A classification of singly curved deployable scissor grids

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Abstract
Deployable scissor structures can easily and repeatedly undergo swift transformations between a compact and an expanded state, making them fit for a broad range of architectural applications. Throughout the years, a myriad of different scissor grid configurations have been proposed for a variety of geometries. The available documentation on these configurations is scattered across many different publications and is often incomplete or not explored to the full, which makes any form of comparison or evaluation a difficult task. Therefore, we have started to collect and complete this information in order to obtain a full overview of the geometrical possibilities of deployable scissor structures and their corresponding kinematical behaviour. This paper presents this overview for deployable scissor grids with single curvature, since singly curved surfaces comprise many useful shapes, e.g., the barrel vault. The different scissor grid configurations are classified according to the geometry of their grid cells and listed along with other relevant parameters regarding the scissor units, the scissor grid and its kinematical behaviour. As such, this classification table forms a handy tool for designers to efficiently compare and assess the different configurations and their fitness for the desired application. It will be extended in future research to include other geometries as well.

Keywords: deployable scissor structure, classification, single curvature, barrel vault, geometry, kinematics, scissor unit, compactness, triangulated grid, quadrangular grid cells.

1. Introduction
Deployable scissor structures have the ability to rapidly and reversibly transform from a stowed state, in which it forms a compact and easily transportable bundle of bars, to an expanded state, in which it fulfils its function. Generally, scissor grids form single-degree-of-freedom mechanisms that are locked in the fully deployed state in order to become load-bearing structures. Their reliable deployment process and large expansion range makes them broadly applicable in the built environment, usually as lightweight mobile structures or as transformable layers for static constructions. Common applications include shelters for disaster relief, tents and stages for travelling or temporary events and transformable roof structures for sports arenas (Gantes [6]). Here, the scissor grid forms the primary load-bearing structure that is suited with a functional cover, e.g., a membrane or foldable plate cover (Escrig et al. [5]).

Deployable scissor grids come in many different shapes, describing linear, planar, singly curved or doubly curved geometry. Singly curved surfaces generally combine simplicity with applicability. An example of such a surface is the barrel vault. Its (usually) semi-cylindrical envelope is well suited for a self-supporting shelter that can rapidly be deployed in a diverse set of situations. In addition, it enables chaining small modules to form larger covers. Not surprisingly, many existing temporary structures for emergency relief or for military purposes comprise the barrel vault geometry.

A variety of scissor grid configurations have been proposed with a cylindrical shape or, more generally, with arbitrary single curvature. These configurations have been presented in numerous papers published over a span of several decades. A large mismatch in the available information on the different configurations and in terminology, as well as the lack of any complete overview, makes comparison and selecting the fittest model an almost impossible task. Therefore, we have investigated the existing proposals for singly curved scissor grids, completed possible missing information and classified them according to morphological features. The
classification lists the main geometrical and kinematical parameters for each configuration: the scissor grid type, the grid cell geometry, the freedom of form, specific geometrical constraints, the kinematical behaviour, the scissor unit types, the uniformity of components and the compactness of the scissor grid in its stowed position. It therefore provides a complete overview of the possibilities, the characteristics and the specific advantages and disadvantages of each configuration. Consequently, it can be used by the designer to efficiently evaluate if a singly curved scissor grid fits the requirements of a given design project, and if so, which of the types is best suited. This classification of singly curved scissor grids is presented in this paper. First the different parameters listed in the classification table are identified. Afterwards, the table itself is given, followed by a detailed description of each scissor grid configuration.

2. Classification parameters

2.1. Scissor unit type

A scissor unit – or scissor-like element (SLE) – forms the basic building block of any deployable scissor grid. Generally, a scissor unit consists of two rods interconnected by a revolute joint, which allows relative rotation of the rods along an axis normal to the scissor unit’s plane. Depending on the proportions and shape of the rods, different unit types can be distinguished, each one displaying a different kinematical behaviour. The most common are the translational, the polar and the angulated scissor unit (Fig. 1). The first two consist of a pair of straight rods, while the angulated unit consists of a pair of kinked rods.

