Tackling the target matrix definition in MIMO Random Vibration Control testing

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ABSTRACT

Nowadays the advantages of performing Multiple-Input-Multiple-Output (MIMO) Random Vibration Control tests are widely accepted by the environmental engineering community. However their practice still needs to grow because of the high degree of expertise needed to perform these tests. The challenges start even before the actual test, with the definition of a realizable full Spectral Density Matrix as test reference (the control target). Defining this matrix with no a-priori knowledge of the off-diagonal terms is challenging: this operation must guarantee that the target can have a physical meaning. This is translated in the algebraic constraint that the target matrix needs to be positive (semi)-definite. On the other hand the requirements of any Random Vibration Control test are Power Spectral Densities (PSDs), representative for the acceleration operational levels at the control locations. In defining the reference matrix, the main challenge is then to guarantee a physically realizable full target spectral density matrix that has fixed PSD terms. The purpose of this work is to show the capabilities and the limitations of different state-of-the-art and more innovative solutions for the MIMO Random Vibration Control target definition in order to aim to a well-defined automatic procedure to be included in the standard practice.


SYMBOLS

In this work most of the derivations are in the frequency domain hence all the arrays are functions of the frequency f (in Hz), if not specified otherwise. Vectors are denoted by lower case bold letters, e.g. \( \mathbf{a} \), and matrices by upper case bold letters, e.g. \( \mathbf{A} \).

An over-bar \( \bar{a} \) is used to indicate the complex conjugate operation and the apex \( a' \) to indicate the complex conjugate transpose of a matrix, e.g. \( \mathbf{b} \) and \( \mathbf{B}' \) are the complex conjugate and the complex conjugate transpose of the vector \( \mathbf{b} \) and the matrix \( \mathbf{B} \), respectively.

The dagger symbol \( \dagger \) is used to indicate the Moore-Penrose pseudo-inverse of a matrix, whereas the hat \( \hat{A} \) is used to emphasize the estimation of a quantity, e.g. \( \hat{A} \) is the estimate of the matrix \( A \).
INTRODUCTION

Vibration Control (VibCo) tests are performed to prove that an aerospace system and all the sub-components will withstand the vibration environment during the operational life. These tests aim to replicate with an high degree of fidelity the structural responses of a Unit Under Test (UUT) in the in-service conditions. This brings the need for exciting the UUT in all the possible axes (MIL-STD 810G – 514.6 2008). Nowadays the common practice, when more than one axis of excitation is needed, is to perform sequential single-axis tests (sequential Single–Output, SISO, tests). This practice has known limitations and drawbacks (MIL-STD 810G MET 527 2014). The most critical aspect is that a sequential SISO test can lead to an unacceptable UUT time to failure overestimation and different failure modes (Ernst et al. 1995), (Gregory et al. 2008). The main reason lies on the fact that (i) an increased number of control points is essential to guarantee that the response levels match also in location that are not controlled (Daborn et al. 2014) and (ii) with these tests it is impossible to guarantee that the cross-axis interaction the structure experiences in the in-service conditions will also be replicated. On top of more practical aspects that make difficult (or even impossible) to perform sequential single-axis tests, this is the main reason why simultaneous Multi-Axial excitation, and more generally Multi-Input Multi-Output (MIMO) vibration control tests, are nowadays the “go for” in the environmental testing community.

There are several types of MIMO VibCo tests, differing from the nature of the excitation environment the specimen needs to be subjected to. For spacecraft systems and subsystems, a random VibCo test is required for all the main electromechanical components (Aggarwal 2010). This type of test is performed to simulate the response of the unit under test to a broadband random Gaussian vibration environment. Typical scenarios are the road excitation that the aerospace system needs to withstand during the transportation or the payload and avionics equipment responses to the acoustic field during a spacecraft launch. For the SISO case, the test specification is a Power Spectral Density (PSD, usually in $\frac{g^2}{Hertz}$) profile that needs to be replicated in a user defined control location by exciting the UUT with a single shaker. In the MIMO case, it is possible to define required test levels for multiple control channels that will be controlled simultaneously. Additional information about the cross-talk between the control channels is also included. This information must be provided in terms of Cross Spectral Densities (CSDs) between the control channels defining, for instance, desired phase and coherence profiles. This feature is essential to also replicate the cross-talk that naturally exists between difference responses. The control target for a multi-exciters random control test is thus a full Spectral Density reference matrix (SDM). In the following test target and reference matrix will be thus used as synonymous.

