A compact wide-spectrum imaging spectrometer using freeform surface

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Abstract: This paper presents a compact imaging spectrometer covering visible-near-infrared (VNIR) and shortwave-infrared (SWIR) spectrum with a large field-of-view (FOV). The design is composed of a curved prism for dispersing beam and two freeform mirrors mainly for balancing off-axis high-order aberrations. In this study, the FOV with a much longer slit of 50 mm from 400 nm to 2500 nm is constructed, three types of different designs using spherical, aspherical, and freeform surfaces are compared under the same specifications. Results show that the combination of freeform surfaces and prism can effectively improve the imaging quality of off-axis fields and compress the volume. The volume of the design based on freeform surface is reduced by 0.55 times than the corresponding spherical design.

Keywords: curved prism; spectroscopy; freeform; large field of view.

1. Introduction

Spectrometer is an important detection method for aeronautics and astronautics applications. With the improvement of detection requirements and the miniaturization development of satellites, the need for an imaging spectrometer with high performance has become an urgent demand [1-4]. However, current miniature spectrometers have numerous limitations in terms of imaging and spectral performances. Besides, the spectral range and resolution are greatly limited. As the incident angle at each element increases with the FOV, introducing lots of off-axis high-order aberrations that cannot be compensated easily by using a compact structure, therefore, the compactness and high performance cannot simultaneously be realized. The FOV of spectrometer is rectangular, the tangential and sagittal aberrations are asymmetrical, and the design based on traditional spherical surface or complicated structure cannot balance the high-order aberrations; thus, the design of miniature spectrometer with large FOV and high performance remains a challenge.

Previous works have been focused on improving FOV and spectrum band by splitting one large spectrometer into multi-system [5-6]. The FOV enlargement is mainly achieved based on the principle of FOV division by using multiple spectrometers. However, extra optical elements are necessary for the multi-device configuration to divide the imaging FOV, which adds to the complexity of the system and increases the size of the spectrometer. Owing to many types of spectrometers, the SNR has improved by lowering F number [7]. The high-order aberrations correlated with increasing aperture, and more lenses or even complex structures may balance these aberrations, which are not beneficial for compactness.

Grating is widely used as the dispersive element and has become dominant in the market. Although grating-based imaging spectrometers can realize a relatively compact system [8-10], the prism-based spectrometers have a high throughput without limitation from diffraction efficiency and ghost images, as well as low sensitivity to polarization. Therefore, the realization of miniature prism-based spectrometer is of considerable value. The advantages of freeform surface in optical systems are well known, freeform surfaces have been successfully used in the fields of illumination and imaging field [11-15]. In the characterization of a freeform surface, multiple parameters can offer more degrees of freedom to the system design, which means that correcting non-symmetric aberrations would be easier. First, usage of freeform surface makes a compact design and achieves low weight. Second, the number of optical elements can be reduced, thus increasing throughput. The geometric aberrations can be balanced, and optical performance can be improved. Therefore, we propose a new design method of prism-based imaging spectrometer using freeform surface.

In this study, freeform is used to extend field-of-view (FOV) across a wide spectrum and reduce the system volume. The combination of freeform surface and prism can correct field curvature and astigmatism with the increase of the FOV by using the non-symmetric characteristics of freeform surface [16]. The system with an F-number of 3.5 and a slit of 50 mm across a wide spectrum including VNIR and SWIR is constructed. Achieving the specification based on Offner system using spherical
surface is generally difficult. The freeform surface plays a key role in enlarging FOV across a wide spectrum and balancing the aberrations in tangential and sagittal directions. Different designs with spherical, aspherical, and freeform surface are compared under the same specifications. The simulation results show that the spectrometer using freeform surface is compact, and the performance is much better. The volume reduced by 0.55 times. Aberration theory and initial optical design are deduced in Section 2. The systems using spherical, aspherical and freeform surface are compared in Section 3. Detailed imaging performance and tolerance analysis for freeform system are presented to further demonstrate the feasibility for fabrication in Section 4.

