Thesis submitted in fulfilment of the requirements for the award of the degree of Doctor of Engineering Sciences (Doctor in de ingenieurswetenschappen)

NEW ACTUATION PARADIGMS WITH HIGH EFFICIENCY FOR VARIABLE LOAD AT VARYING SPEED

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March 2018

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Acknowledgments

Years go by so fast... Especially if they are filled with hard work, exciting friendships and personal growth. The past four years have been truly amazing, and I have many people to thank for that.

First of all, I would like to express my gratitude to Dirk and Bram for offering me the opportunity to pursue a PhD in the Robotics and Multibody Mechanics group. Without your help, I would never have obtained the FWO fellowship which provided me financial support during my PhD, and which was an important condition for me to leave industry for academia. I have never regretted this decision for a single moment. You gave me the support, but also the freedom I so badly need to be the most productive version of myself.

I would like to thank my colleagues Louis, Kevin, René, Karen, Tomislav, Marta, Pierre, Carlos, Matthias, Svetlana, Victor, Laura, Branko, Seppe, Kelly, Dianbiao, Joris, Stein, Elias, Albert, Huong, Ilia, Maarten, Joe, Greet, Pablo, Elahe, Marina, Stefanie, Ward and Tim, for creating such a great working atmosphere, but also for the coffee breaks, receptions, dinners and occasional poker nights. Some of you I have come to consider not just as colleagues, but also as friends. Thanks also to our technicians Marnix, Marc, Geoffrey and Jean-Paul and to our secretaries Jenny and Birgit.

A special mention goes to my co-authors. In particular, I would like to thank Raphaël and Long, who were usually the two first people I would bother when I encountered any practical issues or when I needed someone to bounce ideas off. Thanks also to Glenn, for passing on his ideas and energy, which has been a great motivation, Pablo, for his analytic mind and the great discussions on gears, David and Bryan, for their motivation to improve our working efficiency, and Joost, for the jokes, but also for being a reliable co-worker. And finally, I would like to thank Philipp, not only for co-authoring several papers, but also for inviting me to TU Darmstadt and hosting me at his institution for four months.

Much of my time at the VUB was spent with some great people, who have been my friends since the Bachelor’s/Master’s or became friends during this PhD. Yoachim, Wim, Ruben, Kim, Charlotte, Liesbeth and all the others who have become a part of our continuously growing lunch group: thank you for all the durak games, the many memorable evenings in ’t Complex, the occasional karaoke nights, and so many other
things. The past four years have truly been great, and you have been largely responsible for that.

Another important part of these past four years was Edukado. Before Edukado, I could not imagine that young and inexperienced university students, like me at that time, could make a difference in the world. Now, in the fall of 2017, we have helped hundreds of children gain access to affordable education by constructing school buildings in Nepal, Malawi and Sierra Leone. Edukado has taught me that we all have greatness inside of us – all it takes is a good team to bring it out. Aushim, Camille, Peter and Geert, thank you for allowing me to be a part of such a team. I hope that, in the years to come, we will continue inspiring students and helping people in developing countries.

To Kevin, Thibault and Jens: thank you for your tireless efforts to convince me to spend more time with you. I feel extremely lucky to have such loyal friends, to whom I can always turn back, even after being separated for long times as a result of distance, work, or other issues. You are like brothers to me.

Finally, I would like to thank my parents for taking care of me, for supporting my education, and for all other things you have done for me throughout the years. Without you, I would never have made it this far.
Summary

Energy efficiency is an important concern in the field of robotics. In order to meet their high energy demands, today’s mobile robots are equipped with large and expensive battery packs. Furthermore, the presence of robots in our society is increasing, and so are the costs related to their energy consumption. Analyzing and improving the efficiency of robots allows us to anticipate their future impact on our energy needs. In this work, we focus on the electrical energy consumption of the actuation system of such robots.

An analysis of the state of the art in efficient actuation reveals that model-based optimization is a common technique to optimize designs for energy efficiency. Interestingly, there are considerable differences in the way how electromechanical actuators are modeled. Moreover, these models are limited to the mechanical domain. For this reason, the first part of this work discusses how modeling affects the energy-efficient design of actuators. Our analysis addresses friction, nonlinear gearbox efficiency, speed- and load-dependent motor losses and losses in the drive circuitry. These aspects, which are rarely considered in the optimization of energy-efficient actuators, are shown to have a considerable effect on the actuator’s power and energy consumption. Drivetrain inertia and gearbox efficiency, in particular, play an important role in robotic applications.

The introduction of elastic elements is one way to make an actuator more energy-efficient. Elastic elements are energy buffers, which allow for a direct exchange of energy with the load. Consequently, the power flow in lossy components can be reduced, and the overall efficiency of the actuator will be higher. In this work, we compare the two basic elastic actuator configurations: Series Elastic Actuation (SEA) and Parallel Elastic Actuation (PEA). It is widely known that SEA allows for a reduction of the motor’s speed requirements, while PEA allows for a reduction of torque. From our tests on a pendulum, we conclude that SEA exhibits high-frequency resonance, which leads to an additional decrease of motor torque in highly dynamic applications. Parallel elastic elements, on the other hand, can be used to compensate for static torques. An interesting conclusion of this work is the strong impact of motor and gearbox losses on the optimal spring stiffness. Motor limitations also have a strong influence on the selection of the optimal spring configuration, as demonstrated in a case study on the design of an active ankle prosthesis.

The second energy-efficient actuation paradigm studied in this work is redundancy.
In a redundant actuator, the required output power can be distributed over the two motors. With an appropriate control strategy, the motors can be used in an energy-optimal way, and the overall efficiency of the actuator can be increased. From the different types of redundancy, kinematic redundancy is shown to be a promising option for robotics. Kinematically redundant actuators offer the possibility of changing the reflected inertia, allowing them to achieve high accelerations, even if a high torque is required.

Finally, we study whether series elasticity and redundancy can be combined in a single actuator. Applying this combination in robotics is challenging, especially in terms of control. We demonstrate that a series elastic dual-motor actuator concept can be applied to a hopping robot by implementing and validating a general control framework that generates repetitive hopping, while effectively managing the actuator’s redundant degree of freedom. Considering that hopping is a subtask of legged locomotion, this work paves the way for compliant-redundant actuation of legged robots, prostheses and exoskeletons.

In conclusion, this work demonstrates the potential of elastic and redundant actuation for the reduction of a robot’s electrical energy consumption. To fulfill this potential, a good match is required between the task and the design of the actuator. Creating this match is not an easy task, because the constraints imposed by the additional springs or motors complicate the design process. We believe that the insights from this work can serve as a basis for the successful implementation of elasticity and redundancy in actuators for robotics.
Nomenclature

List of symbols

γ  Speed ratio (Dual-Motor Actuator)
ΔV_{\text{brush}}  Brush-drop loss
η  Efficiency
θ  Angle
θ_a, θ_0  Amplitude of oscillation (pendulum)
θ_o  Offset angle (pendulum); output angle
φ  Knee angle (MARCO Hopper)
Π  Cost function
ρ  Planetary differential ratio
ν  Viscous friction coefficient
ω  Steady-state speed; frequency
B  Brake function
C  Efficiency function
E  Energy (consumption)
F  Force
g  Gravitational acceleration (9.81 m/s²)
k  Spring stiffness
$k_t$ Torque constant of motor

$k_b$ Speed constant of motor

$I$ Current

$i$ Speed ratio (Dual-Motor Actuator)

$J$ Inertia (as parameter); Jacobian (as matrix)

$L$ Motor terminal inductance

$l$ Distance from pendulum axis to center of mass; length

$M, m$ Mass

$n$ Gear ratio

$P$ Power

$R$ Total motor winding resistance

$r$ Radius

$t$ Time

$T_C$ Coulomb friction coefficient

$T$ Torque

$U$ Motor voltage

**List of subscripts**

$a$ Antiresonance

$abs$ Absolute value

$buffer$ Energy buffer

$C$ Carrier

$cable$ Bowden cable

$Coulomb$ Coulomb friction

$d$ Desired value (control)

$drive$ Drive (circuitry)

$elec$ Electrical
eq  Equilibrium position/angle

gearbox  Transmission, gearbox

imp  Imposed

Joule  Joule loss

l  Load

m  Motor

max  Maximum (limit) value

mech  Mechanical

nl  No-load

nom  Nominal value

o, out  Output

p  Parallel spring

R  Ring

ref  Reference drivetrain

refl  Reflected (inertia)

rl  Resonance of link subsystem

RMS  Root Mean Square value

rs  Resonance of system

S  Sun

s  Series spring

source  Power supply

sp  Spindle

tr  Transmission, gearbox

v  Virtual (Virtual Model Control)

viscous  Viscous friction
## List of abbreviations

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<td>First Quadrant Constant Efficiency approach</td>
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<tr>
<td>4QCE</td>
<td>Four Quadrant Constant Efficiency approach</td>
</tr>
<tr>
<td>4QCEI</td>
<td>Four Quadrant Constant Efficiency approach with motor and gearbox Inertia</td>
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<tr>
<td>COG</td>
<td>Center Of Gravity</td>
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<tr>
<td>CPT</td>
<td>Counts Per Turn</td>
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<tr>
<td>CVT</td>
<td>Continuously Variable Transmission</td>
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<tr>
<td>DC</td>
<td>Direct Current</td>
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<td>DMA</td>
<td>Dual-Motor Actuator</td>
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<td>EEC</td>
<td>Electrical Energy Consumption</td>
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<td>EPP</td>
<td>Electrical Peak Power</td>
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<td>FMM</td>
<td>Full DC Motor Model approach</td>
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<td>IVT</td>
<td>Infinitely Variable Transmission</td>
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<td>MEC</td>
<td>Mechanical Energy Consumption</td>
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<td>MPP</td>
<td>Mechanical Peak Power</td>
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<td>OC</td>
<td>Optimal Control</td>
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<tr>
<td>PEA</td>
<td>Parallel Elastic Actuator</td>
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<td>RA</td>
<td>Rigid (stiff) actuator</td>
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<tr>
<td>SEA</td>
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<td>SEDMA</td>
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Chapter 1

Introduction

1.1 Why study energy efficiency?

Autonomous lawn mowers and vacuum cleaners doing our daily household chores. Robotic legs and arms which enable amputees to walk or manipulate objects like a normal person. Exoskeletons that allow paraplegics to move their legs again. Humanoid robots operating in human environments, opening doors, climbing ladders and driving utility vehicles. A robotic dog walking to the kitchen to throw an empty soda can in the trash bin. Self-driving cars bringing us safely to our destination. A drone delivering a package directly to our homes. All these amazing machines exist today, and will most likely become a part of our daily lives in the near future – if that is not the case already. And more exciting things are ahead, because the ongoing revolution in robotics is showing no signs of slowing down. Industrial robot sales increase yearly by approximately 15% [102], while service robots saw increases of 16% (personal and domestic) and 25% (professional) between 2014 and 2015 [103]. And with innovations being presented on a regular basis, new robot designs will most likely flood the market in the coming years, increasing the sales volumes at an even faster rate [141].

Robots are energy-consuming devices. As their numbers are growing, so is the need for an effective management of their energy consumption. Electric motors, which are often used in robots, are typically responsible for 2/3 of the industrial power consumption of a country, or 40% of their overall power consumption [193]. This translates to around 6040 Mt of CO$_2$ emissions per year, a number which is expected to double by 2030 [236]. Currently, most of this energy is consumed by large motors driving fans, pumps, etc [107]. In contrast, robots in comparison only have a minor contribution to the global energy consumption. In Germany, for example, they account for no more than 1% of the total electrical energy consumed in industry [107]. Therefore, as it stands today, improving the energy of robots will only have a minor impact on the global energy consumption [120].

Nevertheless, there are several reasons to pursue more efficient robots. Two types of
1.1 Why study energy efficiency?

**Figure 1.1:** Advantages of energy reduction for mobile robots.

robots in particular can benefit from an increase in energy efficiency: mobile robots and industrial robots.

### 1.1.1 Mobile robots

Figure 1.1 shows a schematic of the most important effects that energy efficiency has on a mobile robot. Firstly, energy losses usually present themselves as a heat loss that needs to be dissipated. In many cases, they also cause vibrations, resulting in a noisy operation of the actuator. These two disadvantages are very unwanted, especially if the robot is intended for use in an environment with humans. In the specific case of wearable robotics, heating and vibrations can cause discomfort to the user, while noise will increase the notion of wearing a robotic device – contrary to the general design goal of such devices. Secondly, the device will have a longer autonomy, meaning that the batteries need less recharging. A reduction in energy losses also allows for a smaller battery pack to be used, as mentioned above. And finally, a more efficient drivetrain can consist of smaller components (motors, gearboxes, drive electronics) because they no longer need to deliver the extra power associated with the energy losses. The reduction of battery and actuator size is very important for wearable robotics, where the robotic device is often desired to be as inconspicuous as possible. Furthermore, a reduction in size usually goes hand in hand with a reduction of weight which, for most mobile robots, is a major determinant for the energetic cost. Consequently, a reduced actuator weight will make the robot and its actuators more efficient again, further increasing the benefits that come with an improved efficiency. In conclusion, it is easy to understand that energy efficiency is a key aspect for mobile robots.
1.1.2 Industrial robots

In 2016, around 1.6 million industrial robots were operational, a number which is expected to increase yearly by 15%. In the automotive industry, especially, robots have become a key component of the production line. Although robots currently consume only 8% of the energy required for production in this sector [143], increasing robot efficiency can still lead to high cost reductions overall. An automotive production plant with a daily output of 1,000 vehicles easily consumes several 100,000 MWh of electricity per year – as much as a medium-sized town [191]. Considering that collaborative robots might soon take their place in the production line alongside human workers, the share of robots in the overall energy consumption can only be expected to increase. The energy consumed by robots is therefore becoming a growing concern for many companies [120]. This is acknowledged by leading players in the robot market, who are heavily involved in the research of energy-efficient robots and control strategies [1, 2].

1.2 What makes achieving energy efficiency so hard?

Most machines run at a constant speed and a constant load. Very often, these machines consist of an electric motor. If sized to the task, brushless DC motors can achieve efficiencies of 85–90% and more, while brushed DC motors with gearing have a maximum efficiency of around 75–80% [101]. The efficiency of electric motors is, however, a complex function of speed and torque, and the maximum efficiency can only be reached for a certain range of output speeds and torques. This is illustrated by Fig. 1.2, which shows how efficiency can drop to very low values at low speeds and low torques.

The speed- and torque-dependent efficiency of motors has serious consequences for their energy consumption. When motors are oversized – something which is often the case in industry [37] – they do not operate at their optimal efficiency. This is a common issue in robotics, where the dynamic motions force motors to deliver varying torques over a wide speed range. As a result, some of its operating points will inevitably lie in the low-efficiency regions, with a detrimental impact on the overall energy consumption.