Defining the target with no a-priori knowledge of the cross-correlation between control channels is very challenging. Filling in the off-diagonal terms must guarantee, in fact, that the full reference matrix will have in the end a physical meaning (can be realizable). This is translated in the algebraic constraint that this matrix needs to be positive semi-definite, without neglecting the pushing constraint that the test has still to guarantee the required PSD levels at the control locations. The solution of a realizable reference matrix with fixed PSD terms is not unique and not all the possible solutions can be exactly reproduced in the laboratory for a given specimen. In this sense, the best information an environmental test engineer could use is a target originated from real life recordings or field data. However, even in this case, when trying to replicate the target on the test rig, there will be inevitable impedance mismatches between the in-field and the
laboratory conditions. There is thus always the risk of trying to force the test item to an unnatural motion that could lead to poor control results and performances (Underwood 2002). A realizable target with high control performances can then be achieved by minimizing the aforementioned impedance mismatches on top of using field data (Daborn et al. 2014). Other considerations could also drive the choice of a reference matrix. For instance a desirable condition would be to minimize (maximize) the power provided to the exciters needed to reach the given target levels (Extreme Drives or Extreme Inputs problem). There are several reasons for achieving this goal: (i) for fixed $g_{\text{RMS}}$ response levels, it guarantees that the exciters are minimally stressed, which means to work in safety with respect to the expensive test equipment and (ii) it allows to reach higher $g_{\text{RMS}}$ levels retarding the maximum limits of the hardware (DACs overload, amplifiers safety switch, …) which means to have the possibility of getting the best out of the exciters performances. The achievable exciter levels are often a limiting aspect for Multi-Exciter test, as also shown in on-going works on Direct Field Acoustic eXcitation (Alvarez Blanco, 2016). The drives minimization problem has been already tackled by Smallwood (Smallwood 2007). Unfortunately, as also mentioned in the standard practice for Multi-Exciter vibration tests (MIL STD 810 G - MET 527 2014), the method proposed by Smallwood does not guarantee a positive semi-definite reference matrix.

Starting from the challenges and the limitations of both previous approaches and techniques based on linear algebra, this work aims to asses a target definition procedure that first guarantee the target to be physically realizable while keeping the test specifications and then find a solution to the Extreme Drives problem. The results of applying the developed method are shown first via simulated data and then driving a six-axis test rig in different test configurations.

**MIMO RANDOM VIBRATION CONTROL PROCESS**

The block scheme of a general MIMO Random VibCo test is illustrated in Figure 1.

![Figure 1: General MIMO Random Control block scheme.](attachment:image.png)
The structure under test is excited driving \( m \) electrodynamic or hydraulic shakers and the system’s response is recorded in \( ℓ \geq m \) control points. In the hypothesis of the structure under test behaving linearly and being time invariant, the system is represented by the Frequency Response Function (FRF) matrix, \( H ∈ ℂ^{ℓ×m} \)

\[
Y = HU
\]

where \( Y ∈ ℂ^{ℓ×1} \) and \( U ∈ ℂ^{m×1} \) are the spectra of the control channels recordings \( y(t) = \{y_1(t), ..., y_ℓ(t)\}^T \) and the input drives \( u(t) = \{u_1(t), ..., u_m(t)\}^T \), respectively. In case of rectangular systems, the impedance matrix \( Z ∈ ℂ^{m×ℓ} \), is generally obtained as the Moore-Penrose pseudo-inverse (Golub and Van Loan 1996), \( Z = H^\dagger \). In all the vibration control tests, a System Identification pre-test is needed to estimate the system transfer function; this is usually performed by running a low-level random test and using the so-called \( H_1 \) estimator

\[
\hat{H} = \hat{S}_{yu} \hat{S}_{uu}^\dagger
\]

where \( \hat{S}_{yu} \) and \( \hat{S}_{uu} \) are spectral density matrices estimated, for instance, via the Welch’s Averaged Periodogram.