![Figure 1: A (a) plane-translational, (b) curved-translational, (c) polar and (d) angulated scissor unit.](image)

The kinematical behaviour of these three scissor unit types is best described by examining their unit lines. The unit lines are the imaginary lines connecting the upper and lower ends of the rods (Fig. 1). For a translational scissor unit, these lines stay parallel throughout all stages of the deployment. Geometrically this means that the rods and the unit lines always form two similar triangles or, in case of a symmetrical unit, two congruent triangles. A translational unit with two rods of equal length describes a planar geometry and is therefore referred to as a plane-translational unit (Fig. 1a). Curvature can be introduced by using units with unequal bar lengths, called curved-translational units (Fig. 1b). Scissor grids composed of translational units have been studied extensively to create all kinds of singly curved and doubly curved surfaces, e.g., by Zanardo [16], Sánchez-Cuenca [15], Langbecker [11] and De Temmerman [2].

In a polar unit, the angle between the unit lines (\(\gamma\) in Fig. 1c) varies during deployment. Most commonly, a polar unit consists of two identical rods, such that the unit lines and the rods form two congruent triangles. The intermediate hinge point is moved away from the midpoint of the rods, thus providing polar linkages with curvature. Unlike translational units, polar units can therefore introduce curvature with rods of equal length. Polar units have also been studied extensively within the field of deployable scissor structures, e.g., by De Temmerman [2], Escrig [3, 4], Gantes [6], Langbecker [11], Sánchez-Cuenca [14] and Zanardo[16].

The unit lines of an angulated scissor unit embrace a constant angle (\(\gamma\) in Fig. 1d) during deployment. The most common angulated unit, as proposed by Hoberman [9], consists of two identical kinked bars that, together with
the unit lines, form two similar isosceles triangles. It is used to generate a myriad of free-form surfaces that retain their shape and only change in scale as they deploy, resulting in so-called shape-invariant scissor grids.

All three scissor unit types possess a single degree-of-freedom, characterised by the deployment angle $\theta$. As $\theta$ changes, the unit deploys. Each value for $\theta$ corresponds with a certain value for the structural thickness $t$, which is measured along the unit lines between the lower and upper end points of the rods (Fig. 1).

2.2. Scissor grid geometry

Scissor structures are formed by linking scissor units in a two- or three-dimensional grid according to a certain pattern. Depending on the type of scissor unit used, and the type of grid on which they are positioned, different geometries can be achieved, displaying different curvature. Curvature is often introduced into a scissor grid to enhance its structural performance, or simply to fulfill functional or aesthetic requirements. A large freedom of form improves the chance that a certain configuration can answer to these design requirements. Even though the possibility to obtain arbitrary curvature is interesting, applying it comes at the cost of an increased diversity of components. For this reason, symmetrical or regular geometry (e.g., the barrel vault) is often more desired.

Unrelated to the curvature of the grid, Hanaor and Levy [8] distinguished three major grid types: a linear grid, a single-layer grid (SLG) and a double-layer grid (DLG). Linear scissor grids usually form masts or spines, e.g., for membrane structures. In a SLG, the scissor units are placed in-plane with the base surface they describe. They require curvature, preferably double curvature, to obtain some structural depth. Applications are currently limited to retractable roofs with a rigid perimeter and dismountable surfaces. The DLGs form the most interesting grid type, as they allow the largest variation in shape and application and display the best structural properties. They have also been the most broadly researched in literature. Here, the scissor units are positioned perpendicular to the base surface that they cover. As a result, their top nodes and their bottom nodes describe two distinct grids. The shape of the grid cells determines the rigidity of the grid: in case of triangular cells, a rigid grid is obtained, while other polygonal cells require additional bracing components to ensure rigidity. These components can be cables, struts (telescopic or folding) or even the functional cover attached at the end (Escrig and Valcárcel [4]).