2. Aberration theory and initial optical design

2.1 Aberration theory of curved prism

The traditional prism with two ingoing and outgoing flat planes is selected as a dispersive element, collimating lens are added to ensure that the incident angles are equal. An imaging spectrometer using flat-plane prism includes collimation systems and dispersion element and imaging systems. The entire system is cumbersome and difficult to realize a miniature and compact design.

The curved prism ensures that the incidence angles of the given point are approximately equal without the collimated lens. The front and rear surfaces of a curved prism can provide certain aberration compensation for the system. Fig. 1 shows the structure, where \( R_1 \) and \( R_2 \) are the radii of the concave and convex surface, \( O_1 \) is the center of the concave surface, and the point \( A \) is on the Roland circle of the front surface. The incidence angles of three rays emitted by the point \( A \) are \( i_1, i_2 \) and \( i_3 \).

![Fig. 1. Optical path of curved prism](image)

In the triangle \( \Delta AO_1a, \Delta AO_1c \) equations are obtained by sine theory:

\[
\frac{R_1}{\sin \angle O_1Aa} = \frac{AO_1}{\sin i_1}, \quad \frac{R_1}{\sin \angle O_1Ac} = \frac{AO_1}{\sin i_3}
\]

Simplification:

\[
\sin \angle O_1Ac = \frac{\sin i_3}{\sin i_1}, \quad \sin \angle O_1Ac - \angle O_1Aa = \frac{ac}{R_1}
\]

\[
i_3 - i_1 = \frac{ac}{R_1}
\]

Therefore, the difference between the incident angles depends on the aperture of the prism, and the incident angles are approximately equal when the aperture is small, and the radius is relatively large.

For a curved prism, when the object point \( A \) is located on the Roland circle, the corresponding meridian image point is also on the Roland circle while the sagittal image point is not found on this circle, resulting into considerable astigmatism that increases with the FOV.

\( s \) and \( t \) represent the sagittal and tangential distances for the intersection point \( a \) at the object space. After refracted by the front surface, the incident angle is \( i_3 \), \( \alpha \) is the dispersion angle of the curved prism, the astigmatism can be deduced as follows:

\[
t_1 = s_1 = -R_1 \cos i_2, \quad \sin i_2 = n \sin i_3
\]

\[
\cos i_2 = \frac{d_1^2 + R_2^2 - R^2 \cos^2 \alpha}{2d_1R_2}
\]

\[
n \cos^2 i_2 - \frac{\cos^2 i_2}{R_1} = \frac{n \cos i_2 - \cos i_2}{R_1}
\]

\[
\frac{n}{s_1} - \frac{1}{s_1} = \frac{n \cos i_2 - \cos i_2}{R_1}
\]
The sagittal and tangential distances $s', t'$ can be expressed as follows:

$$\text{astig} = f(\alpha, R_1, R_2, n, i_2)$$  \hspace{1cm} (8)

### 2.2 Aberration correction of freeform

We present a novel design method by introducing freeform surface to correct high-order aberrations with increasing FOV and wide spectrum [13]. To achieve the large FOV and wide spectrum, the aberrations can be compensated by the non-rotational characteristics of freeform surface. The expression of the freeform is described by Zernike polynomials, the corresponding Zernike aberration terms correct astigmatism, and coma introduced FOV and spectrum.

![Freeform Surface](image)

Fig. 2. The principle of Astigmatism correction using freeform surface

According to wavefront aberration theory, the off-axis third-order wavefront aberration combined with vector theory was deduced by Thompson [14-16], the third term describe the astigmatism and a detail description could be written as follows:

$$W_{ast} = \frac{1}{2} \sum_j w_{222j}((H - \sigma_j) \cdot \rho)^2 = \frac{1}{2} \left( \sum_j w_{222j} H^2 - 2H(\sum_j w_{222j} \sigma_j) + \sum_j w_{222j} \sigma_j^2 \right) \cdot \rho^2$$  \hspace{1cm} (9)

Where $\rho$ is the normalized pupil vector, $\sigma_j$ is the descent vector, $H$ is the normalized field vector, $w_{222j}$ are the coefficients of aberrations.

