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A Hopping Robot Driven by a Series Elastic Dual-Motor Actuator

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Abstract—In this paper, we present the kinematically redundant Series Elastic Dual-Motor Actuator (SEDMA). It consists of two motors, coupled to a series spring through a planetary differential. The redundant degree of freedom of this system can be exploited to optimize a specific aspect of the actuation task. This requires a controller which distributes the required output power among the two different inputs in an optimal way. The closed-loop control design for redundant systems is, however, a challenging topic, especially for strongly dynamic tasks like hopping. In this work, we test the abilities of the SEDMA by using it to actuate a single-leg hopping robot. Its controller features a closed-loop hopping controller and a control allocator, which minimizes the electrical energy consumption of the SEDMA. Tests on a physical setup prove that the actuator and its online controller are able to generate consistent hopping patterns. The work highlights the difficulties in achieving optimality in online control when the internal dynamics of kinematically redundant actuators become relevant.

Index Terms—Legged Robots; Optimization and optimal control; Motion Control; Compliance and Impedence Control

I. INTRODUCTION

D Espite years of research on humanoid robots, current state-of-the-art robotic systems still lack the agility of humans and animals. Our exceptional capabilities are, at least partially, explained by the way our skeletal muscle is built up. One key feature of skeletal muscle is the elasticity of tendons, which act as a spring in series with our motor units. By exploiting the elasticity of the tendons, locomotion can be performed in a much more (energy-)efficient way [1].

Another remarkable feature of skeletal muscles is their motor units. These consist of multiple muscle fibers, which can be activated according to the need [2][3]. In robotics, it has been shown that the redundancy can be exploited for overcoming transmission nonlinearities [4], shaping the operating range [5] and improving position tracking [6], backdrivability [7] and energy efficiency [8][9]. Several authors have, for example, demonstrated that biomedical robots such as exoskeletons and prosthetics, can achieve high reductions in energy consumption by implementing redundant actuation concepts, especially when combined with energy buffers such as springs and flywheels [10][11][12][13].

In spite of the great energetic advantages predicted by these works, practical proof for the energy-efficiency of these systems is still missing. Building redundant actuators is difficult, especially if they need to be compact and energy-efficient [14]. Furthermore, energy-efficient online control of such complex actuators is a difficult problem [15]. Nearly all methods are based on the assumption that actuator dynamics do not have an influence on the control allocation problem. As a consequence, they have difficulties with rapidly varying control commands [16]. This is a problem in robotics, where actuator dynamics often cannot be neglected [17].

In some cases, this problem can be solved by pre-computing the inputs offline, e.g. by means of optimal control theory [18]. The method is, however, unsuitable when the actuator interacts with an unknown environment or when robustness to perturbations is required. Furthermore, the optimization problem is typically solved with the square of the mechanical power (\(P^2\)) or the torque (\(T^2\)) as cost functions [19]. These cost functions, however, do not take the nonlinear dynamics of gearbox and, especially, bearing friction into account. For complex actuators with many components, such as the redundant actuators mentioned earlier, this approach can be questioned. Here, the electrical power consumption – which is what we really want to minimize – becomes a complex function of the speeds and torques in the system [17]. The differences between electrical and mechanical power can be significant [20] and affect the optimization. Ambrose et al., for example, found the mechanical cost of transport of a bipedal robot to increase with speed, whereas the electrical cost of transport actually decreased [21].

Following this discussion, a cost function based directly on the electrical power (\(P_{elec}\)) would be preferable, as the more accurate model can be expected to result in a larger decrease in energy consumption. As mentioned earlier, however, such a cost function will require some approximations regarding the actuator’s dynamics to enable its implementation in an online controller. These approximations will inevitably have an impact on the optimality of the final result. The aim of this paper is therefore to assess experimentally how well such an approach performs in comparison with optimal controllers based on simpler cost functions.