2.3. Kinematics of the scissor grid

Different scissor grids can display different mechanical behaviour. If a scissor grid acts like a pure mechanism, it is referred to as a foldable scissor grid. Its bars remain unstrained throughout all stages of deployment, resulting in a smooth and reliable deployment process. However, often the grid is kinematically incompatible: a geometrical mismatch occurs in the grid during several stages of the deployment, resulting in deformations of the rods and additional strains in the structure. Generally, this effect is unwanted, certainly when the incompatibilities are present at the fully-deployed or the fully-folded state. On the other hand, if the grid is only incompatible during the deployment process, these stresses and deformations can theoretically be turned into an advantage, as shown in Gantes [6]. The snap-through or clicking scissor mechanisms that he presents use these incompatibilities to lock themselves in a strain-free position once deployed. This removes the need to add external locking devices, which is more cost and time effective, especially for larger structures. Nevertheless, ensuring their reliability requires a more complex design process, more complicated and expensive connections and the need very precise manufacturing (Gantes [6]). Consequently, these effects are difficult to control in practice.

Since the geometry and kinematics of a scissor grid are completely related – as well as its structural behaviour –, various geometrical constraints are imposed upon the grid in order to obtain a functioning mechanism. The exact nature of these constraints depends on the scissor grid configuration and the type of scissor units used, and can for example be that all structural thicknesses $t$ have to be equal throughout the grid. For some scissor grids to be foldable, angular distortions have to be allowed between the scissor units in the grid, thus requiring a more complex joint to enable this kinematical behaviour. In 1985, Escrig [3] described the following geometric condition for scissor grids consisting of straight bars (Fig. 2):

$$k_i + l_i = k_{i+1} + l_{i+1}$$

(1)

This equation is often referred to as the deployability equation and ensures that the scissor linkage can be reduced to its most compact state when folded up. In a zero-thickness model of a scissor grid, where joints are represented by points and rods by lines with zero thickness, this means that all rods become collinear ($\theta = 0$ in Fig. 1a to c). For angulated units this equation is no longer valid: in its most compact state, the zero-thickness model of an angulated scissor unit is not reduced to a single line, but takes up a certain area (Fig. 3d).
3. Classification of singly curved scissor grids

Table 1: Classification of singly curved scissor grids

<table>
<thead>
<tr>
<th>Scissor unit type(s)</th>
<th>Quadrangulated double-layer grid</th>
<th>Triangulated double-layer grid</th>
<th>Parallel linkage of linear scissor grids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic module in deployed position</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Polyhedron described by basic module in deployed position</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Basic module in fully folded position</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Non-circular curvature</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Deployment</td>
<td>foldable</td>
<td>foldable</td>
<td>incompatible*</td>
</tr>
<tr>
<td>Needs external bracing</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Compactness ***</td>
<td>++</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>Single rod length</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Single joint hub</td>
<td>x</td>
<td>x</td>
<td>x x**</td>
</tr>
</tbody>
</table>

* Foldable if the angles between the scissor-unit planes in the grid are allowed to vary: special joint hubs required.
** Joint hub has to allow angular distortions of which the magnitude varies along the grid, thus it depends on the joint design.
*** Compactness in the stowed position, given as a scale from most compact (++) to least compact (--).

3.1. Double-layer grids with quadrangular cells

Deployable scissor grids consisting of quadrangular cells pose a very easy and powerful way to obtain a myriad of singly curved geometries. In this two-way grid the primary direction consists of a set of identical planar scissor linkages, which in the secondary direction are interconnected at each node by identical plane-translational scissor units. Consequently, any planar scissor linkage with a constant structural thickness $t$ can be translated into a three-dimensional scissor grid. Aside from this large freedom of form, the major advantage of this grid type is that it is always foldable, thus possessing a smooth and reliable kinematical behaviour. Its major disadvantage is that it requires extra bracing components since its quadrangular grid cells provide no geometrical rigidity.

The large design freedom possessed by this scissor grid type follows from the vast amount of possible forms that can be achieved by planar scissor linkages. Escrig [3] demonstrated these enormous design possibilities for planar linkages of scissor units consisting of straight bars and Sanchez Cuenca [14], Zanardo [16] and De Temmerman [2] have shown how to derive such scissor unit geometry from a base curve. A similarly large potential for planar linkages of angulated elements is demonstrated in Hoberman [9] and in You and Pellegrino [15]. Adding a third dimension to the planar linkage requires all structural thicknesses to be equal, which means that all scissor units of the linkage become symmetrical: point symmetry in case of translational units and line

*Figure 2: The deployability equation.*
symmetry in case of polar or angulated units. This extra constraint still allows a large freedom of form (Fig. 3) and additionally improves the uniformity of components, e.g., polar linkages then consist of a single bar length.