The objective of MIMO Random Control vibration tests is to replicate a full Spectral Density Matrix (SDM), \( S_{yy}^{\text{ref}} \). Theoretically the test target could be directly achieved by sending the input drives that have the specified input spectral density matrix

\[
u(t) = \text{ifft}(U); \quad S_{uu} = E[UU'] = Z S_{yy}^{\text{ref}} Z^\dagger
\]

Nevertheless, due to the possible non-linear behaviour of the unit under test and noise in the measurements, the system estimated in the pre-test phase will inevitably differ from the actual one (\( HZ ≠ I \), where \( I \) is the \( ℓ \times ℓ \) identity matrix)

\[
S_{yy} = (HZ) S_{yy}^{\text{ref}} (HZ)^\dagger ≠ S_{yy}^{\text{ref}}
\]

and a control action is needed to reduce the error

\[
E = S_{yy}^{\text{ref}} - S_{yy}
\]

Nowadays innovative random vibration control strategies have been proposed by few authors (Smallwood 1999), (Peeters 2002), (Underwood 2001) and mainly rely on the work of Smallwood and Underwood. The control algorithms use the possibility of apply the so called Cholesky Decomposition to one of the spectral matrices in the game (either \( S_{yy}^{\text{ref}} \) or \( S_{uu} \)) and then iteratively correct the resultant Cholesky Factor. This operation is necessary to generate a set of multiple drives from the SDMs but is perfectly allowed because of their positive semi-definite nature (Smallwood 1993).

However, in the target definition phase, this property needs to be guaranteed for the reference matrix.
BUILDING THE MIMO RANDOM VIBRATION CONTROL MATRIX

The target of a MIMO Random Control test is a reference SDM $S_{yy}^{\text{ref}}$. The diagonal terms of $S_{yy}^{\text{ref}}$ are usually known PSD levels representing the test specifications. The off-diagonal terms (Cross Spectral Densities, CSDs) are often unknown quantities and the choice of providing the best solution is on the environmental test engineer. Most of the MIMO VibCo software have the possibility of defining element-wise the CSDs in terms of coherence and phase profiles. For computational reasons linked to the control process stability (Underwood 2002), coherence values of 0 and 1 are usually avoided. Typical values of low coherence and high coherence are $0.05 \div 0.08$ and $0.95 \div 0.98$, respectively. All the CSDs are then easily computed via

$$\text{CSD}_{ij} = \sqrt{\gamma_{ij}^2 \text{PSD}_i \text{PSD}_j} e^{i \phi_{ij}} \quad (6)$$

where $i$ is the imaginary unit and $i$ and $j$ are the $i$-th and the $j$-th control channels. The resulting full SDM needs to be positive semi-definite.

For a MIMO test the $\frac{\ell(\ell+1)}{2} - \ell$ off-diagonal terms need also to be set.

Filling in the MIMO matrix element by element in terms of $\frac{\ell(\ell+1)}{2} - \ell$ coherence and phase profiles could result in $S_{yy}^{\text{ref}}$ being non positive semi-definite, following the fact that none of the necessary and sufficient conditions have been taken into account in the completion process (Musella et al. 2017).

Nowadays there are few ways to fill in the MIMO matrix: use measured phases and coherences, assume that the responses are uncorrelated (setting low coherence through the frequency band of interest) or assume that the responses are in phases and fully correlated.

Figure 2: A six-DoFs actuator with two control channels. To replicate a roll motion, the two control channels need to be fully correlated and in phase opposition.