As FOV increases, the center descent vector of incident beam on freeform $\rho$ varied, $\rho' = \rho + \Delta h$, the decenter value $\Delta h$ is corresponding linear to $H$, so the high-order aberrations can be corrected by calculating coefficients of aberrations, then optimizing them to realize aberration balancing.

### 2.3 Initial design

The first step of the design work is to determine the dispersion width of the spectrometer. A wide spectral band from 400 nm to 2500 nm has been selected. The target average spectral resolution is better than 10 nm, which is an acceptable value similar to that of most common products. Table 1 shows the detailed system specifications.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>400–2500</td>
<td>nm</td>
</tr>
<tr>
<td>F number</td>
<td>3.5</td>
<td>-</td>
</tr>
<tr>
<td>Slit length</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Pixel size</td>
<td>10</td>
<td>μm</td>
</tr>
<tr>
<td>Spectral band</td>
<td>200</td>
<td>–</td>
</tr>
</tbody>
</table>

The second step of the design work is to construct the initial configuration and optimize the suitable parameters that can meet the requirements. The optical configuration parameters include the angles of prism, radii of elements, and distances between elements. The initial design of system benefits from Offner coaxial system, as shown in Fig. 3. In the system, $h, h'$ are the distances between object, image and the center. $r_1, r_2$ and $r_3$ are the radii of mirror 1, 2, and 3, respectively.
Based on paraxial theory, the intersection angle of the curved prism can be deduced as follows:

\[ \alpha = \frac{h}{r_1}, \quad u_1 = 2 \frac{h}{r_1}, \quad u_2 = 2 \frac{h}{r_5} \]  

(10)

\( B_t, B_s \) are the tangential and sagittal image points of A, respectively. \( \theta_t, \theta_s \) are the intersection angles between CB_t, CB_s and the optical axis:

\[ CB_t = r_2 \sin \beta = r_3 \sin \gamma', \quad CB_s = \frac{CB}{\cos(\theta_t - \theta_s)} \]  

(11)

The best focus condition for spectrometer is the coincidence of the meridian and the sagittal; therefore, \( \theta_t = \theta_s \).

According to the dispersion formula:

\[ \Delta \lambda = \frac{1}{d\Phi/d\lambda} \cdot \frac{a}{f}. \]  

(12)

In the equation, \( d\Phi/d\lambda \) is the dispersive rate of the prism, and \( a \) is the pixel size of detector. Given the initial value of \( h \) and object distance, the radii of \( M_1, M_2, \) and \( M_3 \) and the angle of the prism can be preliminarily confirmed by eliminating astigmatism conditions and paraxial theory to meet the requirements of the space structure. The following solution is obtained: the radii of \( M_1, M_2, M_3 \) and the angle of prism are calculated as follows:

\[ R_1 = R_3 = -320mm, \quad R_2 = -160mm, \quad \beta_{\text{prism}} = 15^\circ \]  

(13)

3. Design of the spectrometer

3.1 Optical design using spherical surface

The designed spectrometer consists of the slit, the mirrors, the dispersive prism, and the detector. The slit serves as the entrance of the dispersive optics. The slit allows only line scene into the dispersive optics, acting as the field stop. Double reflections through the prism enlarge the spectral dispersion width. The image of the ground projected through the slit is aligned along one dimension in the cross-track spatial direction and the spectrum is dispersed along the other dimension in the spectral direction. We make the optical design of a compact imaging spectrometer system for the solar reflected spectrum (400–2500 nm) with 2.0 mm spectral width, relative aperture F number is 3.5.