Robots that interact with their environment are particularly difficult to design energy-efficiently. Tasks such as turning knobs or manipulating high payloads typically require high torques from the robot’s actuators at low output speeds. When the payload is removed and the manipulator is allowed to move freely, however, the requirement changes to delivering high speeds at low torques. The same problem exists in legged robotics. During the stance phase, the leg needs to carry the robot’s weight, requiring high motor torques. When the foot is lifted from the ground (swing phase), the required motor torque drops considerably since the leg is now only moving its own inertia, but the motors need to work at higher speeds. In both cases, the loaded phase and the no-load phase lead to conflicting torque-speed requirements.

If such a robot is actuated by an electric motor with gear reduction, the motor will mostly be operated in what, according to Fig. 1.2, are the least efficient regions of its efficiency map. In the MIT Cheetah robot, for example, up to 68% of the total energy was
1.2 What makes achieving energy efficiency so hard?

Figure 1.2: Efficiency map of a brushless DC motor (reprinted from [52]).
consumed by heating of the motor windings [199]. Designs focusing on the reduction of motor losses can significantly improve the overall efficiency of a device. With such an approach, Brown and Ulsoy managed to decrease the energy consumption of a passive-assist device by 25%. No less than 90% of the energy savings was attributed to a more efficient use of the motor [34]. These examples illustrate how important the motor is for the overall energy efficiency of a robot.

1.3 How can actuators be made more energy-efficient?

In this work, we study two energy-efficient actuation paradigms: the introduction of energy buffers and the creation of redundancy.

1.3.1 Energy buffers

Energy buffers are elements that are able to store and release energy for a certain amount of time. The battery – a chemical energy buffer – is mostly used for its long-term energy storage capabilities, i.e., as a power source. For short-term energy storage, two types of energy buffers can be applied in actuators: electrical energy buffers (inductors, capacitors) and mechanical energy buffers (springs, inertias). The use of ultracapacitors as an additional power source is common in electric vehicles [74, 162, 131]. In robotics, the latter have a higher potential for energy savings. This can easily be explained in a conceptual way.

Consider a typical actuator consisting of a controller, motor and gearbox (Fig. 1.3a). Each of these components has its efficiency and, as a result, energy will be dissipated when the actuator is delivering power. Roughly, one can assume these losses to be proportional to the power flow through the component. This means that there are two ways of increasing the energy efficiency: either by improving the efficiency of the individual components, or by decreasing the power flow through the components.

In most machines – pumps, fans, compressors, etc. – the motor delivers power to the load. This is commonly referred to as a “positive power flow”. While the power is flowing from the battery to the load, it accumulates the losses of each of the components on its path, reducing the available power at the output. In robotics, however, power flow often takes place in two directions: from the power supply to the load, but also from the load to the power supply (negative power flow). Typically, this is the result of a recirculation of energy, where energy coming from the power supply is temporarily stored in the load as kinetic energy, and then released back at a later time. In such a circular power flow, all components are traversed twice, meaning that energy is dissipated twice, too. This is the reason why, in general, it is best to avoid reverse power flows in actuators, even if negative power can be regenerated.

Energy buffers can be a solution to this problem. In Fig. 1.3b, a spring is added between the gearbox and the load. Now, the load can exchange energy with the spring

---

1 In chapter 2, we will give a more detailed analysis of how losses relate to speed, torque and power.
1.3 How can actuators be made more energy-efficient?

**Figure 1.3:** Power flows in typical actuator architectures. (a) Stiff actuator with motor and gearbox. (b) Series Elastic Actuator with a spring as energy buffer. As illustrated by this figure, the elastic element allows for a direct energy exchange with the load.
– an energy buffer – instead of with the power supply. By doing so, the power flow from power supply to load and vice versa is decreased, and the losses associated with the components in between are avoided. Of course, one may also choose a capacitor – the electrical equivalent of a spring – as an energy buffer. In this case, the capacitor would be placed in between the motor and the drive circuit. The load can then exchange energy with the capacitor, and losses in the drive electronics and battery are avoided. Motor and gearbox losses, however, still remain. These losses, which can be reduced by using a spring instead of a capacitor, are very often a great source of energy losses in an actuated system.

Based on this discussion, it is easy to see how the potential for energetic energy savings is increased by placing the energy buffer as closely to the load as possible. This makes mechanical buffers such as springs and, in some cases, flywheels, the ideal energy buffers for robotic applications.

1.3.2 Redundant actuators

If a single motor is used to move a joint, a direct relationship exists between the torque and speed of the motor and that of the joint. This means that, for a specific motor-gearbox combination, the operating points are defined uniquely – and so are the corresponding efficiencies on the efficiency map. While energy buffers can be used to manipulate this relationship, their impact is directly related to the output trajectory. This lack of versatility is, in many cases a problem.

A way to gain control over the motor’s operating points is to add a second motor to the actuated joint. By doing so, an extra degree of freedom is created, which can be exploited to divide the required energy over both motors. This gives the user some degree of control over the operating points of both motors and, consequently, over their efficiencies. By choosing the distribution in such a way that the overall efficiency is maximized, the actuator can be made more efficient than the single-motor equivalent. This will be demonstrated in chapter 4.

Depending on how the motors are coupled to the output, different types of redundancy can be achieved. When two or more motors are attached to the same output shaft, their torques add up, and the actuator is said to be statically redundant. By coupling them through a differential, the output speed will be a weighted sum of the motor speeds. This is referred to as kinematic redundancy. Another possibility is to use the secondary motor to create a variable transmission, a concept which has especially been successful in the automotive industry [146]. And finally, variable stiffness actuators, which are essentially series elastic actuators of which the stiffness is varied by a secondary motor, can also be considered as redundant actuators.
1.4 Research objectives and scope

This objective of this thesis is to analyze whether energy buffers and redundancy have the potential of improving an actuator’s energy efficiency for typical robotic tasks. This very general research objective is strongly linked to several fundamental research questions, such as

- How do motor efficiency, gearbox losses and friction affect the energy consumption of actuators in robotics?
- What is the role of negative power in robotics?
- How complex must the model of the actuated system be in order to provide a reasonable estimate of the power and energy consumption?
- Can the efficiency of actuators be improved by applying motor and gearbox models in model-based optimizations?
- How does the task affect the energy efficiency of the actuator?
- How can model-based optimizations be used to find the most energy-efficient actuator designs?
- How can elastic elements contribute to energy efficiency?
- Can redundancy be exploited to make actuators more energy-efficient?
- Can series elasticity be combined with redundancy?

Answering these questions requires a broad knowledge of topics such as the natural dynamics of actuators and mechanical systems, control of actuators, drive circuits, friction and speed- and load-dependent motor and gearbox efficiency. Most of these subjects have been studied extensively in literature, but the knowledge is scattered around different fields of research, often transcending the boundaries of robotics. Gathering sufficient knowledge about all these topics is a time-consuming task, which explains why many analyses in literature are reduced to a linear system in the mechanical domain. This has given rise to several misconceptions and oversimplified assumptions in terms of their impact on energy efficiency in robotics. By studying the electrical energy consumption of actuators and by including nonlinear losses in our analyses, we hope to generate more accurate insights regarding the real energy efficiency of different actuation concepts.

Different actuation technologies are used in robotics. When macro-motion is required, the most common are hydraulics, pneumatics and electric motors [98]. In this thesis, we only consider actuators which consist of electric motors. Electric motors are an excellent choice for wearable robots because of their portability, as they do not require a complex network of tubes like hydraulic and pneumatic actuators. Furthermore, they are reliable, require little maintenance and operate silently in comparison with hydraulic and pneumatic actuators [39]. In terms of power range, this work targets applications such as
active lower-limb prosthetics and exoskeletons. Here, motors typically have power ratings of around 50-250 W, the range of powers needed for human activities such as walking [244, 43] and stair ascent and descent [139, 185]. Another important consideration is the power supply which, for wearable devices, usually delivers a low DC voltage (max. 50 V) in accordance with European regulations [61]. Based on these demands, permanent magnet DC motors are a logical choice in terms of actuation technology [147, 101]. Literature surveys indicate that permanent magnet DC motors are, indeed, the preferred actuation technology for active upper-limb and lower-limb orthoses [233, 45, 224]. Both brushed and brushless DC motors are being used in today’s state-of-the-art designs of actuated prosthetics and exoskeletons [42].

1.5 Outline of the thesis

In chapter 2 of this thesis, we give an overview of the different losses in drivetrains with electric motors. We explain how the losses can be modeled, and how these models can be applied to actuators in robotics.

In chapter 3, we discuss energy buffers in actuated systems by analyzing the two most basic spring configurations: series and parallel elastic actuation. These two configurations, while having fundamentally different equations of motion, can both be used to create an energy-efficient actuator. In this work, we will assess the impact of friction and motor losses on the optimal configuration and spring stiffnesses. Furthermore, we will analyze how the type of load – slowly-varying versus highly dynamic – affects the optimal design.

Chapter 4 features a discussion of redundant actuation principles and their potential benefits in terms of energy-efficiency. A detailed study on a kinematically redundant actuator provides insights about the advantages and disadvantages of such actuator concepts. We explain how such an actuator can be combined with locking mechanisms to improve the actuator’s efficiency map. Furthermore, we explore how such a kinematically redundant actuator can provide a solution to the conflicting torque-speed requirements in dynamic applications.

In chapter 5, we demonstrate that a kinematically redundant actuator can be combined with elastic elements by applying this concept to a robotic hopper. We also present a framework for the control of such actuators, which is validated in experiments on a hopper.

A discussion of the research questions and directions for future work are given in chapter 6. Finally, in chapter 7, we present some general conclusions about the energy-efficient use of energy buffers and the exploitation of redundancy.
Chapter 2

Modeling the power consumption of actuators

2.1 Introduction

With the rise of autonomous and mobile robots, the field of robotics has seen an increased interest in energy efficiency. In recent years, several papers have appeared in which actuator designs are compared by simulating their energy consumption for a specific task [87, 135, 95, 151, 240, 250]. In other works, model-based optimization is used as a tool to generate energy-optimal trajectories or controllers [173, 225, 32, 76, 134]. In both cases, a representative model is an important condition to obtain correct or optimal solutions. A review of recent publications reveals that very different models and methods are in use for the calculation of energy consumption. Many authors base their calculations entirely on mechanical energy consumption at the output shaft of the gearbox [221, 240, 105, 86, 225, 163, 19], not taking into account the efficiency of the gearbox and the motor. As most efficiencies are load- and speed-dependent, something which is especially the case for DC motors, one can expect that the most efficient solution in terms of mechanical energy consumption will not necessarily be the best solution in terms of electrical energy consumption. In the MIT Cheetah robot, for example, up to 68% of the total energy was consumed by heating of the motor windings [199]. For this reason, it has become increasingly common to include the motor in the model in order to improve the calculations. In this regard, interesting results were presented in [33], in which an optimization is performed on a robotic arm. Here, the authors claim having achieved two third of their energy savings due to a more efficient use of the DC motor.

Often, the motor model only contains a term representing the losses due to the resistance of the motor windings (Joule losses) [11, 173, 189, 184]. Sometimes a friction

---

2.2 Load-and speed-dependent efficiency

In this section, we give an overview of the losses in typical components of an actuator drivetrain. Most of the contents in this section are taken from Verstraten et al. (2015) Modeling and design of geared DC motors for energy efficiency: Comparison between theory and experiments. Mechatronics, 30, pp. 198-213. [232].

2.2.1 Friction losses

Friction losses result from contact between surfaces and are therefore inevitable in any mechanical system. Unfortunately, they are extremely difficult to model because they depend on several aspects such as lubrication, temperature, contact geometry, surface
CHAPTER 2. MODELING THE POWER CONSUMPTION OF ACTUATORS

Figure 2.1: Simple static friction models with incremental complexity: (a) Coulomb and viscous friction, (b) Coulomb and viscous friction with stiction, (c) Realistic friction behavior displaying the Stribeck effect, which . Reprinted from [10].

finish, material properties, etc. Friction models are therefore often phenomenological rather than based on physically motivated models [158], although these exist as well [4].

In mechanical drivetrains, the shaft is usually connected with the outside world through bearings, making this the main source of friction. Bearing friction, just like friction in general, has been studied for many decades, and various friction models have been developed with varying complexity. They may include complex effects such as temperature and torque-dependency, e.g. [26, 204]. The aim of this section is not to go into detail about these friction models; for this, we refer to the excellent review papers that can be found in literature, e.g. [10, 158, 4]. Instead, we will focus on the basic aspects of friction which capture most of the friction behavior.

In this work, we will always represent the friction torque $T_{frict}$ with a classical friction model which consists of Coulomb friction and viscous friction:

$$T_{frict} = T_C \text{sign} (\dot{\theta}) + \nu \dot{\theta} \quad (2.1)$$

with Coulomb friction coefficient $T_C$ and viscous friction coefficient $\nu$. As illustrated in Fig. 2.1, this model follows the friction behavior quite well at high velocities. At low
velocities, however, it does not account for static friction (stiction), which can be higher than Coulomb friction (Fig. 2.1b), nor does it capture the Stribeck effect (Fig. 2.1c) resulting from the transition from boundary lubrication to hydrodynamic lubrication. In this work, we are mainly interested in the power losses due to friction. Since the dissipated power is the product of speed and torque, the low-speed accuracy of the friction model is of less importance in this work, justifying the use of the simplified formula (2.1).

Furthermore, friction may exhibit complex behavior such as frictional lag, presliding displacement, higher-order speed-dependency, etc. [204]. Eq. (2.1), which is a function of speed $\dot{\theta}$ only, cannot account for these effects. Nevertheless, they can be very relevant in robotics, especially in systems that require accurate positioning [4] or when the small motions are amplified by transmissions [10]. Frictional lag, for example, can be as high as 9 ms [10]. As a result, the impact of friction on the control accuracy can be extremely high, with errors of over 50% being reported for heavy manipulators [113]. Luckily, these effects can be modeled and compensated quite well by dynamic models such as the Dahl model [44] or the LuGre model [46]. But again, in this work, rather than the influence of friction on control, we are mainly interested in the amount of energy dissipated by friction. Considering the type of motions that are imposed to the actuators (cyclic, typically sinusoidal) and the fact that friction often has a small contribution to the overall power in our case studies, Eq. (2.1) was found to give an adequate approximation of friction in our systems, provided that the correct friction coefficients are selected.

