Deep-learning-assisted hologram calculation via low-sampling holograms

Tomoyoshi Shimobaba  
Graduate School of Engineering  
Chiba University  
Chiba, Japan  
shimobaba@faculty.chiba-u.jp

David Blinder  
Department of Electronics and Informatics (ETRO)  
Vrije Universiteit Brussel (VUB)  
Brussels, Belgium  
dblinder@etrovub.be

Peter Schelkens  
Department of Electronics and Informatics (ETRO)  
Vrije Universiteit Brussel (VUB)  
Brussels, Belgium  
pschelke@etrovub.be

Yota Yamamoto  
Graduate School of Engineering  
Chiba University  
Chiba, Japan  
y-yamamoto@chiba-u.jp

Ikuo Hoshi  
Graduate School of Engineering  
Chiba University  
Chiba, Japan  
aeka2345@chiba-u.jp

Takashi Kakue  
Graduate School of Engineering  
Chiba University  
Chiba, Japan  
t-kakue@chiba-u.jp

Tomoyoshi Ito  
Graduate School of Engineering  
Chiba University  
Chiba, Japan  
itot@faculty.chiba-u.jp

Abstract—Digital holograms can be calculated by simulating light wave propagation on a computer. Hologram calculations are used for three-dimensional displays. However, the calculations take a long time, and the data size of the calculated holograms becomes large. This study presents a deep-learning-assisted hologram calculation using low-sampling holograms. We calculate holograms with low-sampling rates, resulting in the acceleration of the hologram calculation and the decrease of the hologram size. However, the low-sampling holograms decrease the quality of the reconstructed images and will occur the aliasing errors when not satisfying the Nyquist rate. The proposed method uses a deep neural network to retrieve the full-sampling holograms from the low-sampling holograms. We show elementary results of the proposed method in numerical simulation.

Index Terms—acceleration, compression, deep-learning, deep neural network, holography

I. INTRODUCTION

Digital holograms can be calculated by simulating light wave propagation on a computer. Such hologram calculations are used for a holographic display that is an ideal three-dimensional (3D) display satisfying all the depth cues. Even though computer technology is advancing, hologram calculations require an enormous computational time [1], [2]. Besides, the resolution of digital holograms has been increased to obtain better image quality from large-scale holograms [3]–[5], increasing in not only the calculation time but also the data size.

To realize holographic displays, many fast calculation algorithms, which include point cloud, polygon, holographic stereograms, and layer-based methods, have been proposed [6], [7]. In the point cloud method, a 3D object is expressed as an aggregation of object points; the light waves emitted from the object points are superposed in a hologram plane. This method provides flexibility expression of a 3D scene and can readily cooperate with other means, for example, holographic stereograms [8]. The superposition of light wave from object points is essential, so a number of acceleration methods for the superposition have been proposed: for example, look-up table methods [9]–[11], methods using symmetric property of light wave [12], and wavefront recording plane methods [13]–[16]. Hardware accelerators such as graphics processing units (GPUs) and field programmable gate arrays (FPGAs) are well suitable for the implementation of point cloud methods and dramatically accelerate the superposition [17]–[21]. Recently, sparsity-based algorithms have been proposed [22], [23]. We have proposed a wavelet transform-based calculation [22], which accelerates a hologram calculation by representing an object light with a few wavelet coefficients.

Using this method, we have succeeded in calculating a large hologram with 4.3 gigapixels and the pixel pitch of 1μm [24]. Thereby, such a hologram can recreate a 3D reconstructed image with a wider viewing angle and larger image size. On the other hand, the calculation time and the data size increase as increasing the hologram size. One of the simple ways to decrease the calculation time and data size is the downsampling to the hologram. If we down-sample holograms by the ratio of $1/2 \times 1/2$ horizontally and vertically, we can accelerate the hologram calculation approximately 16 times faster and reduce the data size to 1/4. However, the low
sampling holograms decrease the quality of the reconstructed images and will occur the aliasing when not satisfying the Nyquist rate.

This study presents a deep-learning-assisted hologram calculation using low-sampling holograms. We calculate holograms with low-sampling rates, resulting in the acceleration of the hologram calculation and the decrease of the hologram size. The proposed method retrieves the full-sampling holograms from the low-sampling holograms via a deep neural network (DNN). We show elementary results of the proposed method in numerical simulation. Section II describes the proposed method. Section III shows results of the proposed method and Section IV concludes this study.