Escrig’s deployability equation ensures maximum compactness for scissor grids consisting of translational or polar units. For angulated linkages this equation is no longer valid. The compactness of angulated scissor units can vary between different units. Since the angled scissor linkage is a single-degree-of-freedom mechanism, it will reach its most compact state when one element (the one with the largest kink) has reached its most compact state (Roovers [12]) (Fig. 3d). The structural thickness corresponding to this state determines the bar length of the plane-translational units in the secondary direction of the scissor grid, so that the depth along this direction is theoretically reduced to zero in the stowed state. As a result, the translational scissor units along the secondary direction can always reach a maximum state of compactness. Thus the overall compactness of the scissor grid will depend on the compactness of the planar linkage forming the primary direction of the grid. Consequently, when comparing a set of scissor grids with similar curvature that cover the same area, the scissor grids containing angulated linkages will be the least compact in its stowed state, while the scissor grids based on polar units will be the most compact, as its constant bar length allows it to be reduced to a box-like bundle of bars. Note that the actual compactness largely depends on the detailing of the rods and joints.

Figure 3: Arbitrary curve converted into a planar scissor linkage with constant structural thickness t in its deployed and most compact state. The linkage consists of: (b) translational units (the curvature is limited: interference at the last scissor unit); (c) polar units; (d) angulated units.

3.2. Double-layer grids with triangular cells

Scissor structures with quadrangular cells provide many design possibilities for foldable scissor grids. However, their low in-plane stiffness has set researchers of to search for triangulated (i.e., three-way) scissor grids, which inherently possess a geometrical rigidity. Escrig [4] has proposed two cylindrical scissor grids with triangular cells consisting of scissor units with straight bars (Fig. 4a and b). All structural thicknesses and all bar lengths of the grid are equal for both types. Warping occurs in the diagonal scissor units, meaning that the grids are geometrically incompatible: in the stowed position all rods are strain-free, but during the deployment the rods of the diagonal units increasingly deform, causing additional strains and stresses which peak in the fully deployed state. As an alternative, Escrig proposed to use pre-bent bars for the diagonal units, which are straightened in the stowed position. As such, the unstrained state is shifted from the stowed to the deployed configuration and the built-up potential energy in the rods can be used to self-unfold the structure to its expanded form. Nonetheless, pre-bending the bars leads to the obvious disadvantage of an increased manufacturing complexity. The first scissor grid type (Fig. 4a) consists of circular arches of polar units (direction A) interconnected by warped polar units in the diagonals (directions B and C). It can only take the shape of cylindrical surfaces, thus limiting the design possibilities. The second type (Fig. 4b) consists of linear linkages of identical plane-translational units (direction A), again interconnected by warped polar units (directions B and C). This grid type also allows other (i.e., non-circular) singly-curved surfaces to be formed.

De Temmerman [2] proposed another type of triangulated scissor grid that combines the ability to obtain a broad range of singly curved surfaces with geometrical compatibility throughout all stages of the deployment (Fig. 4c). The grid is populated solely by translational scissor units: identical plane-translational units along direction A and curved-translational units along directions B and C. The structural thickness is identical for all units in the grid, but the rod lengths vary due to the use of curved-translational units. With its single curvature, its foldability and its rigid triangular cells, this grid type combines three very beneficial properties. However, ensuring foldability requires that the angles between the scissor units are allowed to change (Fig. 5). Consequently, more complicated joints are required that enable the correct kinematical behaviour.
3.3. Parallel linkage of linear grids

Another way of obtaining cylindrical or singly curved geometry with scissor structures is to position a set of identical linear scissor grids side by side, either spaced or interconnected at multiple nodes so that they can be deployed simultaneously. A linear scissor grid fit for this purpose is presented in Escrig [3]. Its basic module consists of one symmetrical plane-translational unit and two identical and symmetrical polar units (Fig. 6). A linear repetition of this module results in a linear grid with constant curvature. Obtaining non-circular curvature is not possible when applying the deployability equation. The linear grid is foldable and has a rigid triangular cross-section, but cannot be folded in a compact way (Fig. 6d). Larger structures can easily be formed by attaching the matching nodes of a set of parallely positioned grids. Adding an extra layer of plane-translational units to interconnect the remaining nodes located at the apex of each linear grid as shown in Escrig [4] leads to a highly incompatible grid, which is unfavourable.
Figure 7: Linear grid of snap-through modules proposed by Gantes [6]: (a) basic module, in which the diagonal units cause the clicking effect; (b) linkage of several linear grids in deployed and folded state.