### 2.2.2 Gearbox efficiency

Although direct-drive solutions exist, most actuator designs use gearing in order to match the output load and speed to the motor’s most efficient operating range. Gears, however, introduce energy losses. Several models have been developed to predict the efficiency of a specific type of gear pair (spur gears, bevel gears,...), usually focusing on a specific type of loss. Typically, gear losses are separated into load-dependent and load-independent losses. An extensive review of load-independent loss models is given in [208]. Many models for predicting load-dependent losses exist, and development of such models has not stopped in recent years [248, 117, 126, 41]. Much work on gear efficiency has been performed in the 80s by Anderson and Loewenthal. Fig. 2.2 presents a typical efficiency plot for a spur gear pair, based on their commonly used model published in 1980 [8]. A strong dependency on torque exists, whereas the dependency on speed is less pronounced. At low torques, efficiency decreases quickly. Regarding speed, low speeds yield higher efficiencies. Experimental evidence shows that these trends apply not only to spur gears [171], but also to other types of gearboxes such as bevel and worm gears [49, 48].

In robotics, the reduction ratios offered by spur gears alone are usually insufficient. Planetary gearboxes are a popular solution because of their ability to achieve large reduction ratios at relatively small size. Their complexity, however, makes it difficult to obtain decent efficiency models. An overview of several well-known efficiency formulas for planetary gearboxes is given in [169], and a systematic method, which is easily programmable, is presented in [47]. These formulas only include meshing losses, meaning that inertia effects and speed-dependent losses are neglected. More detailed models such
Teeth on pinion | 48
---|---
Teeth on gear | 80
Diametral pitch | 8 mm
Face width | 10 mm

Table 2.1: Properties of the gear pair presented in Fig. 2.2.

![Efficiency plot for the gear pair specified in Table 2.1, based on the model of [8]. Gear efficiency decreases quickly at low torques, especially at high speeds.](image)

Figure 2.2: Efficiency plot for the gear pair specified in Table 2.1, based on the model of [8]. Gear efficiency decreases quickly at low torques, especially at high speeds.

as [168] include these losses, but when compared to tests, do not always provide a good estimate of the gearbox efficiency. Furthermore, all of these models require some knowledge of gear design (i.e. module/pitch diameter, number of teeth, etc.) and are therefore of little practical use when choosing a gearbox from a catalog. For these reasons, we will work with the constant efficiency value specified in the catalog for the calculations made in this work.

We will, however, take the dependency on power flow into account. As suggested in [79], we can write the relationship between load torque $T_l$ and motor shaft torque $T_{shaft}$ by defining an efficiency function $C_{tr}$:

$$T_{shaft} = C_{tr} \cdot T_l$$  \hspace{1cm} (2.2)

$$C_{tr} = \begin{cases} 
1/\eta_{tr} & \text{(load driven by motor)} \\
\eta_{tr} & \text{(motor driven by load)} 
\end{cases}$$  \hspace{1cm} (2.3)

in which $\eta_{tr}$ is the gearbox efficiency. Depending on whether power is flowing from the motor to the load or vice versa, the gearbox losses will lead to an increase or decrease of the motor torque. Consequently, whether the efficiency value has to be put in the numerator or denominator of Eq. (2.2) depends on the state of the system. This is accounted for by the definition of the efficiency function $C_{tr}$ (Eq. (2.3)). A more detailed discussion can be found in the test section (section 2.3.4.1), where the effects of this phenomenon on motor current are discussed.
The gearbox efficiency $\eta_{tr}$ strongly depends on the number of gear stages. The gear reduction that can be obtained from a single planetary gear is limited to 10:1, which is why planetary gearboxes often consist of multiple stages [99]. However, adding stages to a gearbox also reduces its efficiency due to the increased number of components and the losses associated with them. For this reason, gearbox efficiency will tend to be lower for high gear ratios [47].

On a final note, just like ordinary gear pairs, planetary gearboxes suffer a sharp decrease in efficiency when used at low torques. The effect is very similar to that of Coulomb friction. The physical reason behind this is that a planetary gearbox consists of multiple bearings which, as we have explained in the previous section, mostly exhibit this type of friction. Experiments have indeed confirmed that bearing losses are dominant in planetary gearboxes operating at low loads [67]. Several authors have therefore suggested to use a Coulomb friction term to model gearbox losses [168, 54, 199]. Assuming that the catalog efficiency $\eta_{tr}$ is specified at the nominal working point, the Coulomb friction coefficient $T_{C, tr}$ would then be given by

$$
\left(\frac{1}{\eta_{tr}} - 1\right) T_{nom} = T_{C, tr}
$$

with $T_{nom}$ the motor’s nominal torque. In terms of power losses, such a model leads to power losses which are proportional with speed:

$$
P_{loss} = \left(\frac{1}{\eta_{tr}} - 1\right) T_{nom} \dot{\theta}_{out}
$$

where $T_{out}$ and $\dot{\theta}_{out}$ stand for the output torque and speed, respectively. With this model, the gearbox friction loss is independent of the output torque. Conversely, with the model given by Eq. (2.3), losses are directly proportional with the input power $T_{out} \dot{\theta}_{out}$:

$$
P_{loss} = \left(\frac{1}{\eta_{tr}} - 1\right) T_{out} \dot{\theta}_{out}
$$

Eqs. (2.5) and (2.6) yield the same results if $T_{out} = T_{nom}$. For lower output torques, the losses will remain constant with a Coulomb friction model, while the model given by Eq. (2.3) results in a decreasing power loss. In other words, a gearbox modeled with Coulomb friction will have an efficiency which decreases with torque, while a gearbox modeled by Eq. (2.3) has a constant efficiency.

\subsection*{2.2.3 Motor efficiency}

\subsubsection*{2.2.3.1 Motor losses}

The losses of permanent magnet DC motors can be split into following contributions:
• **Joule losses**: The motor winding resistance causes a voltage drop proportional to the current $I$ delivered by the motor. This leads to a dissipation of power which amounts to $P_{\text{Joule}} = R \cdot I^2$, where $R$ stands for the total winding resistance. In robotics, this is often the main source of power loss [200]. Joule losses are also known under several different names, such as “resistive losses”, “copper losses”, “Ohmic losses” or “heating losses”.

• **Brush-drop losses**: Brushed DC motors rely on graphite brushes to transfer current from the stator to the rotor. The current transfer results in a power loss of

$$P_{\text{brush}} = I \cdot \Delta V_{\text{brush}}$$

where $\Delta V_{\text{brush}}$ is the voltage drop across the brushes. The voltage drop depends on the composition of the brushes, contact pressure and the condition of the surface of the commutator, and must therefore be determined for every machine. Nevertheless, it is often assumed to be constant, with a value of $\Delta V_{\text{brush}} = 2V$ [154]. The brush-drop losses are therefore often approximated as $P_{\text{brush}} \approx 2I$, i.e., proportional to the current.

• **Mechanical (friction) losses**: Mechanical losses are mostly due to friction in the bearings, aerodynamic drag experienced by the rotor and – in the case of brushed DC motor – friction between the brushes and the commutator. The latter, often called windage losses, increase if there is forced ventilation. In accordance with our discussion in section 2.2.1, friction losses can be considered to be proportional to motor speed $\dot{\theta}_m$ (Coulomb friction) and, to some extent, to $\dot{\theta}_m^2$ (viscous friction). Windage losses are typically considered to be proportional to $\dot{\theta}_m^3$ [154, 80].

• **Core loss (iron loss, magnetic loss)**: These are the losses due to the rotating magnet. They can be subdivided into two main contributions. **Hysteresis (remagnetization) losses** represent the hysteresis in the motor curve, which is caused by the fact that some energy is needed to reverse the direction of the magnetic field. This can be described by a power loss $P_{\text{remagn}} = T_{\text{magn}} \dot{\theta}_m$ with $T_{\text{magn}}$ the torque needed for reversal of magnetization. Hysteresis losses are thus proportional to speed, similar to Coulomb friction. Secondly, there will be losses due to eddy currents in the magnetic core. **Eddy current losses** are proportional to $\dot{\theta}_m^2$ and, can thus mathematically be included in the viscous friction losses.

• **Stray load losses**: These losses account for extra losses occurring at high loads. They are mainly related to pulsations in the flux, caused primarily by the short-circuit current in the coil undergoing commutation. Stray losses are only relevant for large-size machines, and can be neglected for small-size motors.

### 2.2.3.2 DC motor model

A practical way of representing a motor’s electrical power consumption is the following DC motor model ([29]):
\[
\begin{align*}
I &= \frac{1}{k_t} \left( T_m + \nu_m \dot{\theta}_m \right) \\
U &= \Delta V_{\text{brush}} + L \frac{dI}{dt} + RI + k_b \dot{\theta}_m
\end{align*}
\] (2.7)

which gives the relationship between motor torque \( T_m \) and motor speed \( \dot{\theta}_m \) and armature current \( I \) and voltage \( U \). The model includes damping (viscous friction losses) \( \nu_m \dot{\theta}_m \), a supplementary torque proportional to the motor’s speed \( \dot{\theta}_m \) to account for speed-dependent losses such as eddy current losses, friction losses, hysteresis losses, etc. Brush losses are represented by the additional voltage \( \Delta V_{\text{brush}} \) which, for obvious reasons, is zero for brushless DC motors. Furthermore, there are the Joule losses \( RI \), which take up a part of the motor voltage and are proportional to the torque delivered by the motor. There is also a voltage loss \( L \frac{dI}{dt} \) due to the terminal inductance \( L \). Finally, it must be mentioned that the torque due to the acceleration of the rotor’s inertia is not explicitly present in these equations. It must be included in the calculation of \( T_m \) in case of variable speed.

From the above motor model, the electrical power consumption can be calculated as

\[
P_{\text{elec}} = I \cdot U
\] (2.8)

The power of a lossless motor can be found by setting \( R, L \) and \( \nu_m \) to zero:

\[
P_{\text{elec,lossless}} = I \cdot U = \frac{T_m}{k_t} \cdot k_b \dot{\theta}_m
\] (2.9)

and, since the motor’s speed constant \( k_b \) is equivalent to the torque constant \( k_t \),

\[
P_{\text{elec,lossless}} = T_m \dot{\theta}_m
\] (2.10)

which corresponds to the mechanical power at the motor shaft, as expected.

In accordance with our discussion of friction losses and motor losses, a combination of Coulomb and viscous friction would give an even more realistic representation of the speed-dependent motor losses. Some manufacturers indeed state these two friction coefficients in their datasheets \[118\], but the manufacturer of the motors used in this work (Maxon motors) only specifies a no-load speed and current. Therefore, Coulomb and viscous friction will only be used to model the speed-dependent losses in this work if the coefficients are identified in experiments. Otherwise, we will only use damping as an approximation, according to Eq. 2.7. Since usually the motor’s viscous damping coefficient \( \nu_m \) is not specified by the manufacturer, it is common to use the following approximation:

\[
\nu_m = \frac{k_t \cdot I_{nl}}{\omega_{nl}}
\] (2.11)

which ensures that, in no-load conditions, a current equal to the no-load current \( I_{nl} \) is consumed when the motor is rotating at the no-load speed \( \omega_{nl} \).
2.2.3.3 Motor efficiency map

The motor efficiency $C_m$ can be found by dividing the motor’s mechanical output power by the electrical power input or vice versa, depending on the direction of power flow:

$$C_m = \begin{cases} \frac{T_m \dot{\theta}_m}{P_{elec}} & \text{(load driven by motor)} \\ \frac{P_{elec}}{T_m \dot{\theta}_m} & \text{(motor driven by load)} \\ 0 & \text{(motor driven by load, no regeneration)} \end{cases} \quad (2.12)$$

When the load is driving the motor, the mechanical power $P_{mech} = T_m \dot{\theta}_m$ will be larger than the electrical power $P_{elec}$; when the motor is driving the load, the opposite is true. Therefore, depending on the direction of power flow, numerator and denominator are switched in Eq. (2.12) in order to obtain an efficiency value between one and zero. However, even if the load is driving the motor, this does not necessarily mean that the electrical energy can be recovered. The third case in Eq. (2.12) corresponds to two symmetrical pairs of operating regions in the second and fourth quadrant, in which the energy dissipated by the motor’s resistance and by motor friction is higher than the power flowing from the load, and energy is drawn from the power supply in order to compensate for the difference. Because in this case, energy is still consumed from the power source even though the motor is braking, a nominal efficiency of zero has been assigned to this region. The operating regions in which the motor is driven by the load and energy is regenerated, i.e. the second case in Eq. (2.12), are bounded by the operating conditions $P_{elec} = 0$ (i.e. $U = 0$ or $I = 0$) and $P_{mech} = 0$ (i.e. $\dot{\theta} = 0$ or $T_m = 0$) ([217]). This is evidenced by a map showing steady-state motor efficiency as a function of speed and torque (Fig. 2.3) in all four quadrants of operation, based on Eq. (2.12). Such efficiency maps – also called iso efficiency contours – are a very useful tool for the evaluation of motor-driven systems with variable loads [50, 223].

Looking at the first quadrant, the lowest efficiencies occur at low-speed high-torque. A second low-efficiency region can be found at high speeds and low torques, but the efficiency drop is not as pronounced as in the previously mentioned region. This indicates that the resistive losses, which depend on the torque, dominate over the damping losses, which depend on the speed. As a rule of thumb, most motors achieve their maximum efficiency at around 10-20% of stall torque and 90% of no-load speed. This point of maximum efficiency occurs at a lower speed when the torque is decreased. Robotic applications, which run at variable speeds and torques, often in multiple quadrants, typically do not operate in the zone of maximum efficiency at all times, but also pass through low efficiency regions. Hence, the average efficiency for a typical trajectory can be expected to be well below the maximum value.

2.2.4 Drive circuitry

Robotic tasks often require motors to not only deliver energy to a load, but also to absorb it. This happens, for example, when kinetic energy is removed from a load, an act commonly known as braking. The absorption of power by a motor and the storage of this
Table 2.2: Motor properties of a 80W Maxon DCX35L motor.
Figure 2.3: Motor efficiency contour plot for the motor specified in Table 2.2, based on Eq. (2.12). Quadrants are denoted by Roman numerals. The second and fourth quadrant (load driving motor) each contain two zero-efficiency regions (dark blue), in which the negative energy coming from the load is dissipated entirely in the motor. The low-speed zero-efficiency region, bounded by $U = 0$ and $\dot{\theta} = 0$, is the largest. The second zero-efficiency region, which lies in between the $I = 0$ line and the speed axis, is hardly noticeable because both lines nearly coincide. This is why, in order not to compromise readability, the $I = 0$ line is not shown on the plot.
energy in a battery is called regeneration. A common misconception in robotics is that motors cannot regenerate energy or, even stronger, that the absorption of energy requires an (equal) amount of energy from the motor \[240, 225, 59, 87, 114\]. The latter would imply that the motor and drive circuit are receiving power from both the battery and the load, with no path to sink this energy except for dissipation in its components. Such a situation would violate the laws of physics (conservation of energy), and is therefore obviously false. Analyses based on this assumption can therefore lead to very dubious claims, e.g., dissipative elements that decrease an actuator’s energy consumption \[59\].