II. PROPOSED METHOD

Figure 1(a) shows the outline of the proposed method. We calculate low-sampling holograms from 2D or 3D objects using diffraction calculation. As will described in the next subsection, the calculation time of low-sampling holograms can be decreased. However, the quality of the reconstructed image will be degraded. To assist the image quality improvement, the DNN predicts approximated full-sampling holograms from the low-sampling holograms. The DNN is trained using the dataset composed of the low-sampling and full-sampling holograms.

A. Hologram calculation

Figure 1(b) shows the outline of the hologram calculation. 2D and 3D objects are regarded as the aggregation of point light sources. A digital hologram can be calculated by the addition of light waves emitted from each object point on the hologram plane. For the calculation, consider the following equation:

\[ u(x_h, y_h) = \sum_{j=1}^{N} a_j \exp(\frac{2\pi}{\lambda} r_{hj}) \]

where \( i = \sqrt{-1}, N \) is the total number of the point light sources, \((x_h, y_h)\) is the coordinate of the hologram, \((x_j, y_j, z_j)\) and \(a_j\) are the coordinates and the light intensity of the \(j\)-th point light source, respectively, \(\lambda\) is the wave length, \(r_{hj}\) is the distance between \(j\)-th object point and \((x_h, y_h)\), and \(u_{z_j}\) is the point spread function (PSF) at the position \(z_j\) of an object point. When the PSF is distributed over the entire hologram, the computational complexity of the superposition is proportional to \(O(NN_h^2/d_h d_v)\) where \(N_h \times N_h\) is a number of hologram pixels. Therefore, in the hologram calculation, the superposition has the greatest calculation cost. In this study, we calculate holograms from 2D images; that is, we set \(z_j = \text{const.}\)

A straightforward way to decrease the calculation cost is to reduce the number of sampling of the hologram. If we down-sample holograms with the ratio of \(1/d_h \times 1/d_v\) horizontally and vertically, the calculation cost simply decreases \(O(NN_h^2/d_h d_v)\). However, the down-sampling will occur the aliasing error when not satisfying the following relation:

\[ \theta = \sin^{-1}(\frac{\lambda}{2d}), \]

\[ z \geq N_h p \tan \theta, \quad (2) \]

where \(z\) is the distance between a hologram and object and \(\theta\) is the maximum diffraction angle determined by the sampling pitch of the hologram \(p\). In this study, the down-sampling is performed by decimating the hologram pixels with the intervals of \(d_h \times d_v\) and changing the sampling pitch \(p\) to \(pd_h\) and \(pd_v\), horizontally and vertically.

B. Deep neural network

Figure 2 shows the network structure of the DNN. We modified U-Net that is often used for image generation and segmentation problems [25]. We assume that the input hologram size is over \(1,024 \times 1,024\). The size is too large because the number of the network parameter becomes enormous. Therefore, an entire hologram is divided by sub-holograms with \(64 \times 64\) pixels, and we input sub-holograms to the DNN sequentially. The predicted sub-holograms by the DNN are tiled to restore the entire hologram.

The input layer consists of the amplitude and argument of low-sampling complex amplitudes, which are calculated by Eq. (1), with \(in \times in = 64^2 \times 64^2\) pixels. The modified U-Net predicts the full-sampling phase-only hologram from the low-sampling amplitude and argument. This DNN is composed of convolution layers with the activation function “ReLU”, max pooling layers and up-sampling layers denoted as “Conv+ReLU,” “MaxPool” and “UpSampling”. The size of the convolution kernel is \(3 \times 3\) pixels, and the ratio of the down- and up-sampling in “MaxPool” and “UpSampling” is \(2 \times 2\). In the figure, the notation \((N \times N, ch)\) means that the input image size is \(N \times N\) and the number of the convolution filters is \(ch\). Around the last layer, \(1 \times 1\) convolution layer with the linear activation function, the add layer and the sigmoid function are used. The add layer directly receives the input images. The skip connections directly transfer the acquired information by first half convolution layers to the corresponding second half ones, resulting in accelerating the training and improving the prediction performance.

We used the loss function defined as

\[ L = \frac{1}{2} \| Y - X \|_2^2, \]

(3)

where \(Y\) is the ground truth of the phase-only holograms, \(X\) is the predicted holograms, \(\| \cdot \|_2\) denotes \(\ell_2\) norm.