A curved prism is used as the dispersive element to make the system simple and compact. The prism is located as the secondary element. The front surface is then coated by the inner reflection film, and the large dispersive width is achieved through the two reflections. First, the basic parameters of the initial structure are determined according to the dispersive width. Then, the wedge angle of curved prism, eccentricity, and tilt of the element are adjusted, and the optimization process is executed by using optical simulation software ZEMAX.
Before the optimization, reasonable constraints and optimal conditions should be established and controlled. The centroid RMS is selected as the default optimal evaluation function. Additional restrictions are set on the dispersion and spectral distortion in the optimization process. In the process of optimization, as the FOV increases, the aberrations evidently varied. The imaging performances and spectral distortions at the field of 10mm can be balanced using the spherical system. However, when the slit length increased to 50 mm, astigmatism and field curvature sharply increased, the image quality rapidly declined, and astigmatism and MTF are poor. Overall, the system length is 370 mm, Fig. 4 shows the system layout and optimization results, indicating poor astigmatism.

3.2 Optical design using aspherical surface
Owing to the large spatial FOV (50 mm), the spherical prism and mirror cannot meet performance requirement. In the above system, large field curvature and astigmatism dramatically affect the image quality. The two aberrations can be reduced when substituting the spherical mirrors with even aspherical mirrors. Therefore, we firstly set the spherical mirror to be even asphere type, and different FOVs would incident in different sagittal radii, which are beneficial to reduce the aberrations. Different rises from the even aspherical surface, its radii vary with different object heights. The spherical surface of mirror is replaced by even aspherical surface to further reduce astigmatism with FOV increasing. As the FOV is linear, the high-order coefficients of the reflected mirror are set up to 6 orders in the direction X.

The results of even aspherical system, including astigmatism, are shown in the following diagram in Fig. 5. The following diagram indicates that astigmatisms of typical wavelengths are effectively reduced compared with those of the spherical system. The comparison also indicates that even aspherical surface can certainly reduce astigmatism across a wide spectrum and is a good solution to the problem of energy concentration. The volume shown in Fig. 5 is 330 mm.

3.3 Optical design using freeform surface
The residual astigmatism and field curvature remain evidently large based on the preceding aspherical design. We introduce freeform surface. Given that Zernike polynomials are continuous and orthogonal on the unit circle domain, independence exists between polynomial coefficients. Moreover, a good correlativity exists between Zernike polynomial coefficients and wavefront aberrations [17-19]; thus, the relationship between freeform surface and wavefront aberrations can be established easily. Therefore, Zernike polynomials are selected as freeform surface equations to describe the influence of the surface on the aberrations of the optical system. The expression is shown in equation (14).
\[ Z = \frac{cr^2}{1 + \sqrt{1 - (1 + k)c^2 r^2}} + \sum_{i=1}^{N} A_i Z(\rho, \phi) \]  

(14)

where \( N \) is the number of Zernike coefficients in the series, \( A_i \) is the coefficient on the \( i^{th} \) Zernike fringe polynomial, \( r \) is the radial ray coordinate in lens units, \( \rho \) is the normalized radial ray coordinate, and \( \phi \) is the angular coordinate.

The optimization of freeform system is more complicated than the rotational symmetric optical system. Therefore, we choose several sampling FOVs. The optimization process should focus on the balance of each sampling FOV to obtain uniform image quality of the entire field. First, the mirror is set as freeform surface to optimize the system. A total of eight representative wavelengths, namely, 400, 600, 800, 1000, 1200, 1600, 1800 and 2500 nm, are set with equal weights 1.0. Three fields, namely, 0, 0.7, and 1.0, are set. We have used Zernike polynomials up to 10 terms, which can related to aberrations up to the second order. Consequently, the aberrations do not sharply decline with the FOV increased. Then, we adopt a progressive design strategy by gradually increasing field samples. Meanwhile, the second mirror away from the stop is also set to the freeform surface, and the Zernike polynomials are up to 10 terms. The projection area of the beam from different fields on the freeform surface is deviated relative to the projection area at the center of the field on the freeform surface. Therefore, the aberration contribution from different fields has complicated changes with the variation of FOV. Given that it is non-rotational symmetrical and considering the projected deviation of imaging beam on the freeform surface from different FOVs, it can introduce more complex aberrations compared with that of traditional aspherical surface; therefore, the optical system has better aberration correction ability. In the optimization process, when the freeform surface is away from the stop, aberrations for each field introduced by the freeform surface evidently varied with the FOV. Therefore, correcting aberrations associated with the FOV, such as coma, astigmatism, and field curvature, is conducive. Following the design methods discussed, we make further optimization. The design based on freeform surface is compared as follows. The optical length of layout is 165 mm. The layout and performance of the optimized system are shown in Fig. 6.