To investigate this question, we propose a control framework for a kinematically redundant actuator combined with a series
The sun gear is coupled to a 150W Maxon RE40 DC motor with a 15:1 planetary gearbox; the ring gear to a 200W Maxon EC-4-pole motor with a 15:1 planetary gearbox and an additional 3:1 reduction provided by a spur gear pair. The carrier of the planetary differential serves as the output, just like in an ordinary planetary gearbox. A more detailed description of the design of the DMA can be found in previous work [9].

B. MARCO Hopper II

The SEDMA is implemented as the actuator on the modular MARCO Hopper II test bench (Figure 1b). The hopper, introduced in [23], is representative of a human leg with an actuated knee. The shank and thigh are represented by two links of equal length \((l_t = 250 \text{ mm}, \text{ mass } m_t = 0.17 \text{ kg})\). A mass \(m_h = 1.8 \text{ kg} \) attached to the hip joint plays the role of the trunk; a mass \(m_f = 0.30 \text{ kg} \) corresponds to the foot. The SEDMA is connected to the knee joint (mass \(m_k = 0.23 \text{ kg} \)) through a Bowden cable attached to a pulley (radius \(r_{	ext{pulley}} = 4.3 \text{ cm} \)). In order to prevent overextension of the knee, a steel cable is placed between the hip and foot. This cable also has a secondary function: by transferring kinetic energy from the hip to the foot, it enables the foot to leave the ground. Finally, to ensure a purely vertical motion of the hip and foot, these two joints are mounted on a linear bearing.

Bürster potentiometers (S/N 8709-5250) measure the position of the foot relative to the ground, as well as the distance between the hip and foot. Another Bürster potentiometer (S/N 8709-5150) is used to measure the extension of the spring. The force in the Bowden cable is sensed by a SCAIME ZF100 sensor. Finally, the position and speed of the spindle are derived from the readings of the incremental encoders (500 CPT) on the motors of the SEDMA.

III. SEDMA EQUATIONS

A. Spring and spindle

The series spring (stiffness \(k_s = 19.87 \text{ N/mm} \)) and the spindle (gear ratio \(n_{sp} = 314 \text{ rad/m} \)) determine the relationship between \(\theta_C\), the output of the planetary differential, and \(x_o\), the output position of the SEDMA. This relationship is described by the following equation, which also contains the output force of the SEDMA \(F\):

\[
x_o = \frac{\theta_C}{n_{sp}} - \frac{F}{k_s}
\]

The spindle converts the rotary motion at its input, \(\theta_C\), to a linear motion \(x_o\) at the output. Likewise, the carrier torque \(T_C\) is converted to an output force \(F\):

\[
F = C_{sp} n_{sp} T_C
\]

where, in accordance with [17], we defined the efficiency function

\[
C_{sp} = \eta_{sp} \text{sign}(F \cdot x_o)
\]

with \(\eta_{sp} = 85\%\) the efficiency of the spindle.

Fig. 1. The Series-Elastic Dual-Motor Actuator on the MARCO Hopper II setup. (a) Schematic, (b) Actual setup.
B. Dual-Motor Actuator (DMA)

A detailed derivation of the equations of the DMA can be found in [9]. The main results are summarized below.

The planetary differential links the two input speeds \( \dot{\theta}_S \) and \( \dot{\theta}_R \) of the sun and ring motor, respectively, to the speed of the carrier, \( \dot{\theta}_C \). Defining the state vector \( q = \left( \dot{\theta}_S, \dot{\theta}_R \right)^T \), we can write

\[
\dot{\theta}_C = \frac{1}{n_C} \left[ \frac{1}{n_S (1 + \rho)} \frac{\rho}{n_R (1 + \rho)} \right] q = C_0 q
\]  

(4)

Here, \( n_S \) and \( n_R \) are the reductions of the gearboxes of the sun and ring drivetrain, respectively, as specified in Table II. The reduction offered by the planetary differential is characterized by \( \rho \). Its parameters are listed in Table I.