All electric motors are inherently capable of regenerating energy \[101\]. A possible limitation, though, is the drive. In order to enable regeneration, a 4-quadrant controller is needed. However, the cost of such a drive is usually considerably higher than a single-quadrant drive. A more economical alternative might be to use a single-quadrant controller and to dissipate the energy in a braking resistor. The price to pay is a lower energy efficiency, but also additional heat which must be dissipated to the environment.

Datasheets usually do not contain sufficient data to reconstruct the complex dynamics of the circuit, and therefore, we will not attempt to calculate the voltage and current at the input of the PWM, i.e. at the output of the power source. Instead, we will calculate the consumed power directly by using the efficiency of the drive circuitry, \(\eta_{\text{drive}}\). This property, which is not considered by most authors, can be retrieved from manufacturer’s datasheets.

Two types of losses occur:

- **Continuous losses due to controller electronics:** If the controller does not have a separate power supply for the electronics, these losses also need to be added to the power drawn from the source. Some manufacturers specify a standby or idle current consumption in their datasheets. This is the current which is continuously drawn from the power source in order to power the electronics. Small, versatile 4-quadrant controllers up to 50W typically consume around 50 mA. By multiplying this current with the power source voltage, the constant power loss \(P_{\text{standby}}\) due to controller electronics can be calculated. Obviously, in low-power applications, this loss - typically 1 to 2W - can be detrimental to the total energy consumption.

- **Losses related to the power flowing through the controller:** In general, datasheets mention a controller efficiency \(\eta_{\text{drive}}\), typically 90-95\%. \(\eta_{\text{drive}}\) can be used to calculate the power drawn from the power source, \(P_{\text{source}}\), by applying following formula:

\[
P_{\text{source}} = C_c P_{\text{elec}} + P_{\text{standby}}
\]  

in which \(P_{\text{standby}}\) corresponds to the losses due to controller electronics and \(C_c\) is the controller efficiency function.

For a regenerative drive, \(C_c\) is defined as

\[
C_c = \begin{cases} 
1/\eta_{\text{drive}} & (P_{\text{elec}} > 0) \\
\eta_{\text{drive}} & (P_{\text{elec}} < 0)
\end{cases}
\]  

(2.14)
Notice how, just like in Eq. (2.3), the drive circuitry efficiency switches from the denominator to the numerator as the electrical motor power changes sign.

Alternatively, the negative power can be dissipated in resistors in the drive circuitry. In this process, no energy is drawn from the power source [188]. The power flowing from the power source is then given by

\[
C_c = \begin{cases} 
\frac{1}{\eta_{\text{drive}}} & (P_{\text{elec}} > 0) \\
0 & (P_{\text{elec}} < 0) 
\end{cases} \tag{2.15}
\]

Eq. (2.15) also applies when electronic circuitry is preventing the controller from sending current into the battery. Such protection circuits are common with high-end batteries such as Li-Ion batteries, which may get damaged or even explode if too high reverse currents are applied [9].

Finally, the actuator’s energy consumption \(E_{\text{elec}}\), with drive circuitry included, is modeled as the integral of the power drawn from the power source, \(P_{\text{source}}\):

\[
E_{\text{elec}} = \int P_{\text{source}} dt \tag{2.16}
\]

with the integration interval spanning the entire duration of the motion.

2.2.5 Power source losses

The work only considers the energy consumed at the input of the controller. There is, however, also an efficiency associated with the power source. This is particularly relevant if the power source is a portable unit such as a battery. Lead-acid battery cycle efficiencies are typically around 80% [209], while the lightweight Li-ion batteries reach efficiencies of around 90% [192]. Additional circuits such as equalizing circuits may further decrease these efficiencies. Due to these losses, it will not be possible to entirely reuse the energy regenerated by the motor. This will be discussed further in section 2.3.3.3.

2.2.6 Backlash

Backlash appears when play exists between mechanical components in the drivetrain. It is often associated with geared systems, although other components such as couplings, belts and actuators may also introduce backlash. Some degree of backlash is required in order to prevent jamming or to allow for lubrication. Nevertheless, in dynamic systems, one generally tries to minimize backlash in dynamic systems, as it may degrade control performance by introducing steady-state errors or limit cycles [156]. Although backlash is mostly seen as a control problem, it also introduces additional energy losses. Backlash is characterized by impacts between two parts, which are inevitably accompanied by impact losses [249]. Furthermore, oscillations induced by the controller may lead to additional energy dissipation [133]. Due to the complexity of these loss mechanisms, however, their impact on the overall energy consumption is hard to quantify [15]. Consequently, losses due to backlash will not be treated in this work.
2.3 Case study: pendulum

In this section, we will study the different loss mechanisms at variable loads and speeds on a pendulum to which different swinging frequencies are imposed. The actuator considered here only consists of its two most essential components: a motor and gearbox. The contents of this section are based on the journal paper Verstraten et al. (2015) Modeling and design of geared DC motors for energy efficiency: Comparison between theory and experiments. Mechatronics, 30, pp. 198-213. [232].

2.3.1 Equations

The forced pendulum, driven by a torque $T_l$, can be described by its classical equation:

$$T_l = J \ddot{\theta} + \nu \dot{\theta} + Mgl \sin \theta$$

(2.17)

which contains an inertial term $J \ddot{\theta}$, a gravitational term $Mgl \sin \theta$, and a damping term $\nu \dot{\theta}$ characterizing the friction losses of the system (denotations in Table 2.3) – although Coulomb friction would be a more realistic representation of friction, as explained in section 2.2.1. In order to find the natural motion of the pendulum ($T_l=0$), this nonlinear second order system is typically linearized by making a small-angle approximation:

$$T_l = J \ddot{\theta} + \nu \dot{\theta} + Mgl \theta$$

(2.18)

However, it is also possible to obtain the exact solution to the nonlinear differential equation (2.17) by using Matlab’s ellipj function. Defining $\omega_{rl}$ as the natural (resonance) frequency of the undamped pendulum (link),

$$\omega_{rl} = \sqrt{\frac{Mgl}{J}}$$

(2.19)

the solution for $\theta$ in the no-damping case ($\nu = 0$) is given by the Matlab equation

$$\theta(t) = 2 \cdot \arcsin \left( \sin \left( \frac{\theta_0}{2} \right) \cdot \text{ellipj} \left( \omega_{rl} \cdot t, \sin \left( \frac{\theta_0}{2} \right) \right) \right)$$

(2.20)

and the natural acceleration and speed can be calculated from $\theta$ by

$$\dot{\theta}(t) = \omega_{rl} \cdot \sqrt{2(\cos \theta - \cos \theta_0)}$$

(2.21)

$$\ddot{\theta}(t) = -\omega_{rl}^2 \sin(\theta)$$

(2.22)

If the pendulum is driven by a DC motor with gearbox, the motor will need to deliver not only the required torque $T_l$, but also the torque to compensate for the gearbox losses and the inertia of the gearbox and rotor, since the latter was not included in the DC motor model (2.7). Hence, the torque required from the motor will be

$$T_m = (J_m + J_{tr}) n \dot{\theta} + \frac{C_{tr}}{n} T_l$$

(2.23)
in which $J_m$ is the motor inertia, $J_{tr}$ is the gearbox inertia specified at the input shaft, $n$ is the gear ratio, and $C_{tr}$ is the efficiency function of the gearbox (2.3).

Defining the total inertia $J_{tot}$ as

$$J_{tot} = J + \frac{n^2}{C_{tr}} (J_{tr} + J_m)$$  \hspace{1cm} (2.24)

the motor torque can be rewritten as

$$T_m = \frac{C_{tr}}{n} (J_{tot} \ddot{\theta} + v \dot{\theta} + Mgl \sin \theta)$$  \hspace{1cm} (2.25)

In order to calculate the energy consumption, a trajectory for the pendulum must be defined. In other works which use a pendulum as a test load ([222, 20, 17]), a sinusoidal trajectory is imposed by the actuator, typically with limited amplitude for the small-angle approximation to be valid. Unlike those works, we will impose the exact solution of the pendulum problem by varying $\omega_{rl}$ in Eq. (2.20). In the case of a sinusoidal trajectory, there will be a small residual torque which will appear in the results as an unmodeled loss. The advantage of our approach is that this residual torque is eliminated and thus, theoretically, the motor will only need to compensate for the losses at the resonance frequency of the system.

Assuming a lossless system by omitting the damping term, and assuming perfect tracking by replacing $\ddot{\theta}$ by Eq. (2.22), the motor torque (2.23) can be rewritten as

$$T_m = \frac{C_{tr}}{n} (Mgl - \omega_{imp}^2 \cdot J_{tot}) \cdot \sin(\theta)$$  \hspace{1cm} (2.26)

$\omega_{imp}$ is the imposed value of $\omega_{rl}$, and corresponds to the angular frequency imposed by the actuator. Damping is often omitted in design calculations because it is very difficult to obtain a good estimate of the damping coefficient without measuring it. This is the reason why we have neglected it in our simulations. However, we will study its effects experimentally in the section 2.3.4.1.

Eq. (2.24) allows to write the system’s natural frequency $\omega_{rs}$ as

$$\omega_{rs} = \sqrt{\frac{Mgl}{J_{tot}}}$$  \hspace{1cm} (2.27)

When the pendulum is swinging at its natural frequency, motor torque is reduced to zero, even in the presence of gearbox losses. This can be seen by introducing the system’s natural frequency (2.27) into Eq. (2.26). This is however only true if there are no losses before the gearbox. If there are, the motor will need to deliver some torque in order to compensate for these losses; moreover, the losses will be amplified by the gearbox efficiency.

Motor speed, finally, is given by

$$\dot{\theta}_m = n \dot{\theta} = n \cdot \omega_{imp} \cdot \sqrt{2(\cos \theta - \cos \theta_0)}$$  \hspace{1cm} (2.28)

By introducing Eqs. (2.26) and (2.28) into Eq. (2.7), motor current and voltage can be calculated as a function of $\omega_{imp}$ at any point in time.
2.3 Case study: pendulum

Figure 2.4: Test set-up used for efficiency measurements on the Maxon motor and gearbox. A torque sensor (b) and an encoder (c) are placed in between the pendulum load (a) and the gearbox-motor (d), which allows for the calculation of the energy consumption at this particular point.

2.3.2 Test setup

To validate the simulations, a simple test set-up was made (Fig. 2.4). Its physical properties, derived from a 3D CAD model, are listed in Table 2.3. The parameters of the set-up are presented in Table 2.3. The pendulum swings symmetrically around $\theta = 0^\circ$, which corresponds to a vertical line. It is driven by an 80W Maxon DCX35L motor, of which the specifications are listed in Table 2.2, in combination with a 338:3 ratio Maxon GPX42 planetary gearbox (Table 2.3). An ETH Messtechnik DRBK torque transducer at the primary shaft is used to measure the motor torque. It can measure up to 20Nm with an accuracy of 0.5% and has an inertia of 136 gcm², which is also accounted for in the simulations. US Digital E6 series optical encoders on the secondary and primary shaft measure the angular position of the pendulum and the gearbox shaft. The encoders both have a resolution of 2000 counts per turn and an inertia of 0.073 gcm² which, although negligible, was also included in the simulations. For the current measurements, an Allegro ACS712 current sensor with measurement range from -5A to 5A and total output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ($M$)</td>
<td>1.568 kg</td>
</tr>
<tr>
<td>Moment of inertia in rotation axis ($J$)</td>
<td>0.102 kg m²</td>
</tr>
<tr>
<td>Distance from rotation axis to COG ($l$)</td>
<td>188.6 mm</td>
</tr>
<tr>
<td>Release angle ($\theta_0$)</td>
<td>45°</td>
</tr>
</tbody>
</table>

Table 2.3: Pendulum and gearbox properties.
error of 1.5% was used, while the motor voltage was measured directly with a National Instruments sbRIO 9626 board. Both measurements were conducted at the motor terminals, i.e. converter losses are not measured. The resistance of the current sensor (1.2 mOhm) and the additional cables (0.228 Ohm) were also added to the simulation in order to get a correct estimate of the Joule losses. At each swinging frequency, ten periods were recorded by the sbRIO 9626 and subsequently averaged to obtain the final result.

The pendulum is position controlled by a Maxon EPOS2 controller, using the integrated Maxon ENX16 EASY encoder with a resolution of 1024 counts per turn for position feedback. It receives pre-computed position set values from the sbRIO 9626 through a CAN bus. The controller consists of a high-speed (10 kHz) PI current control loop and a low-speed (1kHz) PID position control loop with constant-gain velocity and acceleration feedforward, as sketched in Fig. 2.5. The parameters of the PI and PID controller and the feedforward gains are tuned using the EPOS2’s autotuning function. Finally, the EPOS2 controller and the motor are powered by two identical 12V lead-acid batteries placed in series. The batteries not only deliver power to the drive, but also store any energy that might be available from regeneration.

2.3.3 Simulations

The imposed trajectory is a periodic motion. Since the load is moving in a gravitational field – a conservative force field – the amounts of negative and positive energy are equal, and therefore no net mechanical energy input is required. The actuator will only need to deliver some energy to compensate for energy losses. As a criterion for energy efficiency, the energy consumption per cycle is used. To characterize the electrical losses, this value will be compared to the following value

$$E_{mech} = \int_{cycle} P_{mech} dt$$

(2.29)
2.3 Case study: pendulum

Figure 2.6: Energy consumption per cycle as a function of pendulum frequency $\omega_{imp}$, assuming zero drive inertia. When nearing 0 rad/s, the cycle time goes to infinity, and so does the electrical energy consumption, which is heavily affected by the rapid decrease in motor efficiency at low speeds.

In which the mechanical power $P_{mech}$ is given by

$$P_{mech} = T_m \dot{\theta}_m$$

(2.30)

$E_{mech}$ is representative of the mechanical energy consumption of the motor during one cycle of the pendulum. Unlike in [19], the results are not normalized by cycle time, and are therefore expressed in J rather than in J/s.