These methods are clearly not general since measured data could not be available and/or there might be the necessity of replicating partially or fully correlated out of phase responses. This is the case, for instance, of a Multi-Axial shaking table that needs to be used to excite mainly a single degree of freedom (DoF) as shown in Figure 2.

Since a standardization of the CSD terms is not meaningful (nor possible), the idea pursued by this work is to provide a method rather than a set of values to complete the target.
The starting point of the method are the given PSD levels that are assumed to be known terms (standard profiles). The CSD terms are unknown and need to be defined by choosing a set of meaningful coherences, in the sense that the resulting full SDM is positive semi-definite.

The solution is not unique.

Between the infinite solutions an interesting feature would be to pick the one that minimizes the required exciters power (Minimum Drives problem). An indicator for the overall power required by the exciters, for a Multi-Input system, can be the sum of the single independent drive powers. In algebraic terms, this quantity is actually related to the trace of the drives SDM (drives trace). The problem of minimizing the drives trace has been already tackled by Smallwood (Smallwood 2007). Unfortunately the method does not guarantee positive semi-definite matrices in case of more than two control channels (Musella et al. 2017). Putting aside the fact that such a target could never be reached (being not physical), this translates in the practical issue of making the method not suited for testing purposes, because the nowadays MIMO Random Control technologies make use of the Cholesky Decomposition to generate a set of multiple drives with the desired cross-correlations (Smallwood 1993).

It is understandable thus, that the first aspect to be taken into account in the target definition process is its physical realizability. In case of fully coherent responses, it is possible to obtain a positive semi-definite matrix by randomly selecting $\ell - 1$ phases and derive the remaining $\frac{\ell(\ell-3)}{2} + 1$ terms according to the Phase Pivoting method (Musella et al. 2017), as shown in Figure 3. This simple idea can be used to generalize the Extreme I/O Method proposed by Smallwood.

![Figure 3: Phase Pivoting principle. Giving the phases between two pairs of control channels, the phase between the remaining pair is automatically defined.](image)

**EXTREME DRIVES METHOD**

The idea of the Extreme I/O Method proposed by Smallwood is to find, with fixed PSD levels, the set of coherences and phases between the control channels that minimize (maximize) the trace of the drives SDM (Smallwood 2007), i.e.

$$\text{given } \text{diag}(S_{yy}^{\text{ref}}) \text{ find } S_{yy}^{\text{ref}} \text{ so that } \text{Tr}(S_{uu} = ZS_{yy}^{\text{ref}}Z') \text{ is minimum(maximum)}$$

By considering the basics equations of Linear Time Invariant systems
It is possible to write the diagonal terms of eq. as

\[ S_{\text{ref}} = HS_{uu}H' \]  

\[ S_{uu} = ZS_{yy}^\text{ref}Z' \]  

(7)  

(8)

The trace of the drives SDM is by definition the sum of the diagonal terms

\[ P = \text{Tr}(S_{uu}) = \sum_{i=1}^{m} \left( \sum_{j=1}^{\ell} \sum_{k=1}^{\ell} Z_{ik} S_{yy}^j Z_{kj} \right) = \sum_{i=1}^{\ell} \sum_{j=1}^{\ell} S_{yy}^j \sum_{i=1}^{m} Z_{ik} Z_{ij} \]

(9)  

(10)

By defining the hermitian matrix \( F = Z'Z \) (carrying information about the system behaviour) and noticing that \( S_{yy}^\text{ref} \) needs to be hermitian too, eq. (10) can be rewritten as

\[ P = \sum_{j=1}^{\ell} S_{yy}^j F_{jj} + 2 \sum_{i=1}^{\ell-1} \sum_{l=j+1}^{\ell} |S_{yy}^j| |F_{ij}| \cos(\phi_{ij} - \theta_{ij}) = \]

\[ = \sum_{j=1}^{\ell} S_{yy}^j F_{jj} + 2 \sum_{i=1}^{\ell-1} \sum_{l=j+1}^{\ell} \gamma_{ji} \sqrt{S_{yy}^j S_{yy}^l} |F_{ij}| \cos(\phi_{ij} - \theta_{ij}) \]