The speed ratio \( i \) describes the speed distribution between the ring and sun. It is defined as

\[
i = \frac{\dot{\theta}_R}{\dot{\theta}_S} = \frac{n_R}{n_S} \]  

(5)

This allows us to invert the speed equation (4):

\[
q = \left[ \frac{n_S (1 + \rho)}{1 + \rho} \frac{1}{n_R (1 + \rho)} \right] \dot{\theta}_C
\]

(6)

The accelerations are found by differentiation of (6):

\[
\dot{q} = \left[ \frac{n_S (1 + \rho)}{1 + \rho} \frac{1}{n_R (1 + \rho)} \right] \ddot{\theta}_C + \left[ \frac{n_S (1 + \rho) \rho}{n_R (1 + \rho)^2} \right] \dot{\theta}_C \dot{\theta}_C
\]

(7)

The vector of motor torques \( T_m = (T_{mS}, T_{mR})^T \) is given by the following equation:

\[
T_m = J \dot{q} + B q + C \text{sign}(q) + D (T_C + T_{CC} \text{sign}(C_0 q))
\]

(8)

\[
J = \begin{bmatrix} J_S & 0 \\ 0 & J_R \end{bmatrix} + J_C D \begin{bmatrix} 1 \frac{1}{n_S (1 + \rho)} \\ \frac{1}{n_R (1 + \rho)} \end{bmatrix} + J_{CC} D \begin{bmatrix} 1 \frac{1}{n_S (1 + \rho)} \\ \frac{1}{n_R (1 + \rho)} \end{bmatrix}
\]

\[
B = \begin{bmatrix} \nu_S & 0 \\ 0 & \nu_R \end{bmatrix} + \nu_C D \begin{bmatrix} 1 \frac{1}{n_S (1 + \rho)} \\ \frac{1}{n_R (1 + \rho)} \end{bmatrix}
\]

\[
C = \begin{bmatrix} T_{CS} & 0 \\ 0 & T_{CR} \end{bmatrix}
\]

\[
D = \begin{bmatrix} \frac{C_S}{n_S} \left( \rho C_{PG} + 1 \right)^{-1} \\ \frac{C_R}{n_R} \left( \rho C_{PG} + 1 \right)^{-1} \end{bmatrix}
\]

Here, we defined efficiency functions for the gearboxes

\[
C_\lambda = \eta_\lambda^{-\text{sign}(T_{\lambda S} \theta_\lambda)}
\]

(9)

and for the planetary differential:

\[
C_{PG} = \eta_{PG}^{-\text{sign}(T_{PG} \theta_C)}
\]

(10)

with \( \lambda = R, \theta \) an index corresponding to the ring and sun, respectively, and with \( \eta_\lambda \) the meshing efficiency of the corresponding gear. \( J_S \) and \( J_R \) are the inertias of the sun and ring branch; \( J_C \) is the inertia of the carrier. Friction is modelled as a combination of Coulomb friction (coefficients \( T_{CS}, T_{CR}, T_{CC} \)) and viscous friction (coefficients \( \nu_S, \nu_R, \nu_C \)) on the sun and ring branch, as well as on the output of the DMA, i.e. the carrier. All the aforementioned coefficients, as well as other relevant parameters, are listed in Tables I and II. Friction coefficients were obtained experimentally in prior work [9].

Finally, the electrical power of the SEDMA, \( P_{elec} \), is

\[
P_{elec} = U_S I_S + U_R I_R
\]

(11)

where motor current \( I_\lambda \) and motor voltage \( U_\lambda \) are determined from the motor model

\[
\begin{cases}
I_\lambda = T_{m\lambda} / k_{T\lambda} \\
U_\lambda = k_{T\lambda} \theta_\lambda + R_\lambda I_\lambda
\end{cases}
\]

(12)

with \( k_{T\lambda} \) the torque constant of the motor and \( R_\lambda \) the motor’s winding resistance. The electrical energy consumption of the SEDMA can be found by integrating (11).