2.3.3.1 Excluding drive inertia

In this paragraph, we will study the behavior of the system without drive inertia by setting $J_r$ and $J_m$ to zero. Fig. 2.6 shows the mechanical energy $E_{mech}$ and the consumed electrical energy $E_{elec}$, both for one cycle of the pendulum, as a function of the imposed pendulum frequency. The mechanical energy plot has one minimum at a pendulum frequency of 5.3 rad/s, which corresponds to the pendulum’s natural frequency $\omega_{nl}$. At this frequency, the actuator does not deliver any torque or power to the load. Any other frequency will require a torque from the actuator; the further away from natural frequency, the higher this torque will be, which consequently results in a higher power consumption. Actuator speed is proportional to the imposed pendulum frequency $\omega_{imp}$, as shown by Eq. (2.28). This explains the quick rise in energy consumption at high frequencies. The large gap between the mechanical and electrical energy consumption at low frequencies demonstrates the rapid decrease in motor efficiency at low speeds. Note that the energy consumption per cycle is still significant at low frequencies because of the longer cycle times.

Fig. 2.6 also shows how the load- and speed-dependency of motor efficiency affects the performance of the actuator. It can be observed that the largest difference between
mechanical and electrical energy consumption, i.e. the worst efficiency, occurs at the lowest pendulum frequencies. This corresponds to a low-speed high-torque operating region, which, as already mentioned in section 2.2.3, is the most unfavorable region. Once above the natural frequency, as the speed increases, the losses increase at a slower rate than the power consumption, and efficiency improves as well.

Motor torque and speed are influenced by the gearbox, and consequently, so is motor efficiency. Fig. 2.7 shows the energy consumption per cycle for different gear ratios.

- Low gear ratios lead to an increase in motor torque and a decrease in motor velocity. As a result, the damping losses (proportional to velocity squared) will decrease at the cost of increasing Joule losses (proportional to torque squared). There is some benefit to low gear ratios near natural frequency $\omega_{ri}$, where torque is low and Joule losses are negligible. However, they come at the cost of a strong increase in electrical energy consumption away from natural frequency, where high torques are demanded.

- High gear ratios achieve the opposite: they lead to a decrease in motor torque and an increase in motor velocity, meaning that the contribution of Joule losses is decreased relative to damping losses. This results in high motor efficiencies at low speeds at the cost of efficiency loss at higher speeds.

The effect of gear ratio choice can also be studied by plotting the operating trajectory on the efficiency map of the motor. In Fig. 2.8, the operating points at which the motor
2.3 Case study: pendulum

Figure 2.8: Operating points at which peak power occurs for gear ratios of $n=20$ and $n=150$, visualized on a motor efficiency map. A high gear ratio spreads the operating points over a wider range of speeds, decreasing the torque and increasing the energy efficiency. A small gear ratio demands high motor torques at low speeds, an unfavorable situation in terms of motor efficiency.

consumes maximum power for various frequencies are plotted on the efficiency map of the motor, for gear ratios of 20 and 150. Low gear ratios will push the motor’s operation points into the low-speed high-torque region, where efficiency is lowest. High gear ratios push the operation points towards better efficiency regions, although very low torques are supplied less efficiently. This is in agreement with our previous findings.

From this discussion, it might seem that high gear ratios are more favorable. It is worth noting though that, as discussed in section 2.2.2, high gear ratios require multiple-stage gearboxes, which come at very poor efficiencies [99]. For this reason, a well-chosen gear ratio is generally low, but still sufficiently high in order to allow the motor to operate in a region with decent efficiency.

2.3.3.2 Effect of drive inertia

Designers will generally try to avoid large inertias because of the reduction in acceleration capacity associated with them. Even though the inertias of the motor and gearbox are rather small compared to that of the load they are driving, one must take into account that their contribution to the acceleration torque is amplified by gear ratio squared (Eq. (2.24)). Therefore, the effect of drive inertia certainly cannot be neglected for high gear
Figure 2.9: Energy consumption per cycle for different pendulum frequencies $\omega_{imp}$, drive inertia included. Except for the highest gear ratios $r$, a clear minimum is observed at the pendulum’s natural frequency $\omega_{nl}$.

ratios. From Eq. (2.23), it can be derived that the optimum gear ratio in order to achieve the maximum acceleration is

$$n = \sqrt{\frac{C_{tr}J}{J_m + J_{tr}}}$$

(2.31)

i.e. the gear ratio which matches the reflected drive inertia with the load inertia. In Eq. (2.31), the load- and speed-dependent efficiency function $C_{tr}$ is usually taken to be the constant catalog efficiency $\eta_{tr}$ (reverse drive) or its inverse (forward drive). Utilizing this specific gear ratio not only maximizes acceleration capability [165]; it also has the benefit of avoiding resonance problems [57]). For these two reasons, this technique – known as “inertia matching” or “impedance matching” – is widely applied in the sizing of servo applications. However, by increasing gear ratio, drive inertia is increased, which also has an effect on energy consumption. The optimum gear ratio in terms of energy efficiency may therefore be different from Eq. (2.31), something which was already pointed out in [104].

Fig. 2.9 shows the simulated energies for different gear ratios, assuming a regenerative drive and including drive inertia into the calculations. Drive inertia does not lower the frequency of minimum power consumption, which is still observed at the pendulum’s resonance frequency $\omega_{nl}$. However, energy consumption clearly increases at higher pendulum frequencies, while there is a slight decrease at low frequencies for high gear ratios. This is because the system’s losses are roughly proportional to the powers flowing through the system. As demonstrated in Fig. 2.10, an increasing drive inertia leads to a decrease
Figure 2.10: Peak power as a function of pendulum frequency $\omega_{imp}$. If drive inertia is neglected (green), minimum mechanical peak power will occur at the pendulum’s resonance frequency $\omega_{rl}$. The addition of drive inertia decreases the resonance frequency of the system, and consequently lowers the frequency at which minimum mechanical (blue curve) and electrical (red curve) peak power occurs.
in the resonance frequency. This is associated with an increased peak power for high frequencies, and consequently, increased losses. It is noteworthy that adding inertia to the system is not necessarily undesirable from an energy efficiency point of view. It is well-established that inertia, usually in the form of a flywheel, can be useful as a reservoir for energy storage in periodic motions. The increased drive inertia has the exact same effect as the addition of a flywheel, lowering the resonance frequency of the entire system. If this resonance frequency matches the one of the imposed motion, no torque, hence no power, will be required from the actuator, except for compensation of the losses. In this way, the drive inertia can be used to tune the system for a more energy efficient performance. As we will discuss in chapter 3, parallel springs – another energy storage method – have a similar effect: they will increase the resonance frequency of the system.

### 2.3.3.3 Regenerative vs. non-regenerative drive circuitry

The previous simulations were conducted under the assumption that 100% of the negative energy can be regenerated. But what consequences does full or partial dissipation of this energy have for the total energy consumption? To answer this question, we have simulated the pendulum’s energy consumption using Eqs. (2.16), (2.14), (2.15). The results are plotted in Fig. 2.11.

- The mechanical energy consumption, which is calculated at the motor shaft, solely consists of the energy needed to compensate for the gearbox losses. At the resonance frequency $\omega_{rl}$, there is no torque and, since gearbox losses are modeled to be proportional to torque, no power consumption.
2.3 Case study: pendulum

In case of a non-regenerative drive, minimum energy consumption is achieved when a minimum amount of energy is cycled through the motor. In other words, it is imperative to keep the required power as low as possible at each moment. For the pendulum problem, this corresponds to the frequency of minimum peak power, which lies at the natural frequency of the entire system, $\omega_{rs}$. Any other frequency will require some energy input in order to provide the positive power required for the motion.

In case of a regenerative drive with 100% regeneration efficiency, all of the negative energy available before the controller is recovered. The pendulum’s motion itself requires no net energy input; only the losses contribute to the total energy consumption. The optimal swinging frequency therefore lies at the point where the electrical and mechanical losses are minimized. As mentioned before, the gearbox losses demonstrate a clear minimum at $\omega_{rl}$. Motor losses are minimal at 2.9 rad/s, which is far below $\omega_{rl} = 5.3$ rad/s. This minimum is, however, much less clear than the minimum in gearbox losses, which is why the optimal frequency still tends to lie close to $\omega_{rl}$. High gearbox efficiencies can lower the optimal frequency, and in more general cases, one can expect the lowest energy consumption to occur at a frequency slightly below $\omega_{rl}$.

It must be noted that not all energy which is transferred to the power source can be recovered. There are always some losses associated with a charging-discharging cycle. For batteries, the losses depend on several factors such as the type of technology, charge/discharge rate and whether the battery is fully charged or not. Lead-acid batteries such as those used in the test set-up have typical efficiencies of around 80% ([192], [209]). In order to take this into account, one can assign a battery efficiency $\eta_{battery}$ by which the negative power is decreased. Source power would then be calculated as:

$$P_{source} = \begin{cases} 
\frac{P_{elec}}{\eta_{drive}} & (P_{elec} > 0) \\
\eta_{drive}\eta_{battery}P_{elec} & (P_{elec} < 0)
\end{cases}$$  \hspace{1cm} (2.32)

Taking $\eta_{battery}$ to be 80%, the minimum already clearly shifts towards $\omega_{rs}$ instead of $\omega_{rl}$.

In conclusion, a regenerative drive consumes less energy than a non-regenerative one, but the difference in energy consumption becomes smaller as the motion approaches the system’s natural frequency $\omega_{rs}$, a point at which both perform equally well.

2.3.4 Experimental results

2.3.4.1 Empirical adjustments to the model

Figures 2.12, 2.13 and 2.14 show angle, speed, voltage, current, torque and power plots for one pendulum period, at frequencies of 1 rad/s, 3 rad/s and 5 rad/s. While the measurements at 1 rad/s and 3 rad/s match the model quite nicely, there are significant deviations
Figure 2.12: Measurements at a swinging frequency of 1 rad/s. Modeled values are indicated in blue, measurements in green. The gray curves indicate the 1 standard deviation interval.
Figure 2.13: Measurements at a swinging frequency of 3 rad/s. Modeled values are indicated in blue, measurements in green. The gray curves indicate the 1 standard deviation interval.
Figure 2.14: Measurements at a swinging frequency of 5 rad/s. Modeled values are indicated in blue, measurements in green. The gray curves indicate the 1 standard deviation interval.
in the torque and current signal at 5 rad/s because, at this speed, the coupling started slipping, leading to high peaks in the torque signal. As the speed increases, the controller also starts ringing more violently, i.e., oscillations in input torque occur.

Before discussing the peak power and energy measurements, we will first make two empirical adjustments to the model. First, motivated by the measurements, a different value is assigned to the gearbox efficiency in negative power flow. Second, two damping terms are added to the model to represent the losses which have not yet been modeled, such as bearing losses.

**Directional efficiency of gearbox** The gearbox (or any mechanical component inducing losses) will convert some of the input energy to heat. If the motor is driving the load, this can be accounted for by subtracting the gearbox losses from the motor torque. However, in case of the load driving the motor, these losses have to be subtracted from the load torque instead of the motor torque, since it is the load which now supplies the input energy. This was accounted for in the simulations by Eq. (2.3), using the gearbox efficiency or its inverse depending on the direction of power flow. Fig. 2.12 clearly demonstrates how strongly the current is affected by this nonlinearity. The measurements follow the

![Figure 2.15: Difference between modeled and measured current at $\omega_{imp} = 1$ rad/s. The catalog-based model (blue curve, Eq. (2.3)) assumes $\eta_{tr} = 72\%$ in both negative and positive power flow, whereas in the adjusted model (magenta curve, Eq. (2.33)) $\eta_{tr}$ is reduced to 55\% in negative power flow. In the positive power region, the difference in current can almost completely be attributed to oscillations caused by the controller. In the negative power regions, however, an offset is observed, which is corrected by the adjusted model.](image)

model quite well with respect to this phenomenon, although the drop is a bit sharper. A possible explanation would be that the gearbox efficiency is in fact lower than the one specified in the catalog. With the gearbox being loaded at less than 13\% (1 rad/s) and 6\%
(4 rad/s) of its maximum continuous torque, this would be in accordance with our discussion on gearbox efficiency (section 2.2.2), where we mentioned the sharp decrease in gearbox efficiency at low loads and speeds. The torque measurements remain unaffected by the phenomenon, since they were taken before the gearbox. Another explanation can be derived by looking at the difference between the measured and modeled current (e.g. Fig. 2.15, at 2 rad/s). While the model seems to be quite accurate in positive power flow, a difference of about 0.10A is observed in negative power flow. To improve the gearbox model, we adjust the catalog-based gearbox efficiency model (2.3) by assigning different gearbox efficiencies depending on the direction of power flow:

\[
C_{tr} = \begin{cases} 
1/\eta_{tr}^+ & \text{(load driven by motor)} \\
\eta_{tr}^- & \text{(motor driven by load)} 
\end{cases} \tag{2.33}
\]

By setting the gearbox efficiency \(\eta_{tr}^-\) in reverse power flow to 55% and maintaining \(\eta_{tr}^+ = 72\%\) in forward power flow, the correspondence between measurements and model can be increased significantly. The observation that gearbox efficiency is lower in the negative direction of power flow is backed by theoretical and practical evidence in [237, 238], where the author measured directional differences in power losses up to 17%.

**Addition of friction losses** Close to the pendulum’s natural frequency \(\omega_{rl}\) (low-torque region), the measurements demonstrate a higher energy consumption, indicative of losses proportional to speed. Such losses may occur due to bearing friction, or they may reflect the speed-dependent losses of the gearbox. Whatever their cause, they can be represented by the damping term in Eq. (2.17), which was previously omitted from our calculations. Even though it is common not to take any damping into account in actuator design calculations, a damping term is often included in empirical models of structures. The reason for this is that the damping coefficient is a very generalized measure of various hard-to-quantify energy dissipating effects such as material hysteresis, bearing friction, joint friction etc. This makes it very structure-dependent, hence hard to predict, and therefore more suited for empirical rather than analytical modeling.

In order to determine to what extent damping can improve the model, the damping term in Eq. (2.17) is no longer neglected:

\[
T_{l,ext} = J \ddot{\theta} + v_{d1} \dot{\theta} + Mgl \sin \theta \tag{2.34}
\]

The linearized version of this equation is a second order system, which is fairly accurate at small angles and therefore often used to characterize the pendulum for control purposes ([20]). Continuing our quest for a more accurate model of the system, we will retain the nonlinear form of Eq. (2.34) and attempt to calculate the damping coefficient \(v_{d1}\) empirically by fitting this equation onto the torque measurement. The measured output angle and its first and second derivative are used as inputs to Eq. (2.34) rather than the imposed angle, which will not give a close fit due to the deviation in torque caused by controller ringing.

Eq. (2.34) still does not account for the bearing on the shaft between the torque sensor and the gearbox. The equation for the motor current is therefore extended to include
Table 2.5: Absolute and relative damping energy losses at different pendulum frequencies $\omega_{imp}$ ranging from 1 to 5 rad/s.