Another suitable linear grid was proposed by Gantes. He has conducted extensive research on snap-through modules that are used to form snap-through scissor structures. One of these modules comprises a truncated square pyramid, which can be linked to form spherical geometries, such as a single arch or a dome. It is not possible to adapt this snap-through module to fit a continuous double-layer grid with a cylindrical shape: this requires two of the polyhedron’s faces to become parallel, which results in geometrical incompatibilities (Gantes [6]). However, similar as for Escrig’s linear grid it is possible to generate cylindrical surfaces by linking a series of circular arches at their top nodes (Fig. 7). Strips of modules with non-circular curvature can also be obtained, though this involves a highly complex design process, which includes numerically solving a large set of non-linear equations (Gantes and Konitopoulou [7]).

3.4. Other grids

The classification above does not comprise an exhaustive list of singly curved scissor grid types. It does however contain the most feasible grid types for practical application as a load-bearing structure. For example, single-layer scissor grids of translational scissor units can be provided with single curvature, but this grid lacks the stiffness needed for most practical applications as it will not be able to act as the primary load-bearing element. Application where this grid is of great use is in the Mongolian yurt, where it is part of the wall assembly. Other possible configurations not incorporated in the classification make use of other, less common types of scissor units. A first example of such a unit is the generalised angulated element (GAE) (You and Pellegrino [15]), which forms a generalisation of the angulated unit and could thus take its place in a quadrangulated scissor grid. However, since the freedom of form provided by the basic angulated unit already is sufficiently large, the resulting added value would be small compared to the increased geometrical complexity. A second example is found in scissor units in which extra kinematical degrees-of-freedom have been introduced through additional hinges or sliding elements (Kokawa [10], Rosenberg [13]). The resulting scissor structures therefore no longer have a single deployed state, but can take up a number of different geometries once deployed. However, when these additional degrees-of-freedom are locked, the proposed scissor structures again fit in one of the categories mentioned in the classification. A last example is the radial scissor unit. This consists of three, four or even more rods interconnected at the intermediate hinge point, giving rise to a three-dimensional scissor unit inscribed in a polyhedron. These units are most suited to form planar or spherical scissor grids and, as shown by Escrig in Candela et al. [1], produce unwanted effects when fitted to a cylindrical shape, such as large deformations or highly complex joints. Further research is required to establish their usefulness for singly curved geometries.

4. Conclusions and future work

The classification presented in this paper provides a clear overview of the different scissor grid configurations that are able to generate singly curved surfaces, together with a list of their main geometrical and kinematical characteristics. As a result, it demonstrates the large potential of scissor grids by gathering the many design possibilities they offer to obtain a certain shape. Additionally, designers can use this classification as a tool to easily evaluate and compare these different possibilities in an efficient way, in order to select the best solution for their project. However, in order to get a full spectrum of all characteristics of the scissor grids essential to decide the fittest solution in the conceptual design phase, it might be useful in future work to also take the structural performance into account, as this will largely determine the lightness of the structure. The ability of a
classification to bring clarity in the complex matter of scissor grids lies at the basis of more general research conducted at our department which aims at maximising and mapping the entire geometrical and kinematical potential of deployable scissor grids.

By examining the classification it becomes clear that each scissor grid has its own advantages and disadvantages and that there is no best solution. Nevertheless, there might be solutions that work better than others depending on the design requirements. The foldable triangulated scissor grid composed of translational scissor units has caught our interest by combining three highly beneficial scissor grid characteristics: (i) foldability, resulting in a smooth and reliable deployment process, (ii) a geometrically rigid grid and (iii) a large freedom of form. We are therefore further developing this concept into a full scale deployable scissor structure, spanning approximately 6m. The success or failure of this grid type will lie in the ability to find a functional and efficient design for its joints to allow the required angular distortions of the grid, without compromising its structural behaviour.

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