C. Constraints

In order to prevent damage of the components or unrealistic control inputs, following constraints are implemented in the controller:

1) Maximum speed: Motor speeds \( \dot{\theta}_S \) and \( \dot{\theta}_R \) were limited to the maximum values specified in their datasheets, i.e. \( |\dot{\theta}_S| < 12,000 \text{ rpm} \) and \( |\dot{\theta}_R| < 25,000 \text{ rpm} \).

2) Maximum motor torque: Motor torques were limited to values of \( |T_{mS}| < 350 \text{ mNm} \) and \( |T_{mR}| < 250 \text{ mNm} \) roughly twice the maximum continuous torque \( T_{\lambda\text{max,cont}} \) of the respective motor.

3) Maximum voltage: Motor voltages \( U_S \) and \( U_R \) were limited to 48 V, the voltage of the power source.

IV. Control Framework

The control framework is depicted schematically in Fig. 2. Its two main parts, the high-level virtual model control and the
A. Virtual Model Control

In previous work, Virtual Model Control (VMC) was successfully used to control the MARCO Hopper II, actuated by means of a traditional geared DC motor [23]. The basic idea is that the dynamics of the hopper are replaced with that of a single-mass harmonic oscillator. This model, known as the spring-loaded inverted pendulum (SLIP) model, describes the dynamics of human running and hopping well [24]. In this work, the same VMC controller is used as high-level controller to determine the force required from the SEDMA.

In practice, the VMC is implemented as a state machine. A schematic, which also shows the equations governing the state transitions, is depicted in Figure 3.

The state machine distinguishes between the flight phase and the stance phase. The latter, in turn, is split into two parts based on the state of the virtual spring: an extension and a compression phase. In the flight phase, the commanded force is zero to allow the hopper to move freely. As soon as the stance phase is reached, the commanded spring force is

\[ F_d = \frac{2l_f \cos \left( \frac{\phi}{2} \right)}{r_{pulley}C_{cable}} \left[ \frac{m_{hip} + \frac{1}{2} m_l}{m_v} k_v \left( l_0 - l_{leg} \right) + \frac{1}{2} m_l g \right] \]

where \( \phi \) is the knee angle as defined in Fig. 1, \( l_0 = 40 \) cm the maximum (extended) length of the virtual spring, \( l_{leg} \) the distance between hip and foot, and \( C_{cable} \) the efficiency function for the Bowden cable, defined as

\[ C_{cable} = \begin{cases} \eta_{cable} & \text{(stance-extension)} \\ 1/\eta_{cable} & \text{(stance-compression)} \end{cases} \]

with \( \eta_{cable} = 85\% \). Other parameters were defined in section II. The mass of the virtual harmonic oscillator \( m_v \) is set to 1 kg, the virtual stiffness \( k_v \) to 175 N/m. According to [23], these parameters are appropriate to enable hopping.

B. SEDMA force control

The force control is handled by a cascaded controller consisting of an outer force loop and an inner speed loop. This is a well-established control approach for series elastic actuators, which works well even when friction is high [25]. The force loop consists of a PI controller which translates the force error \( F_d - \hat{F} \) to a desired output speed \( \dot{\theta}_{C,d} \). This is the input for the speed control loop. The details of the speed loop, which consists of the control allocation and the low-level control, are explained below.

1) Control allocation: The control allocator translates the desired output speed \( \dot{\theta}_{C,d} \) to motor speeds \( \dot{\theta}_S \) and \( \dot{\theta}_R \). We calculate the optimal speed distribution offline and implement it in a three-dimensional lookup table.