<table>
<thead>
<tr>
<th>$\omega_{imp}$ (rad/s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{damping1}$</td>
<td>0.058 J</td>
<td>0.061 J</td>
<td>0.064 J</td>
<td>0.060 J</td>
<td>0.052 J</td>
</tr>
<tr>
<td>$E_{damping2}$</td>
<td>0.544 J</td>
<td>0.692 J</td>
<td>0.865 J</td>
<td>1.251 J</td>
<td>1.863 J</td>
</tr>
<tr>
<td>$E_{damping1}/E_{tot}$</td>
<td>2.03%</td>
<td>2.43%</td>
<td>2.53%</td>
<td>2.33%</td>
<td>1.81%</td>
</tr>
<tr>
<td>$E_{damping2}/E_{tot}$</td>
<td>18.9%</td>
<td>27.7%</td>
<td>34.0%</td>
<td>49.0%</td>
<td>64.8%</td>
</tr>
</tbody>
</table>

Table 2.4: Empirical estimates of the damping coefficients $\nu_{d1}$ and $\nu_{d2}$ as defined in Eq. (2.34), for pendulum frequencies $\omega_{imp}$ ranging from 1 to 5 rad/s. The $R^2$ value was added in order to give an idea of the quality of the fit.

<table>
<thead>
<tr>
<th>$\omega_{imp}$ (rad/s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{d1}$</td>
<td>0.032</td>
<td>0.017</td>
<td>0.012</td>
<td>0.008</td>
<td>0.006</td>
</tr>
<tr>
<td>$\nu_{d2}$</td>
<td>0.295</td>
<td>0.188</td>
<td>0.156</td>
<td>0.172</td>
<td>0.201</td>
</tr>
<tr>
<td>$R^2(T_{l,ext})$</td>
<td>0.9995</td>
<td>0.9938</td>
<td>0.9869</td>
<td>0.9793</td>
<td>0.5658</td>
</tr>
<tr>
<td>$R^2(I_{ext})$</td>
<td>0.9763</td>
<td>0.9788</td>
<td>0.9412</td>
<td>0.8127</td>
<td>0.8214</td>
</tr>
</tbody>
</table>

$\nu_{d1}$ decreases slowly with pendulum frequency, although this is also true for the quality of the fit and, hence, the accuracy of the coefficients. The decreasing coefficients may indicate that a damping term overestimates the speed-dependency of the losses between the load and the torque sensor, which are mostly due to the bearing, and that another friction model (e.g. Coulomb friction) might be more suitable in this case. $\nu_{d2}$ has a value of around 0.17, which is reasonably constant except at 1 rad/s, where it reaches a value of 0.295.

By multiplying Eq. (2.34) by the output velocity and integrating over an entire period of motion, one can obtain the total mechanical energy consumption per cycle $E_{mech,ext}$ based on the extended model. The losses due to damping are given by

$$E_{damping} = \int \nu_{d} \dot{\theta}^2 dt$$  \hspace{1cm} (2.36)

Table 2.5 shows $E_{damping}$ for different pendulum frequencies ranging from 1 to 5 rad/s, for both damping coefficients. The measurements demonstrate a constant damping loss.
of around 0.06 J per cycle for the first damping coefficient. Relatively speaking, the contribution of this damping term never exceeds 2.6% of the measured electrical energy consumption. The second damping coefficient, however, does contribute largely to the total energy consumption, with losses up to 65% at the highest speed. The bearing on this side of the torque sensor, which is likely to be the main contributor to damping, indeed runs quite roughly compared to its counterpart on the other side, which explains the large difference in damping coefficients and their associated losses. In conclusion, the measurements provide evidence for the presence of damping, and depending on the construction of the setup, the addition of damping terms to the model may prove essential to achieving a better estimate of the total energy consumption of the system.

2.3.4.2 Peak power and energy consumption

Fig. 2.16 presents the modeled and measured energy consumption per pendulum cycle, both for the purely catalog-based model presented in sections 2.2-2.3 as for the empirically adjusted model (section 2.3.4.1). In the latter, the damping coefficients have been set to constant values of $\nu_{d1}=0.01$ and $\nu_{d2}=0.17$ to simplify the calculations. Because torque and speed measurements were taken before the gearbox, no net mechanical energy consumption was expected. The measurements, however, indicate a more or less constant loss of 0.06 J/cycle. This corresponds exactly to the energy loss we predicted in section 2.3.4.1 by adding the damping term $\nu_{d1}\dot{\theta}$ to the catalog-based model. The mechanical energy consumption is, however, quite small, and the error made by the catalog-based model is still acceptable. There is, however, a large difference between the electrical energy...
Figure 2.17: Measured and modeled peak powers as a function of pendulum frequency $\omega_{imp}$. Mechanical power is plotted in green, electrical power in red. The measurements (dots) are plotted alongside the catalog-based model (full line) and the adjusted model (dashed line).

consumption obtained from the catalog-based model and the measurements. This difference, which clearly increases with pendulum frequency, can almost entirely be attributed to damping. The adjusted model, which includes a damping term, indeed succeeds in estimating the energy consumption quite accurately. It predicts the shift in resonance frequency due to damping, and yields a maximum error of no more than 3.3%.

Looking at the peak powers (Fig. 2.17), one can see a close resemblance between the measurements and the model, although the measurements are consistently higher due to ringing of the controller. As predicted, the mechanical peak decreases near $\omega_{rl}$. The catalog-based model predicted minimum electrical peak power at $\omega_{rs} = 3.8$ rad/s, but no such minimum is seen in the measurements. However, the adjusted model follows the same trend as the measurements, indicating that the absence of this minimum is due to friction. These results demonstrate that the minimum mechanical peak power does not necessarily occur at the same frequency as the minimum electrical peak power. This was already noted in section 2.3.3.2, where drive inertia was identified as the cause for this phenomenon. This inertia affects the electrical but not the mechanical energy measurements, since the latter were taken before the gearbox. In conclusion, the measurements clearly prove our claim that the acceleration of the drive’s inertia can have a significant influence on the total power consumption of the actuator.

2.3.4.3 Electrical energy consumption and controller losses

Up till this point, we have measured the electrical power consumption at the motor terminals, because it corresponds to the power calculated by means of the electrical motor model (2.7). However, the current and voltage profiles change radically when going
through the controller, and depending on the control circuitry, so does the power profile. As discussed in paragraph 2.2, there are some misconceptions regarding negative energy in the robotics community. Since the Maxon EPOS2 50/5 controller allows for regeneration, it would be interesting to see to what extent the controller is able to recover the energy provided by the load.

In this section, measurements were taken at a swinging frequency of $\omega_{imp}=1$ rad/s. In order to obtain a higher power consumption, the weight of the pendulum was increased to 2.439 kg, and the amplitude of the swing was increased to $\theta_0=75^\circ$. Parameters of the set-up can be found in Table 2.6. Because the Maxon EPOS2 controller utilizes the same power source for the motor circuit and electronics, it is continuously drawing power from the source in order to power the electronics. This power consumption is not mentioned in the data sheet; it was measured to be 1.5W when the motor is not running. This was included in the model in order to increase its accuracy. The sensors were carefully calibrated for the measurement range, paying special attention to the zero-point calibration of the current sensor. Just like in the previous sections, ten periods were recorded and subsequently averaged to obtain average measurements for one cycle. These are presented in Fig. 2.18.

As mentioned earlier, the current and voltage profiles measured at the battery terminals – or, equivalently, at the input of the controller – are very different from the ones obtained from the model, since these were calculated at the output of the controller. The torque-current and speed-voltage relationships no longer apply at the battery terminals. Instead, voltage and current are related to the energy flowing from or to the battery. The voltage remains close to the battery voltage of 25V; it will increase above this voltage when energy is sent back to the battery and decrease when energy is consumed. Because of the constant voltage, the current is much lower than the one at the controller output. Its sign, as mentioned earlier, determines whether the battery is being charged or discharged. This is exactly why a good zero-point calibration of the current sensor is so important for this measurement.

The mechanical power measurements (Fig. 2.18) indicate that there is some negative energy available from the motion (8.4 J per period). About one tenth of this energy is dissipated in the gearbox, motor, bearings and controller, but the model predicts that there should still be a small but significant amount left (0.85 J per period) which can be stored in the battery. The measurements at the battery terminals clearly demonstrate zones of negative current (up to 40 mA) and power (up to 1.0 W), i.e. charge flowing to the battery, confirming that the available energy is sent back to the power source.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ($M$)</td>
<td>2.439 kg</td>
</tr>
<tr>
<td>Moment of inertia in rotation axis ($J$)</td>
<td>0.224 kg m$^2$</td>
</tr>
<tr>
<td>Distance from rotation axis to COG ($l$)</td>
<td>253.3 mm</td>
</tr>
</tbody>
</table>

Table 2.6: Pendulum parameters for the measurements presented in section 2.3.4.3 and 2.4.2.
Figure 2.18: Measurements conducted at the battery terminals, at a swinging frequency of 1 rad/s. Modeled values are indicated in blue, measurements in green. The modeled current and voltage waveform is the one observed at the motor terminals; this is why it differs from the measurements conducted at the battery terminals.
2.3.4.4 Overview

Model vs. measurement  Fig. 2.19 demonstrates how the different types of losses contribute to the total energy consumption. Since the measurements were retrieved with limited equipment, the loss distribution displayed in the figure is actually a semi-empirical one, drawing all its information from torque, position, current and voltage measurements, data sheets, and the damping coefficients obtained in section 2.3.4.1. The modeled energy consumption is based on the position imposed by the controller, and includes all the contributions studied in this section which can be retrieved without having to build the actual system, i.e. everything except damping. The predicted energy consumption of 35.1J was only 9.7% higher than the measured consumption of 31.7J. The fact that the modeled energy consumption is higher than the measured energy consumption is somewhat unexpected. Tracking errors are the most likely cause for this observation. Indeed, in Fig. 2.18, the measured mechanical energy consumption does not completely reach the amplitude of the modeled energy consumption.

On a final note, it must be mentioned that the measurements itself do not allow to distinguish between gearbox and motor losses, because this would require a torque sensor in between both. The distribution is therefore an estimate based on the motor model.

Distribution of losses  The motor (42% of modeled losses, 46% of measured losses) is responsible for most losses, even though its maximum efficiency of 87.8% is higher than the gearbox’s maximum efficiency of 72%. The reason for this observation is that the gearbox is modeled with a constant efficiency, whereas a load- and speed-dependent model was applied to the motor. As a result, the average efficiency of the motor will be lower than its maximum efficiency, especially when the motor is operated far from its rated values. Fig. 2.20 demonstrates that the motor is definitely not operated in high-
2.3 Case study: pendulum

Figure 2.20: Operating points on the motor efficiency map (white), for the pendulum specified in Table 2.6, at 1 rad/s. Motor torque and speed are way below the rated values (rated torque: 120 mNm, rated speed: 6640 rpm). Consequently, the motor is operating mostly in low efficiency regions.

efficiency regions for most of the trajectory. In the measurements presented in this section, on average, the motor is moving at 10.6% of the rated speed and delivering 32.4% of the rated torque. The average motor efficiency was measured to be only 28.7%, confirming the theory. Gearbox losses are also load-dependent and, to a lesser extent, speed-dependent. With the gearbox loaded 28.3% on average and operating at a low speed, it will not leave the nearly-flat efficiency region for most of the motion; the estimated gearbox losses can therefore be assumed to be fairly accurate.

The second source of energy losses, accounting for 39% of modeled and 29% of measured power losses, is the controller. With the datasheet mentioning a controller efficiency of 94%, this may come as a surprise. The reason why its power consumption is so high, is that it has supplementary electronics on board which continuously draw 60mA (standby current). With a source voltage of 25V, this corresponds to 1.5W, or 4.5% of the total losses. In low-power applications such as this one, controller electronics can constitute a significant portion of the energetic cost. Of course, as most energy losses are proportional to the power required to move the load whereas the consumption of the controller electronics is nearly constant, the latter’s share in the total energy consumption will quickly decrease. Moreover, by using a more dedicated controller than multi-purpose Maxon EPOS2, the energetic cost could most likely be reduced significantly.

Damping is only a minor contributor to the total energy consumption (3%). Looking
back at the results from section 2.3.4.2, where damping caused about 20% of the losses at $\omega_{imp}=1$ rad/s, this may come as a surprise. The total amount of consumed energy is, however, very different for both sets of measurements. In the measurements presented in this section, the energy consumption at the motor terminals is 22.6 J. In section 2.3.4.2, only 2.87 J was consumed for the same output trajectory. Because damping losses depend only on the output speed – which is identical in both cases – the energy lost through damping is the same, but the relative losses are much lower when higher torques are applied. Consequently, whether or not damping can be neglected in the model depends not only on the construction of the actuator and the speed of the trajectory, but also on its load.

Other losses account for the controller (ringing and insufficient tracking). They represent less than 0.4% of the measured energy consumption.

### 2.3.5 Discussion

From our simulations and measurements, we were able to construct a set of recommendations regarding actuator modeling and the calculation of energy consumption:

- Negative power can be regenerated with a suitable choice of drive circuitry and power source.
- The inertia of the motor and gearbox cannot be neglected in most dynamic applications, especially when the gears apply a substantial reduction.
- A speed- and load-dependent motor model is able to estimate the decrease in motor efficiency in case of low torques and speeds. This is of great importance in robotics, where the actuators often operate in exactly these areas.
- In low-power applications, the current needed to power the drive electronics can provide a major contribution to the total power consumption.
- The introduction of friction terms can improve the quality of the model significantly, especially for low-power applications at high speeds and low torques. From a design point of view, this highlights the importance of a low-friction design in this type of application.

The tests in this section have demonstrated that, if all the above recommendations are taken into account, an accurate estimation of the energy consumption can be made. In the low-speed high-torque test presented in section 2.3.4.4, the model overestimated the actuator’s actual energy consumption by less than 10%. In section 2.3.4.2, in which the motor was operated at high speeds and low torques, errors of less than 4% were obtained provided if a damping coefficient was added to the model. Without this empirical improvement, errors could reach up to 63%. This indicates that a large part of the losses in this operating regime are due to friction, and that the model needs to be adjusted accordingly to generate satisfactory results.
2.4 The influence of the actuator model

In the previous section, we have shown that accurate models may help to improve the energy-efficiency of actuator designs by revealing load- and speed-dependent losses which are generally overlooked. The aim of this section is to assess how less adequate models affect the estimation of the energy consumption for a simple task - again, an 80W geared DC motor applying a sinusoidal trajectory to a pendulum. In Section 2.4.1, we present four models commonly found in literature, and we give some background about the calculation of energy from power. An experimental comparison of the models, based on the power profiles and energy consumptions they yield, is presented in Section 2.4.2. Finally, we will discuss to what extent the models are suited to predict the energy consumption of a geared DC motor, depending on the load and trajectory (Section 2.4.3).


