knapsack problem

New objective only changes for items 1 and 2!

```
Original values: [5 0 3 3 7 9 3 5]
new costvector = [5 0 3 3 7 9 3 5] New cut = [ True False False  True  True  True  True  True]
new costvector = [5 0 6 3 7 9 3 5] New cut = [ True False  True False  True  True  True  True]
new costvector = [5 3 3 3 7 9 3 5] New cut = [ True  True False  True  True  True  True  True]
new costvector = [5 3 6 3 7 9 3 5] New cut = [ True  True False  True  True  True  True  True]

array([5, 3, 6, 3, 7, 9, 3, 5])
```

Demo

Counterfactual explanations

examples/tutorial_ijcai22/7_counterfactual_explain.ipynb

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Conclusions

- Several applications in CP require repeated solver calls
  - Use CP/SAT/MIP-solvers as an oracle
  - Big efficieny gains using incremental solving
  - Use complementary strengths of several solvers

- Many applications in explanation generation:
  - MUS enumeration/extraction
  - OUS/SMUS Implicit hitting set algorithms
  - OCUS Implicit hitting set algorithms with constraints
  - Inverse optimization with cutting planes
  - …

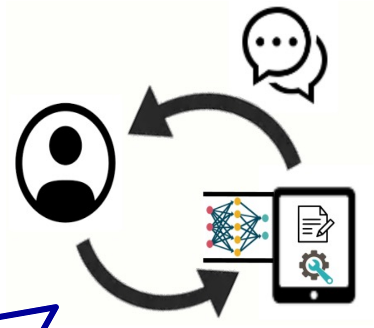
Conclusions

- CPMpy is the right tool for the job:
  - Easy prototyping in Python/Numpy
  - Supports many solver paradigms
  - Incremental solving
  - Open source
    - We welcome all contributions/feedback!

https://github.com/CPMpy/cpmpy

European Research Council
Established by the European Commission

CHAT-Opt:
Conversational **H**uman-**A**ware **T**echnology for **Opt**imisation



Towards **co-creation** of constrained optimisation solutions

- Solver that learns from user and environment
- Towards conversational: explanations and stateful interaction

https://people.cs.kuleuven.be/~tias.guns
@TiasGuns

Hiring post-docs!