The coefficients of freeform surface for mirrors 1 and 2 are shown in Table 2.

<table>
<thead>
<tr>
<th>Item</th>
<th>Coefficient of mirror 1 ( Ai )</th>
<th>Coefficient of mirror 2 ( Ai )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z5</td>
<td>5.785e-005</td>
<td>2.082e-005</td>
</tr>
<tr>
<td>Z6</td>
<td>1.131e-005</td>
<td>5.732e-006</td>
</tr>
<tr>
<td>Z7</td>
<td>-7.867e-007</td>
<td>-9.502e-008</td>
</tr>
<tr>
<td>Z8</td>
<td>-1.143e-006</td>
<td>-3.505e-008</td>
</tr>
<tr>
<td>Z9</td>
<td>4.708e-009</td>
<td>-1.218e-011</td>
</tr>
<tr>
<td>Z10</td>
<td>1.848e-007</td>
<td>-6.041e-008</td>
</tr>
</tbody>
</table>

4. Simulation and analysis of the spectrometer

4.1 Imaging performances of three systems

The MTF curves of the two typical wavelengths, namely, 400 nm and 2500 nm, for the three different structures are shown as follows.
By contrast, the MTF of spherical system rapidly decreases with the FOV increased. The MTF of aspheric system demonstrates relative improvement, and the MTF of the entire FOV is 0.2 @ 50 lp/mm. The MTF of freeform system is more than 0.3 @ 50 lp/mm.

The spectral smiles and keystones are calculated by tracing ray. ZPL program can automatically obtain the values of coordinate Y for sampled rays. We compared the maximum of each type, and the spectral keystone of freeform system is less than 0.1 pixels. However, the value of spherical system is 3 pixels.

The maximum astigmatism value of spherical, aspherical and freeform systems are 800, 400, and 30 um, respectively; therefore, the freeform system has the minimum value.

The length of spherical system is 370 mm, and the optimized optical length of the freeform surface system is 165 mm. Compared with the spherical system, the value of freeform system is reduced by 55%, which can realize miniature for loads.

4.2 Performance analysis of freeform system

The critical parameters for evaluating imaging performances are the spot diagrams and the point spread function (PSF). The spot diagrams for the VNIR and SWIR across the slit are shown in Fig. 8. The radii at the entire spectrum are within the airy disk except 400 nm.
The spectral resolution curve of the freeform system was obtained by real ray tracing and they are shown in Fig. 9. To illustrate the resolution limit for spectral resolutions at 400 and 2500 nm, the PSF of the two nearest points for different wavebands are shown in Fig. 10(a) and (b), where the x-axis is the position of wavelength at the image plane and y-axis is the relative irradiance of the corresponding wavelength. According to the Rayleigh criterion, the design results show that the spectral resolutions at 400 and 2500 nm are nearly 2 and 11 nm respectively.

![Spectral resolution curve](image)

**Fig. 9. Spectral resolution of the freeform system**

The spectral distortions include spectral smile and keystone. The five typical object heights and the eight wavelengths are traced to achieve the spectral distortions. The spectral distortions are shown in Fig. 11. In Fig.11 (a), where the horizontal axis indicates the object height and the vertical axis indicates the value of the spectral smiles. In Fig. 11(b), the horizontal axis indicates wavelength and the vertical axis indicates spectral keystone.