2.4.1 Basic models

2.4.1.1 Catalog-based motor and gearbox models

**First Quadrant Constant Efficiency approach (1QCE)** In the first approach, we will calculate the energy consumption as if the drive were operated at steady-state conditions in the first quadrant of operation (positive torque and speed). The relation between the torque at the gearbox shaft $T_l$ and the torque at the motor shaft $T_m$ can be calculated if gearbox efficiency $\eta_{tr}$ and gear ratio $n$ are known:

$$T_l = n\eta_{tr}T_m \quad (2.37)$$

Assuming that losses do not affect voltage but only current, we can estimate the motor current $I$ based on the motor’s torque constant $k_t$ and the catalog efficiency $\eta_m$:

$$T_m = k_t\eta_m I \quad (2.38)$$

The motor’s voltage $U$ is a function of the motor’s speed constant $k_b$ and motor speed $\dot{\theta}_m = n\dot{\theta}_l$:

$$U = k_b\dot{\theta}_m \quad (2.39)$$

$$= k_b n \dot{\theta}_l$$

Consequently, the consumed electrical power $P_{elec}$ will be
\[ P_{elec} = UI \]  
\[ = \frac{k_b}{k_t \eta_m} \dot{\theta}_m T_m \]  
\[ = \frac{1}{\eta_m \eta_{tr}} \dot{\theta}_l T_l \]  
\[ = \frac{1}{\eta_m \eta_{tr}} P_{mech} \]

This last formula, which is correct if the motor is constantly operating at its maximum efficiency, is perhaps the most common way of calculating the energy consumed by a motor.

**Four Quadrant Constant Efficiency approach (4QCE)** In four-quadrant operation, power can flow from the motor to the load (quadrants I and III) or vice versa (quadrants II and IV). In the latter case, the load is driving the motor, and the losses must be deducted from the energy of the load instead of the energy at the motor shaft. As proposed in [79], this can be implemented by defining a gearbox efficiency function \( C_{tr} \):

\[ C_{tr} = \begin{cases} \eta_{tr} & \text{(load driven by motor)} \\ 1/\eta_{tr} & \text{(motor driven by load)} \end{cases} \]  

Equation (2.37) becomes

\[ T_l = C_{tr} n T_m \]  

Similarly, we can define a motor efficiency function \( C_m \) to rewrite Eq. (2.38):

\[ C_m = \begin{cases} \eta_m & \text{(load driven by motor)} \\ 1/\eta_m & \text{(motor driven by load)} \end{cases} \]  

\[ T_m = k_t C_m I \]  

Motor voltage and electrical power can be calculated by applying equations (2.39) and (2.40).

**Four Quadrant Constant Efficiency approach with motor and gearbox Inertia (4QCEI)** The next step is to add the motor and gearbox inertia to the model. If gearbox inertia \( J_{tr} \) is specified at the input shaft, the shaft torque becomes

\[ T_m = \frac{1}{n C_{tr}} \cdot T_l + J_{tr} \dot{\theta}_m \]  

and a term containing motor inertia \( J_m \) is added to Eq. (2.44):

\[ I = \frac{1}{k_t C_m} \left( T_m + J_m \dot{\theta}_m \right) \]
Full DC Motor Model approach (FMM)  In this approach, we will use a full DC motor model instead of the motor efficiency function $C_m$. The equations for motor current (2.46) and voltage (2.39) are replaced with the motor model proposed in Section 2.2.3, where, for reasons stated there, the induced voltage due to inductance $L$ is neglected:

$$\begin{align*}
I &= T_m + \nu_m \dot{\theta}_m \\
U &= RI + k_b \dot{\theta}_m
\end{align*}$$  (2.47)

For the motor’s viscous damping coefficient $\nu_m$, we will use the estimate

$$\nu_m = \frac{k_i \cdot I_{nl}}{\omega_{nl}}$$  (2.48)

2.4.1.2 Controller losses

Because we are interested in the total energy consumption of the actuator, experiments will be carried out by measuring the power consumption at the battery terminals. This means that, in addition to the above model, the losses due to the controller need to be calculated. As explained in section 2.2.4, these losses can be modeled as

$$P_{source} = C_c P_{elec} + P_{standby}$$  (2.49)

in which $P_{standby}$ corresponds to the continuous losses due to controller electronics and $C_c$ is the controller efficiency function, given by Eq. (2.14). This model will be applied for the 4QCE, 4QCEI and FMM cases. Only in the 1QCE case, $C_c$ will be set to $1/\eta_{controller}$ at all time, consistent with the way efficiency is treated in the model.

2.4.1.3 Energy consumption models

Integration of power  Following the basic relationship between power and energy, we can calculate the electrical energy consumption $E_{elec}$ from the source power $P_{source}$ by integration of the latter w.r.t. time:

$$E_{elec} = \int P_{source}(t) dt$$  (2.50)

with the integration interval spanning the entire duration of the motion. This is the approach which is followed in most branches of physics and engineering, and which should be applied to controllers which allow for regeneration.

Integrated absolute power model  In robotics, it is common to make the assumption that the energetic cost of absorbing power is as high as supplying power to the load. Practically, this means that in Eq. (2.50) the integrand $P_{source}$ should be replaced by its absolute value:
\[ E_{\text{elec,abs}} = \int |P_{\text{source}}(t)| \, dt \]  

(2.51)

Physically, this formula states that, in case of negative power flow, the motor and the controller are receiving energy from both the load and the power source. This would mean that all the energy is dissipated somewhere in these components. While this is rather illogical, the formula has the benefit of yielding a systematically higher energy consumption than Eq. (2.50), which will consistently underestimate energy consumption if any loss mechanisms are missing in the calculation of \( P_{\text{source}} \). Consequently, the physically incorrect equation (2.51) may in some cases be closer to the actual energy consumption than the physically correct equation (2.50). Nevertheless, as we will demonstrate in Section 2.4.2, Eq. (2.51) is also strongly dependent on the model which is used, and will therefore produce more arbitrary results than Eq. (2.50).

**Integrated positive power model**  
A final approach, suggested in [188] but only followed in a limited number of papers [84, 243], is the integration of positive power:

\[ E_{\text{elec,pos}} = \int \max(0, P_{\text{source}}(t)) \, dt \]  

(2.52)

This approach would be most suitable if the controller does not allow for regeneration, but braking is possible through the use of braking resistors, or if electronic circuitry is preventing the controller from sending current into the battery. Such protection circuits are common with high-end batteries such as Li-Ion batteries, which may get damaged or even explode when overcharged [9].

Comparing Eq. (2.52) to Eqs. (2.50) and (2.51), one can easily prove that

\[ E_{\text{elec,pos}} = \frac{1}{2} (E_{\text{elec}} + E_{\text{elec,abs}}) \]  

(2.53)

In other words, \( E_{\text{elec,pos}} \) is simply the average of \( E_{\text{elec}} \) and \( E_{\text{elec,abs}} \).

### 2.4.2 Tests

In order to assess the quality of the models established in Section 2.4.1, they are applied to the setup described in 2.3.2. This time, the motor, again the 80 W Maxon DCX35L motor with a planetary gearbox of ratio \( n=113 \) specified in Table 2.2), will impose a sinusoidal trajectory to the output

\[ \theta(t) = \theta_0 \sin(\omega t) \]  

(2.54)

with frequencies \( \omega \) of 0.5, 1, 2 and 5 rad/s and an amplitude \( \theta_0 \) of 80°. This dynamic task spans all four quadrants of operation and has a net mechanical energy consumption of zero.
The relation between the output angle $\theta$ and the gearbox output torque $T_l$ is

$$T_l = J\ddot{\theta} + Mgl\sin(\theta) + \text{sign}(\dot{\theta}) \cdot T_C + \nu \dot{\theta} \quad (2.55)$$

of which the relevant parameters can be found in Table 2.6. Friction is represented by Coulomb friction $\text{sign}(\dot{\theta}) \cdot T_C$ and viscous friction $\nu \dot{\theta}$, a generally accepted classic model [158]. The values for $T_C$ and $\nu$ were obtained experimentally, and were found to be 0.064 Nm and 0.081 Nms/rad.

Finally, the mechanical output power can be obtained by multiplying the derivative of Eq. (2.54) with Eq. (2.55):

$$P_{\text{mech}} = T_l \dot{\theta} \quad (2.56)$$

### 2.4.2.1 Mechanical output power

In order to validate the model of the setup, the power profile at the motor terminals was measured. The results at 0.5 rad/s are shown in Fig. 2.21. The good match between the measurement and the model indicate that the parameters of the setup (Table 2.6) are estimated correctly, and that Eq. (2.55) and Eq. (2.54) describe the torque and position of the output well. The symmetry of the curve also demonstrates that the amounts of negative and positive work forced upon the load are nearly equal. This symmetry is maintained at higher pendulum speeds.

### 2.4.2.2 Source power

We will now compare how well the modeled and measured source powers match. Fig. 2.22 shows the measured and modeled electrical power at the battery terminal, for a pendulum swinging at 0.5 rad/s and 5 rad/s. Note that, unlike the mechanical power, none of the electrical power profiles is symmetrical w.r.t. the power axis. This is mainly caused by the - mostly constant - controller losses, which add an offset to the electrical power curves.

At 5 rad/s, regeneration occurs between 0.1-0.3s and 0.7-0.9s. At 0.5 rad/s however, negative power is completely consumed by the losses, so that none of it is left at the battery terminals. This demonstrates how hard it can be to design a low-power system which can recover energy from the load. To have a considerable amount of negative power available at the load is, obviously, an important condition for energy regeneration, but secondly, the system should also be designed to have a high efficiency, so that this energy is not lost when power is flowing through the actuator. Only if both conditions are met, the negative energy can be used to recharge the battery. A successful example can be found in [199], where the authors performed an experiment in which they managed to regenerate 63% of the negative work. Another example is the transfemoral prosthesis presented in [212], where the authors regenerated approximately 50W of energy with their knee actuator during the late swing phase of gait.

The discontinuity in the power profiles is not predicted by the IQCE model because the directionality of gearbox efficiency, which is the cause of this phenomenon, is not
Figure 2.21: Measured and modeled mechanical power consumption at the output, for one period of the pendulum at 0.5 rad/s. There is a good match between both, indicating that the mechanical model of the setup is accurate.

incorporated into this model. The results obtained by using this model can therefore be discarded, especially when low-efficiency gearboxes are used. The 4QCE and 4QCEI models yield very similar power profiles at low speed (0.5 rad/s), because the gravitational torque is rather high compared to the inertial torque caused by the motor’s acceleration. At higher speeds and accelerations (5 rad/s), however, the power model obtained by the 4QCE model is far from accurate. The peak in electrical power in the 0.5 rad/s measurement occurs at a later time in than predicted by the 4QCE and 4QCEI models, and its amplitude is higher. The FMM model, in which the motor efficiency is load- and speed-dependent, is able to predict this shift, even though it still does not fully match the measurements. A possible cause is the load- and speed dependency of gearbox efficiency, something which is not accounted for in any of the models. Finally, the notches predicted by the FMM and 4QCEI models are not clearly visible in the 5 rad/s power measurement, because the closed loop time constant was too low to track the discontinuity at this speed. Nevertheless, the measurements clearly follow the general trend predicted by the FMM and 4QCEI model. Both models do not differ much in this measurement, because here, the motor operates mostly in regions with near-maximum efficiency. In any case, the measurements indicate that load- and speed-dependent losses affect the input power of an actuated system, and that they can be predicted provided that an adequate model is used.
Figure 2.22: Measured and modeled electrical power consumption at the battery terminals for one period of the pendulum at (a) 0.5 rad/s and (b) 5 rad/s. While the discontinuities in the power profile due to the gearbox efficiency (2.43) are clearly visible in the measurements at 0.5 rad/s, they are hardly distinguishable in the 5 rad/s measurement. This is due to the time constant of the closed loop system, which is too slow to track the required discontinuous current at this frequency.
2.4.2.3 Consumed energy

Although the comparison of power profiles already gives a lot of information, it is still interesting to see how they translate to energy consumption. The measured and modeled energy consumption of the pendulum at frequencies of 0.5, 1, 2 and 5 rad/s is displayed in Table 2.7. Both equations (2.50) and (2.51) are used to derive the energy consumption from the powers obtained by all models presented in Section 2.4.1.1.

Energy consumption decreases with frequency, as the measurements are increasingly nearer to the pendulum’s resonance frequency (4.4 rad/s) at which energy consumption will be minimal [232], and because the time for the pendulum to complete one period increases as the frequency decreases. The measurements and all models confirm this general trend. The energy consumed by the idle current of the controller is a linear function of time, and so, at low speeds, it will represent a large portion of the energy consumption. At 0.5 rad/s, the idle current causes a loss of 18.6 J (36.4% of the total consumption), whereas at 5 rad/s, this is only 1.9 J (15.7% of total energy consumption). This demonstrates that an energy-efficient controller and, more generally, energy-efficient electronics, can also contribute to the reduction of energy losses, especially in low-power applications.

We will now discuss more specific trends related to the models and the way energy is calculated.

Table 2.7: Measured and modeled energy consumption for one pendulum period, at frequencies of 0.5, 1, 2 and 5 rad/s.