The electrical power consumption \( P_{elec} \), as modeled in Section III, can be written as a function of 5 variables: \( \dot{\theta}_C \), \( \dot{\theta}_C \), \( i \), \( \dot{i} \), and \( T_C \). In other words, the optimal speed ratio depends on the previous speed ratio. To simplify the problem, we approximate (7) as follows:

\[ \begin{bmatrix} \dot{\theta}_S \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{n_S (1+\rho)}{1+\rho_1 + \frac{\rho}{n_S (1+\rho) + \frac{\rho}{1+\rho_1}}} \\ \frac{1}{1+\rho_1} \end{bmatrix} \dot{\theta}_C \]

(14)

We thus neglect the effect of the rate of change of the speed ratio \( i \) on the acceleration. As a result of this approximation, the electrical power \( P_{elec} \) becomes a function of only the output variables \( \dot{\theta}_C \), \( \dot{\theta}_C \) and \( T_C \), as well as the speed ratio \( i \). In other words, \( P_{elec} \) becomes independent of previous speed distributions. This simplifies the problem considerably, but the approximation needs to be validated. This is done in section IV-C.

By minimizing \( P_{elec} \) with respect to \( i \), we can find an optimal speed ratio

\[ i_{opt} = f \left( \dot{\theta}_C, \dot{\theta}_C, T_C \right) \]

(15)

which is only a function of the DMA’s output. Combining this function with (6) allows us to find a relationship between the motor speeds and the output speed. Consequently, the control allocation problem is solved.

In practice, the function (15) is implemented as a 30x30x30 lookup table (LUT) with evenly spaced speed, torque and acceleration breakpoints. The lookup table is calculated offline with a parameter sweep on \( \dot{\theta}_R \) (step size 1 rpm) for every combination of speed, torque and acceleration. Since the speed ratio \( i \) can be derived from \( \dot{\theta}_R \) and the output speed \( \dot{\theta}_C \) of the DMA by means of (4) and (5), the entire dynamics of the drivetrain can subsequently be determined with the equations listed in section III. Each solution is checked against the constraints specified in Section III-C. The value of \( i \) leading to the smallest electrical power \( P_{elec} \) is then added to the LUT as the optimal speed ratio for that particular combination of speed, torque and acceleration.
Linear interpolation and extrapolation are used to calculate intermediate points. The desired speed \( \dot{\theta}_{C,d} \) and the resulting acceleration \( \ddot{\theta}_{C,d} \) are fed to the LUT, along with the estimated output torque \( T_o \). This implicitly assumes perfect tracking at the output, but allows the controller to adapt to changes in output torque. The output of the LUT, the ring speed \( \dot{\theta}_{R} \), is then used to calculate the sun speed \( \dot{\theta}_{S} \) by means of (4). Consequently, the vector \( \dot{q}_d \) is completely defined.

2) Linearized feedback: The equations for feedback linearization are derived from the SEDMA’s equations of motion (8), where we consider \( \dot{q} \) as the control variable and \( T_m \) as the input. Defining the control error \( e = \dot{q}_d - \dot{q} \), we can write the suggested control law

\[
T_m = J\dot{q}_d + Pe + I \int e \, dt + Bq_d \\
+ C\text{sign}(\dot{q}_d) + D(\dot{T}_o + TC\text{sign}(C\dot{q}_d))
\]

with P and I 2x2 diagonal matrices which contain the controller gains. This controller combines feedforward acceleration and friction compensation with feedback of the sensed torque \( \dot{T}_o \), which is calculated from the estimated force \( \dot{F} \) by means of (2). Finally, the torques obtained from (16) are used as setpoints for the Maxon EPOS3 controllers in the sun and ring branch, both operated in Cyclic Synchronous Torque mode. The setpoints are capped at the maximum torque values specified in section III-C. Furthermore, the built-in Pt protection of the EPOS3 ensures that the motor windings do not exceed their maximum temperature during long-term operation.

V. EXPERIMENTS

A. Hopping pattern

Figure 4 shows the first ten seconds of a hopping trial. A constant hopping height (24 mm) and frequency (1.02 Hz) are achieved after only a short transient. The height is in line with the experiments previously presented in [23], although the hopping frequency is lower (1.02 Hz compared to 1.5Hz). This is most likely due to the way friction is compensated in [23], i.e., by increasing the virtual spring stiffness during the upward motion. The resulting energy injection may be higher than strictly needed for compensating the friction, and lead to an increase in hopping frequency.