(11)

Where \( \phi_{ij} \) and \( \theta_{ij} \) are the phase angles (in radians) of the ji-th off diagonal term of \( S_{yy}^\text{ref} \) and \( F \), respectively. Since the PSD terms are fixed, for a given structure, the first term on the right hand side of eq. (11) is always positive and fixed. The second term contains the unknown quantities \( \phi_{ij} \) and \( \gamma_{ij} \) and can be negative because of the cosines contained in the double sum. The theoretical minimum (maximum) trace, is obtained when the second term is minimum (maximum), i.e. when the coherences are all unitary and the cosines all equal -1 (1). This condition can be easily superimposed by choosing \( \phi_{ij} = \theta_{ij} + \pi \) (\( \phi_{ij} = \theta_{ij} \)) as phase shift between the ij-th control channels.

The main issue of the method proposed by Smallwood derives by the fact that fully coherent responses need to follow the Phase Pivoting principle (Musella et al. 2017). This means that out of the \( \frac{\ell(\ell+1)}{2} - \ell \) relative phases to be set, just \( \ell - 1 \) can be independently picked, the remaining \( \frac{\ell(\ell-3)}{2} + 1 \) terms need to be pivoted from the ones set.

In order to solve this issue and to obtain the realizable target that still requires the minimum drives, the idea is to choose ad-hoc the pivot phases. By looking at eq. (11), the term that contributes to minimize (maximize) the drives trace is the double sum in the right hand side.

This term is a summation of contributions coming from the ij-th cross terms with different magnitudes \( 2\gamma_{ji} \sqrt{S_{yy}^j S_{yy}^l} |F_{ij}| \) scaled by the terms \( \cos(\phi_{ij} - \theta_{ij}) \). The best solution would then be to use as phase pivots the \( \ell - 1 \) terms with the biggest amplitude (i.e. assign their phases according to the Smallwood’s method), that thus will decrease by the maximum amount the fixed positive term appearing in eq. (12). The phases of the remaining terms will be pivoted from
the remaining ones. An easier from the implementation point of view, but sub-optimal solution would be to keep the phases \( \phi_{1(j=1:\ell)} = \theta_{1(j=1:\ell)} + \pi \) and use then the first row as phase pivot (Musella et al. 2017).

**SIMULATION AND TEST RESULTS**

The algorithm has been tested first in a simulation environment. The system tested comes from a 3-controls 3-drives electromechanical equivalent of a real system, the so called FRF Box\textsuperscript{TM}. The system’s FRFs are shown in Figure 4. Four tests have been run by just changing the off-diagonal terms of the reference matrix, as shown in Table 1.

<table>
<thead>
<tr>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phases [deg]</td>
<td>Min drives</td>
<td>Max drives</td>
<td>( \phi_{12} = 70, \phi_{13} = -70 )</td>
</tr>
<tr>
<td>Coherences [-]</td>
<td>0.98</td>
<td>0.98</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The reference profiles and the simulations results (in terms of control SDM) are shown in Figure 5 for the cases where the developed method has been used to fill in the reference matrix. It is worth to point out that for the different runs (i) the overall PSD levels for the control channels are fixed (just the phases and coherences between pairs of control channels change) and (ii) the reference matrices are positive semi-definite being the SDMs output of an actual control process. Figure 6 shows the drives traces, used to quantify the drives power required for each run. As expected, the drives traces of the simulations run by setting the reference matrix with the Extreme Drives Method, define a range containing the drive traces coming from the other settings. This result has shown to be general and not limited to the few cases considered in this application case.