![Spectral distortions](image)

**Fig. 11 Spectral distortions. (a) Smiles of different wavelengths (b) Keystone of different object heights**
The plot shows that the spectral smiles are symmetrical to the center of the FOV, and the values increase with variation in wavelengths. The maximum value of smiles is 13.8 µm at the wavelength of 2500 nm, the maximum value of keystone is 21 µm at 2500nm. As the spectrum is wide and the FOV is large, the spectral distortions are almost one pixel, but the data can be corrected by data processing [20].

4.3 Tolerance analysis

The tolerance analysis is conducted for the freeform system to check the manufacturability. The tolerance procedures are performed for surface/element tilt, surface/element decenter, thickness, and surface irregularity. The diffraction MTF is used as the tolerance criterion. All tolerances values are obtained after 1000 Monte Carlo runs. For cross comparison, the degradation of criterion is received to be 80%.

Table 3. Tolerance analysis of freeform system

<table>
<thead>
<tr>
<th>Type</th>
<th>Mirror 1</th>
<th>Mirror 2</th>
<th>Prism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface tilt (degree)</td>
<td>-0.02~0.02</td>
<td>-0.02~0.02</td>
<td>-0.015~0.015</td>
</tr>
<tr>
<td>Surface decenter (mm)</td>
<td>-0.02~0.02</td>
<td>-0.02~0.02</td>
<td>-0.02~0.02</td>
</tr>
<tr>
<td>Element tilt (degree)</td>
<td>-0.015~0.015</td>
<td>-0.015~0.015</td>
<td>-0.01~0.01</td>
</tr>
<tr>
<td>Element decenter (mm)</td>
<td>-0.02~0.02</td>
<td>-0.02~0.02</td>
<td>-0.015~0.015</td>
</tr>
<tr>
<td>Surface irregularity (mm)</td>
<td>-0.0022~0.0022</td>
<td>-0.0022~0.0022</td>
<td>-0.0013~0.0013</td>
</tr>
</tbody>
</table>

We utilize the average MTF at wavelength 632 nm as the nominal criterion. The Monte Carlo calculation is performed in ZEMAX. The analysis results are shown in Table 4.

Table 4. Tolerance analysis of freeform system

<table>
<thead>
<tr>
<th>Type</th>
<th>Surface</th>
<th>Change of MTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface tilt (degree)</td>
<td>The back surface of prism</td>
<td>0.08</td>
</tr>
<tr>
<td>Element tilt (degree)</td>
<td>prism</td>
<td>0.051</td>
</tr>
<tr>
<td>Element decenter (mm)</td>
<td>prism</td>
<td>0.039</td>
</tr>
<tr>
<td>Element tilt (degree)</td>
<td>mirror 1</td>
<td>0.018</td>
</tr>
<tr>
<td>Element tilt (degree)</td>
<td>mirror 2</td>
<td>0.012</td>
</tr>
</tbody>
</table>

The tolerance of prism is evidently sensitive. Tolerances of the mirrors are more relaxed than those of the prism. Therefore, the alignment precision should be guaranteed. The final analysis shows that 90% MTF decreased by 0.074 and 80% decreased by 0.052 for all tolerances.

The system based on the freeform surface is beneficial for correcting aberrations. The surface item of M_4 is a 10-order freeform surface described by generalized Zernike polynomials. The assembly and alignment of freeform surfaces can be assisted by simultaneously manufacturing auxiliary mirrors on system elements. From the preceding analysis, we can obtain that the sag height of the freeform surface gently varied, and the processing level can be finished at present.

5. Conclusion

In this study, a method to design a spectrometer with freeform surface is presented. Compared with the conventional spherical and aspherical system, the volume of the freeform system is reduced by 55%. Aberration theory is first deduced based on the Offner structure, and the high-order residual astigmatism is corrected by combining the freeform surface and curved prism. Further, the performances of three systems were analyzed, the MTF is improved by more than twice based on the freeform design. Finally, we show the tolerance analysis for the freeform system. The reasonable analysis of tolerances indicates the feasibility of manufacturing and assembly with current technologies.

References