![Foot and hip position for the MARCO II hopping robot.](image)

Fig. 4. Foot and hip position for the MARCO II hopping robot.

B. Online speed distribution

Figure 5 shows how the online controller distributes the required carrier speed between the sun and ring. The speeds of the ring and carrier converge quickly to a stable cycle, after a small transient at the start. According to the speed equation (4), the ring contributes \( \rho = 9 \) times more to the carrier speed than the sun. This is evident in Figure 5a, where the ring speed almost follows that of the carrier. The sun mostly serves to make the overall motion more energy-efficient and to provide speeds and accelerations beyond the ring drive’s capabilities.

In terms of speed ratio (Figure 5b), we see both negative and positive speed ratios appearing. This means that, at some time instants, one motor is delivering power while the other is absorbing it. Such internal power flows are generally not considered energy-efficient [26], although we have recently shown they can actually be desirable in strongly dynamic applications [27]. In this specific case, the negative power flows did not come out of the optimization, as the control allocation LUT only contains positive speed ratios. Instead, the motors get pushed into low-efficiency operating points because the low-level controller cannot track the desired speeds. This is an inevitable downside of the proposed control architecture and the proposed approximation: as the control allocator is unaware of previous motor speeds and accelerations, it may demand large accelerations from the motors, beyond what they can provide.

C. Accelerations

In section IV-B1, we proposed a simplification to the equation for the acceleration (7). In this subsection, we will examine the validity of this approximation by comparing the estimated accelerations to the actual accelerations during the hopping experiment. The results are shown in Figure 6.

![Graph showing speed and acceleration data.](image)

Fig. 5. (a) Speeds of the sun, ring and carrier; (b) Speed ratio as defined by (5). The speed of the carrier, which serves as the output of the dual-motor actuator, is a weighted sum of the sun and ring speeds (see (4)).
other way around. This can also be seen in Figure 5b, where the speed ratio jumps from +10 to -10 approximately every 0.5 s. The strong variations in $i$ lead to a strong contribution of the neglected term in (14), which is proportional to the variation of the speed ratio $i$.

As mentioned in the previous section, the ring’s contribution dominates the output speed, and this also applies to the acceleration in Figure 6. The acceleration of the ring is nearly the same as that of the output - albeit scaled by the planetary differential with a ratio of 9:10. As a consequence, the approximation of the ring acceleration is quite good, too, because the simplified formula for the acceleration essentially assumes that the accelerations of the individual motors are proportional to the output acceleration. Conversely, the acceleration profile of the sun is very different from that of the output. This results in an estimated acceleration which is only good at low accelerations. At higher accelerations, the neglected term, which is proportional to the carrier speed $\dot{\theta}_C$, becomes significant and makes the approximation invalid.

D. Comparison with optimal control

In the proposed control architecture, the approximation of the acceleration has an influence on the speed distribution, but not on the output speed. As a consequence, the accuracy of the hopping controller is not affected. But what about the optimality of the speed distribution? To answer this question, we took the measured output speed and torque of the DMA and optimized the speed distribution offline based on this data. Using the commercial software package GPOPS [28], we applied optimal control (OC) with three cost functions:

1) $OC - P_{mech}$: The cost function is the sum of mechanical powers $(T_{mR} \dot{\omega}_R)^2 + (T_{mS} \dot{\omega}_S)^2$, which aims at reducing the gearbox losses by reducing the power flow in both branches.

Friction and gearbox efficiencies are neglected in the calculation of the torques $T_{mR}$ and $T_{mS}$.

2) $OC - T$: The cost function is the sum of motor torques $T_{mR}^2 + T_{mS}^2$, which aims at reducing the Joule losses in both motors. Friction and gearbox losses are neglected in the calculations.