**Figure 4:** FRFs of the 3-control 3-drive system used for the simulations.

The very promising results coming from the simulations motivated a series of experiments that has been carried out on a 6-DoFs hydraulic shaking system, the so-called CUBE\textsuperscript{TM} (De Coninck et al. 2003), (Peeters et al. 2003). The six actuators are arranged in pairs, one pair on each of X,Y and Z-axis, allowing a full 6-DoFs motion in modal control: the six drives (Output 1,2,3,4,5 and
6) are therefore the three shaker’s translations and the three rotations (Tx, Ty, Tz, Roll, Pitch and Yaw, respectively).

The test set-up and the two different sensors configurations tested are shown in Figure 7. In the Configuration 1 (the FRFs between accelerations control channels and drives voltages shown are in Figure 8) the sensors are placed at three corners of the shaker’s top surface; in the Configuration 2 six accelerometers are placed in correspondence of the six actuators’ heads. For the sake of simplicity just the most representative FRFs are shown in Figure 9, out of the entire FRF matrix. In all the performed tests the required levels are 0.4 g$_{RMS}$ white-pink noise in the band [18.75 - 150] Hz, for all the channels. These levels will be fixed for all the tests, being representative for the test requirements. The frequency resolution is 3.125 Hz. For safety reasons all the tests are performed in three sequential steps (-9 dB, -6 dB, -3 dB and full level). All the results are shown at normal ends, meaning that the tests have been safely (no abort whatsoever) run for 1 minute at the full level.

![Figure 5: Control SDM (solid blue) by using the Extreme Drives Targets, (a) Minimum Drives and (b) Maximum Drives. In solid green, solid red the reference profiles and the abort and alarm limits, respectively.](image)

Before the actual test a system verification can be performed in order to check, with the information coming from the System Identification, the predicted levels for the responses and the drives. Figure 10 clearly illustrates how, setting the phases with the proposed Minimum Drives method, the exciters’ $V_{RMS}$ drastically decreases. An uniform reduction of the single drives is not guaranteed by the method that actually optimizes the sum of the required power (in Figure 10 is also reported the sum of the RMS, giving an indication of the actual reduced quantity). On the other hand setting the phases with the Maximum Drives method returns much higher predicted drives levels. The figure also illustrates that uncorrelated responses (i.e. low coherences) require more energy than fully correlated responses with an ad-hoc choice of phases.
Figure 6: Drives traces by setting different CSDs for the MIMO Random Tests. The PSDs are fixed levels.

In Figure 7 the SDM of the control channels is shown for the normal end test performed by setting the reference matrix with the Minimum Drives Method. Similar results are obtained when running a test with the Maximum Drives Target.
Figure 8: Configuration 1 FRFs from voltages to accelerations.

Figure 9: Configuration 2, most significant FRFs from voltages to accelerations.

Figure 10: Predicted drives $V_{RMS}$ from a system’s verification before running the actual test.

The focus, more then on the achieved control results, needs however to go on the required drives power. The drives traces calculated for these tests are shown in Figure 12, together with the single drives’ PSDs. There is clearly a big gap in terms of achieved drives traces obtained using the Extreme Drives method (minimum and maximum) for setting the off-diagonal terms of the target. This gap reflects the overall behavior of reducing the drives PSDs, predicted in the system verification phase. In Figure 12 additional simulations results, marked with dotted lines, are also included to show a very good agreement between test results and simulations, the mismatch to be attributed to the random process and to the control performances mismatch between the off-the-shelf software used for the tests and the implementation of a Multi-Axial Random Control used for the simulations.
It is worth to notice that neither zero phases nor low coherences are generally returning a target with Minimum Drives. In the Extreme Drives targets, the phase information between pairs of control channels is actually retrieved by taking information coming from the identified system: any other phase choice (most probably) and coherence (certainly) will generally return drives traces that fall in between the Extreme Drives ones, as also shown from Figure 12. For three control channels the algorithm’s implementation has been straight forward and guarantees the exact solution to the problem. A generalization of the Phase Pivoting method (included in the procedure to derive the Extreme Drives reference matrices) for $\ell$-control channels ($\ell > 3$) is quite challenging (Musella et al. 2017). A first approximate tested solution has been to use as a phase pivot the phases of the first row coming from the Smallwood’s reference matrix.