C. Dataset

The dataset was numerically generated as follows: we used Caltech256 dataset [26] as the original images. First, these images were resized to \(1,024 \times 1,024\) pixels. After multiplying the resized images with random phases, we calculated complex amplitudes on the hologram plane using diffraction calculation
of Eq. (1). We converted the complex amplitudes to the amplitude \( a(x_h, y_h) \) and argument \( \theta(x_h, y_h) \) using

\[
a(x_h, y_h) = |u(x_h, y_h)|, \tag{4}
\]

and

\[
\theta(x_h, y_h) = \tan^{-1} \frac{\text{Im}\{u(x_h, y_h)\}}{\text{Re}\{u(x_h, y_h)\}}, \tag{5}
\]

where \( \text{Re}\{\cdot\} \) and \( \text{Im}\{\cdot\} \) are operators to take the amplitude and the argument of the complex number. We down-sampled them with the ratio \( 1/d_h \times 1/d_v \), and normalized them with 256 gradations. The dataset consists of a set of the low-sampling complex amplitudes and the full-sampling phase-only holograms.

The calculation condition is that the wavelength is \( \lambda = 633 \) nm, the pixel pitch \( p = 8 \mu m \), and the propagation distance is set to 0.3 m. We prepared 500 holograms in which we divided 450 holograms (115,200 sub-holograms) as the training dataset and 50 holograms (12,800 sub-holograms) as the validation dataset. Our wave optics numerical library [27] performed all calculations of the hologram calculation. We used Keras [28] as the DNN framework.

### III. RESULTS

We show elementary results of the proposed method in numerical simulation. Figure 3 shows (a) the original image (Mandrill), (b)-(d) the reconstructed images from the full-sampling phase-only hologram, the hologram interpolated from the down-sampling hologram and the predicted hologram from the DNN. These down-sampling ratio is \( d_h \times d_v = 1/2 \times 1/2 \). We used two image quality indices: peak signal-to-noise ratio (PSNR) and structural similarity (SSIM). We measured the image quality by the PSNR and SSIM between the reconstructed image of the original hologram and the other holograms. The PSNR and SSIM of the predicted hologram from the DNN are 32.2dB and 0.978 and are superior to those of the interpolated hologram.

Figure 4 shows other results: (a) the original image (Peppers), (b)-(d) the reconstructed images from the full-sampling phase-only hologram, the hologram interpolated from the down-sampling hologram and the predicted hologram from the DNN. The PSNR and SSIM of the predicted hologram from the DNN are 32.0dB and 0.963 and are superior to those of the interpolated hologram.

Figure 5 shows the loss value of Eq.(3) when increasing the epoch. In the curves of the training and validation losses with the down-sampling ratio of \( 1/2 \times 1/2 \), these two curves were gradually decreased without over-fitting even if we did not use the batch normalization and dropout techniques. In addition, we verified the loss value of Eq.(3) when we used the down-sampling ratio of \( 1/4 \times 1/4 \). Although these two curves gradually decreased without over-fitting; however, the loss value was higher than the case in the down-sampling ratio of \( 1/2 \times 1/2 \).

Figure 6 shows the reconstructed images of the Mandrill
image from the full-sampling hologram, the interpolated hologram and the predicted hologram from the DNN in the down-sampling ratio of $1/4 \times 1/4$. The bottom column shows the corresponding power spectra. In the interpolated hologram of Fig. 6(b), the aliasing errors are observed because the calculation condition does not satisfy the Nyquist rate. In the predicted hologram of Fig. 6(c), the aliasing seem to be suppressed compared to Fig. 6(b); however, the reconstructed image contains noises.

The proposed method can merely reduce the data size by the down-sampling ratio. The calculation time of holograms can reduce in proportional to the down-sampling ratio. Note that the additional calculation time for the DNN is required, which depends on only the hologram size. Therefore, when the hologram calculation is performed in a large number of the object points, the proposed method would be useful.

IV. CONCLUSION

We proposed a DNN-assisted hologram calculation via low-sampled holograms. In this method, low-sampling rates helped to accelerate the hologram calculation and to decrease of the hologram size. However, the low-sampling holograms decrease the quality of the reconstructed images and will occur the aliasing if the calculation condition will not satisfy the Nyquist rate. The proposed method used a DNN to retrieve full-sampling holograms from the low-sampling holograms. In this study, we verified only holograms generated from 2D images. The future work will investigate the effectiveness of the proposed method in holograms made from 3D images.

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