3) $OC - P_{elec}$: The cost function is the electrical power consumption $P_{elec}$, given by (11). The calculation includes all friction coefficients and works with the exact value of the acceleration. Assuming the model to be exact, (11) would minimize the actual electrical energy consumption of the actuator.

The first and second strategy are common in literature and have the advantage of not requiring prior knowledge of friction coefficients. The third strategy is equivalent to the online controller, but uses the exact acceleration instead of the approximation. The results from this optimization should reflect the actual energy-optimal distribution, and can therefore be used as a baseline.

In Figure 7, the optimal speed distributions resulting from the three optimizations are compared to the speed distribution obtained from the online controller. As discussed in section V-B, due to the specific design of the DMA, the ring speed resembles the output speed. This is found to be true for all optimizations, which explains the resembling ring speed profiles. Conversely, because the sun has less impact on the output speed, there is more room for changes in its speed profile. This is why sun speed differs a lot among the different optimizations.

Apparently, the $OC-P_{elec}$ optimization yields a solution where the sun branch is not used. This situation arises when the required speed can be delivered by a single motor, and accelerations are insufficiently high to favor distributing the
In the OC-optimization follows this trend by yielding low sun speeds. In the OC-P_mech optimization, however, the sun speed is consistently brought to saturation. As shown in Table III, this is the most energy-inefficient solution. This comes as no surprise, considering that the hopping motion requires high speeds and accelerations from the DMA which was, in fact, designed for high torques at relatively low speeds [9]. In such cases, OC-T, which includes the inertia of the drivetrains, performs better than OC-P_mech, which does not.

The sun speed profile of the measurement displays an additional oscillation during the upward motion. The resulting acceleration has an adverse effect on the energy consumption. In fact, Table III shows that the online controller has the highest energy consumption of all control strategies. This is partially explained by the approximation of the acceleration, but also by the fact that the online controller does not predict future states. As a consequence, the resulting speed distribution may yield high accelerations, leading to a higher energy consumption.

Note that, in applications where the required accelerations are small or where the inertia of the drivetrain components is low, the impact of the error in the approximated acceleration will decrease. The online controller will then outperform the \( T_m^2 \) or \( P_mech^2 \) optimal control approaches thanks to the detailed model of friction in the drivetrain. In other words, the case study presented here is a worst-case scenario for the proposed control allocator, and the results must be interpreted as such.

<table>
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<th>TABLE III</th>
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<td>Energy consumption for the different cost functions.</td>
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<tr>
<td>OC - ( P_{elec} ) (simulation)</td>
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<tr>
<td>OC - ( T ) (simulation)</td>
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<tr>
<td>OC - ( P_{mech} ) (simulation)</td>
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<tr>
<td>Online controller (measurement)</td>
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VI. CONCLUSION

In this paper, we presented a single-leg hopping robot actuated by a Series Elastic Dual-Motor Actuator, a novel bio-inspired actuation concept. We proposed a control framework which solves the problem of distributing the required output powers among the redundant actuator’s motors, while generating a repeatable hopping behavior. The control allocation problem is tackled by using a detailed model of the electrical power to generate an optimality map for the speed distribution. One minor simplification enabled us to implement this map in a three-dimensional lookup table. In tests on a physical setup, the controller successfully generated a hopping motion. One might expect the more complex optimality map to yield better results than simpler cost functions. Its implementation in an online controller is, however, not straightforward. Approximations are required to do so for a dynamic system, and these can undermine its effectiveness in reducing the energy consumption. In this work we indeed found that, in comparison with the proposed controller, a smaller energy consumption could be obtained with a controller which utilizes a simple cost function based on motor torque. The result highlights that dealing with the additional control complexity entailed by the internal dynamics is imperative in order to obtain optimal performance in dynamic applications.

Future work will therefore consist of developing more advanced energy-efficient control allocation strategies for dynamic applications. In these approaches, internal dynamics will be exploited rather than neglected, with the aim of turning their negative impact on power consumption into a positive one.

REFERENCES