![Figure 11: SDM of the control channels for the Configuration 1, Minimum Drives Reference matrix set as test target.](image)

Figure 11 clearly shows that even this approximate solution gives a clear advantage in terms of achievable test levels. In these demo case the test PSDs have been scaled up to get a $3 g_{RMS}$ level for each of the control channels. These levels are not achievable in case the reference SDM is filled in to get fully uncorrelated responses, because the voltage to be sent out from the acquisition system’s DAC that controls the pitch is out of the safety threshold (DAC overload). The solution proposed is thus to set the phases between the control channels with the Minimum Drives Method. In this case, the voltage predicted for the pitch DAC is approximately two times lower and 1 V below the DAC overload threshold. The figure also shows that the situation gets worse by setting the phases with the Maximum Drive Method.

Having shown the capability of the approach, the actual tests have been run by setting for all the control channels the same response levels used for the tests with the configuration 1. Figure 14
and Figure 15 show the control results in terms of PSDs and phases for the test run by setting the reference CSDs with the first row approximation of the Minimum Drives Method. This was the only method implemented at the time of the test for the general \( \ell \)-controls case. A further improvement has then been developed to return the best approximation. The algorithm consists in sequentially picking \( \ell - 1 \) phases, for each spectral lines, out of the \( \frac{\ell(\ell+1)}{2} + \ell \) coming from the Smallwood’s Method and calculating the drives traces for each combination. At each spectral line is then chosen as phase pivot the combination of element that will return the extreme trace.

Simulations have been run by setting the target by using this method and the resulting drive traces together with the test results obtained by using the first row approximation are shown in Figure 16.

**Figure 12:** Drives PSDs and drives traces for the Extreme Drives Method application (Configuration 1) and simulated results.

**Figure 13:** Predicted drives \( V_{RMS} \) for a 3 \( g_{RMS} \) test.
The advantage in terms of power required in the overall band is substantial even if it is not guaranteed that a minimum will be achieved for each spectral line. This is shown for instance by decreasing the coherence of all the channels and keeping the phases equal to the ones retrieved with the maximum drives. The overall level decreases (as expected) but there are frequency lines where the drives trace is higher than the maximum trace. A solution has been found, as explained, by selecting the phase pivot that at each spectral line returns the most extreme trace. As shown by Figure 16, the range spanned by the drive traces obtained by using this method is much wider than the previously achieved result, is included in the range. It is also possible to see how the intermediate cases considered in this example (simulated low coherence and the test results from the maximum drives target with coherence reduced to 0.8) are limited by the two curves. In the figure’s legend the RMS values of the drives traces are also reported in order to quantify with one indicator the overall power in the entire band. It is clear from these values how a minimization (maximization) of the drives power is already achieved with the Minimum Drives Method approximated by using the first row as a phase pivot but that a drastic reduction is achieved by choosing the best approximation for each spectral line. Other typical sets of coherence and phases will return drive traces with RMS greater (smaller) than the one obtained with the developed method.

![Figure 14: Control channels PSDs, Minimum Drives Reference matrix set as test target.](image1)

![Figure 15: Control channels phases (detail).](image2)
CONCLUSIONS

This work proposes the *Extreme Drives Method* as an answer to one of the most challenging questions in the Multi-Axial testing community: how to fill in the MIMO Random Control reference matrix so that it is physically realizable and the target levels are the ones given as test specification? By making use of information retrieved from the system identification (a step performed anyhow before the actual MIMO Random Control test) the proposed method returns the phase and the coherence profiles that can minimize (maximize) the sum of the required drives power. The capability of the method are demonstrated first in a simulated environment and then via actual tests on a six-axis shaker in two different configurations. The results show the capability either in terms of physical realizability of the targets retrieved and in terms of drive powers reduction. During the tests, in fact, it has been recorded a reduction. The implementation limitation of the method for the general case have also been overcome by proposing a best approximation approach that showed via simulation to further improve the already satisfactory results of the previous implementation.

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