<table>
<thead>
<tr>
<th>Model</th>
<th>0.5 rad/s</th>
<th>1 rad/s</th>
<th>2 rad/s</th>
<th>5 rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured</td>
<td>51.08 J</td>
<td>29.90 J</td>
<td>18.30 J</td>
<td>11.85 J</td>
</tr>
<tr>
<td>1QCE</td>
<td>$E_{elec}$</td>
<td>46.73 J</td>
<td>23.35 J</td>
<td>11.50 J</td>
</tr>
<tr>
<td></td>
<td>$E_{elec, abs}$</td>
<td>49.71 J</td>
<td>36.54 J</td>
<td>29.16 J</td>
</tr>
<tr>
<td></td>
<td>$E_{elec, pos}$</td>
<td>48.21 J</td>
<td>29.95 J</td>
<td>20.33 J</td>
</tr>
<tr>
<td>4QCE</td>
<td>$E_{elec}$</td>
<td>45.92 J</td>
<td>27.99 J</td>
<td>17.81 J</td>
</tr>
<tr>
<td></td>
<td>$E_{elec, abs}$</td>
<td>45.92 J</td>
<td>30.67 J</td>
<td>22.76 J</td>
</tr>
<tr>
<td></td>
<td>$E_{elec, pos}$</td>
<td>45.92 J</td>
<td>29.33 J</td>
<td>20.29 J</td>
</tr>
<tr>
<td>4QCEI</td>
<td>$E_{elec}$</td>
<td>45.73 J</td>
<td>27.57 J</td>
<td>16.91 J</td>
</tr>
<tr>
<td></td>
<td>$E_{elec, abs}$</td>
<td>45.73 J</td>
<td>29.99 J</td>
<td>20.44 J</td>
</tr>
<tr>
<td></td>
<td>$E_{elec, pos}$</td>
<td>45.73 J</td>
<td>28.78 J</td>
<td>18.68 J</td>
</tr>
<tr>
<td>FMM</td>
<td>$E_{elec}$</td>
<td>52.55 J</td>
<td>29.80 J</td>
<td>17.25 J</td>
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<tr>
<td></td>
<td>$E_{elec, abs}$</td>
<td>52.55 J</td>
<td>31.92 J</td>
<td>20.67 J</td>
</tr>
<tr>
<td></td>
<td>$E_{elec, pos}$</td>
<td>52.55 J</td>
<td>30.86 J</td>
<td>18.96 J</td>
</tr>
</tbody>
</table>

$E_{elec}$ vs. $E_{elec, abs}$ Comparing $E_{elec}$ to $E_{elec, abs}$, we observe no difference at all at a frequency of 0.5 rad/s, because the negative power from the load is entirely consumed by the losses. The difference increases at higher frequencies, at which more negative power is available. At 5 rad/s, the $E_{elec}$ value obtained by the FMM model clearly provides the best prediction.
Table 2.7 demonstrates a striking trend in the way how modeled energy consumption evolves as the complexity of the model increases. As we move from the simple yet incorrect 1QCE model to the more complex FMM model, which contains more loss mechanisms, we see that, in general, the energy consumption predicted by $E_{elec}$ is increasing. The inertia of the motor and gearbox may counter this effect (as evidenced by comparing the 4QCE and 4QCEI model at 0.5-2 rad/s) by slightly redistributing the power over the pendulum cycle, but apart from this, there is a clear trend towards increasing energy consumption with increasing model complexity. This is in line with what one would expect: the more losses are considered, the higher the predicted energy consumption will be. Looking at $E_{elec,abs}$, however, there is little consistency in how energy consumption evolves with model complexity. At 5 rad/s, $E_{elec,abs}$ follows the logical trend of increasing energy consumption with increasing model accuracy, whereas at 1 and 2 rad/s, the opposite is true. The least accurate model, the 1QCE model, yields the highest energy consumption $E_{elec,abs}$ at 1 rad/s and 2 rad/s, overestimating the actual consumption by no less than 22.2% and 59.3%. It is the combination of two inaccuracies that causes the predicted energy consumption to boom. First, Eq. (2.37) will lead to negative power losses in negative power flow, increasing the amount of negative energy instead of reducing it. This conflicts with the “passive sign convention", which states that dissipated power is a positive quantity [85]. Second, this error is amplified by the use of Eq. (2.51), which converts the - overestimated - negative energy into a positive contribution to the total energy consumption. In conclusion, the results presented in Table 2.7 prove that Eq. (2.51) can lead to serious errors, especially if losses are modeled incorrectly in negative power flow.

**Comparison between models** The 1QCE model yields very inaccurate energy consumption values, underestimating energy consumption (in case of $E_{elec}$) by up to 44.1% at 2 rad/s, or overestimating it (in case of $E_{elec,abs}$) by up to 59.3% at 2 rad/s. This comes as no surprise, as in Section 2.4.2.2 we already pointed out that the measured power profile corresponded very badly with the one obtained from the 1QCE model.

The only difference between the 4QCE and 4QCEI models is the addition of gearbox and motor inertia to the model. Inertia acts as an energy buffer, so intrinsically it does not cause additional energy losses. This is why both models produce very similar energies, especially at low frequencies. One can notice that, moving from the 4QCE to the 4QCEI model, energy consumption decreases at 0.5, 1 and 2 rad/s, but increases at 5 rad/s. By adding gearbox and motor inertia to the model, the total inertia of the modeled system increases, and consequently its resonance frequency $\omega_0$, which is given by

$$\omega_0 = \sqrt{\frac{MgI}{J + n^2 \eta tr (J_m + J_{tr})}}$$

(2.57)

for the linearized system, decreases. With $\omega_0 = 4.4$ rad/s in this particular setup, the 0.5, 1 and 2 rad/s measurements are performed below resonance frequency. In this case, the lowered resonance frequency due to the additional inertias will lead to lower motor currents and powers. The Joule losses being proportional to current squared, this in turn will lead to lower energy losses. Conversely, if a frequency above resonance is imposed,
as in the 5 rad/s measurement, the torques and powers will increase, and so will the energy losses in the system.

Finally, there is the FMM model, which incorporates load- and speed-dependent motor losses and is by far the most detailed of the four models. Because the catalog efficiency used in the other models is in fact the maximum efficiency $\eta_m$, the energy consumption predicted by the FMM model will always be higher than the 4QCEI model, which assumes a constant motor efficiency of $\eta_m$. Looking at $E_{elec}$, the FMM model produces very decent results, with a maximum error of 8.1% at 5 rad/s. Its underestimation of power consumption indicates that there is still some room for improvement by adding yet unmodeled losses, e.g. speed- and load-dependent gearbox losses, or by improving the current model, e.g. by modeling the influence of motor heating on the motor winding resistance or by increasing the complexity of the friction model, as suggested in Section 2.3.2.

2.4.3 Discussion

The aim of this section was to study how well different modeling approaches commonly found in literature can predict the energy consumption of a geared DC motor performing a dynamic task. If one thing stood out clearly from the measurements presented in this section, it is the importance of defining efficiency functions based on the direction of power flow. If the equations which apply to the 1st quadrant of motor operation are maintained - something which is all too often the case - this can lead to serious errors in the estimated energy consumption. A DC motor model can help to cover more of the load- and speed-dependent losses, and so may a load- and speed-dependent gearbox efficiency model. The latter is more difficult to generate from datasheet information though, and for this reason, it was not studied in this section. Gearbox and motor inertia have an impact on the resonance frequency of the system, making them particularly relevant for systems which require high accelerations at low torques. While the inertia itself does not cause additional losses, it may cause the total system loss to drop or increase by changing the current flowing through the motor.

Even though the motion demanded equal amounts of positive and negative work to be done on the load, and even though the amount of negative work was substantial, almost none of it was retained at the battery terminals in the slowest measurements. This demonstrates how hard it can be to regenerate negative energy in a simple actuator system. Calculating the energy consumption is by integrating the absolute value of power was shown to be an unreliable method which produces inconsistent results depending on the model which is used to calculate the losses. The physically correct approach of integrating the power itself leads to consistently good results, provided that the system is modeled sufficiently well.
2.5 Modeling for energy optimizations

So far, the focus of this chapter was on obtaining accurate models. An important field where these models are applied, is the field of optimization. In optimizations, a cost function is minimized based on a model which – in the case of energy minimization – will represent the actuator’s energy losses. The results from section 2.4 have clearly indicated that the outcome of the optimization will depend strongly on the quality of the model, as well as on the choice of the cost function. Before discussing the choice of the cost function (section 2.5), we first look into an important characteristic of the actuator’s losses: their relationship to the output speed and torque.

### 2.5.1 Speed- and torque-dependency of losses

In section 2.2, we gave an overview of the losses in a typical actuation system consisting of an electric motor and gearbox. We related these losses to the components of the actuator, and presented the models that can be used to describe them. All these models depend on speed, torque, or a combination of both. Based on this observation, the power losses in an actuator can be divided into four groups, summarized in Table 2.8.

<table>
<thead>
<tr>
<th>Description</th>
<th>Dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joule losses (motor)</td>
<td>$P_{\text{joule}} \sim T^2$</td>
</tr>
<tr>
<td>Gearbox losses</td>
<td>$P_{\text{gearbox}} \sim P_{\text{mech}} = T\dot{\theta}$</td>
</tr>
<tr>
<td>Friction losses – Coulomb</td>
<td>$P_{\text{Coulomb}} \sim \dot{\theta}$</td>
</tr>
<tr>
<td>Friction losses – viscous</td>
<td>$P_{\text{viscous}} \sim \dot{\theta}^2$</td>
</tr>
</tbody>
</table>

*Table 2.8:* Relationship between different loss mechanisms and the torque $T$ and speed $\dot{\theta}$ experienced by the component, based on the models presented in section 2.2 for an actuator composed of a motor and planetary gearbox.

In this table, $T$ refers to the torque experienced by the component itself. This means that losses downstream of the power flow path need to be added to this torque as well. For example, the motor – which is the most upstream component if the motor is delivering power to the output – will also need to provide a torque to compensate for the friction losses in the system, which depend on the output speed. As a result, the Joule losses of the motor are not only described by a term proportional to the square of the output torque, but also to several mixed terms consisting of torque and speed. Although, in practical cases, many of these mixed terms will have a negligible contribution to the overall Joule losses, it illustrates how the total power consumption can become a very complex nonlinear function of the output torque and speed, with dependencies on speed and torque that are not yet listed in Table 2.8. Still, the four classes of losses listed in this table serve as an excellent guideline for the choice of a cost function, which we will discuss below.
CHAPTER 2. MODELING THE POWER CONSUMPTION OF ACTUATORS

2.5.2 Cost functions

For optimizations, a simple cost function is usually preferred. The two most common cost functions are based on mechanical power ($P_{\text{mech}}^2$) and motor torque ($T_{\text{m}}^2$), where friction is usually neglected. If the imposed trajectory and torque are continuous, so are these cost functions. This will facilitate the optimization, decreasing computational time. Furthermore, friction parameters, component efficiencies etc. might be difficult to obtain, especially for complex actuation systems, which makes a direct optimization of the electrical power difficult. Based on the information in Table 2.8, however, actuator losses are not simply proportional to the mechanical power or torque, but depend on various combinations of speed and torque. Below, we will discuss how this affects the quality of the optimization.

2.5.2.1 Mechanical power flow reduction

The reduction of mechanical power flow is probably the most common criterion for energy reduction. Here, the output torque $T$ and output speed $\omega$ are assumed to be known based on the imposed output trajectory. This input trajectory is typically the subject of the optimization, either in a direct way (trajectory optimization) or in an indirect way (optimization of mechanism kinematics). The optimization cost function $\Pi_{\text{mech}}$ is then based on the mechanical output power $P_{\text{mech}} = T \omega$:

$$\Pi_{\text{mech}} = \int P_{\text{mech}}^2 dt \quad (2.58)$$

Sometimes, this cost function is said to minimize the mechanical energy which, by definition, is given by

$$E_{\text{mech}} = \int P_{\text{mech}} dt \quad (2.59)$$

and, as a result, can become negative. This is the case if, over the course of the task, the load is returning its energy to the actuator. The cost function $\Pi_{\text{mech}}$, however, does not necessarily minimize the mechanical power consumption. Instead, it is a measure of the power flow through the actuator, and mostly prevents high power peaks – negative or positive, both are treated equally, even though negative power decrease the energy consumption. This is why we chose to name this optimization strategy “mechanical power flow reduction” rather than “energy reduction”, as it is often called.

In section 1.3.1, we already gave a brief conceptual explanation of how a reduction of mechanical power flow can increase the energy efficiency of an actuator with energy buffers. This explanation can now be elaborated in a more formal way. If the actuator does not have an energy buffer, $P_{\text{mech}}$ will correspond to the power required to move the load, $P_{\text{load}}$. Conversely, if an energy buffer such as a spring is used to make the actuator more efficient, the optimization’s objective will typically be to find the optimal design parameters of the buffer (e.g. spring stiffness), and $P_{\text{mech}}$ will be defined as the power at the gearbox output:

$$P_{\text{mech}} = P_{\text{load}} + P_{\text{buffer}}$$
In this case, the energy optimization will attempt to minimize the power flow through the gearbox and motor by using the energy storage capability of the buffer. According to Table 2.8, this would minimize the gearbox losses. Motor and friction losses, to some extent, will also be reduced, because these depend on either torque or speed, and the mechanical power flow is the product of both.

Based on this observation, one might roughly state that a component’s losses correlate with the power flow through the component, and thus, ultimately, the electrical energy consumption of an actuator depends on the mechanical power that flows through its components. Therefore, the cost function $\Pi_{\text{mech}}$ can be expected to minimize the actuator’s energy consumption as long as $P_{\text{mech}}$ is a good representation of the internal power flows. This assumption will hold if internal energy buffers such as inertias (e.g. motor or gearbox inertia) and elasticities (e.g. cables, torsion stiffness of axles,...) are accounted for in the model.

### 2.5.2.2 Joule losses reduction

Another common approach is to use the square of the motor torque $T_m$ as a cost function:

$$\Pi_T = \int T_m^2 \, dt$$

(2.60)

According to Table 2.8, this cost function reduces the Joule losses, but completely neglects speed-dependent losses in the motor. Furthermore, gearbox losses are usually accounted for in $T_m$, but friction losses are often not.

Joule losses are often a major source of power losses [199, 174]. In such cases, this cost function can be expected to give good results. Just like for the cost function $\Pi_{\text{mech}}$, and perhaps even more so, a correct model of all energy buffers is crucial towards achieving good results. An important contribution, which is often neglected, is the inertia of the motor and gearbox. The importance of the drivetrain inertia is was discussed in section (2.3.3.2). It is evidenced by the principle of inertia matching, which states that, for a purely inertial load the most energy-efficient design of a drivetrain is one where the inertia of the load matches the reflected inertia of the actuator [104]. For more complex loads typical in the field of robotics, the reflected inertia of the actuator is often even higher than that of the output link [251]. Not taking it into account the drive inertia can therefore lead to a serious underestimation of the motor torque and, consequently, the Joule losses and the cost function $\Pi_T$.

### 2.5.2.3 Electrical power reduction

The goal of the optimization is to reduce the energy drawn from the power supply. Therefore, from a conceptual point of view, the goal is most likely to be achieved if electrical energy consumption itself is chosen as the optimization goal:

$$\Pi_{\text{elec}} = \int P_{\text{elec}} \, dt$$

(2.61)
Note that, here, the cost function does not integrate the quadratic form of the power, but corresponds to the actual definition of energy consumption. As explained in section (2.4), an optimization based on the square of the electrical power would lead to an overestimation of the actual power consumption.

The main challenge here is to model the electrical power $P_{elec}$. This is a non-trivial task, because ideally, it requires an identification of all losses in between the load and the batteries. As discussed in section (2.4), any unmodeled loss will lead to an underestimation of the energy consumption of the actuator. This is a major disadvantage with respect to the cost functions $\Pi_{elec}$ and $\Pi_r$, which are less prone to modeling errors. Realistic loss models are also often discontinuous, and require continuous approximations to work. Because of the more complex cost function, the optimization might also become non-convex, such that a global minimum can no longer be guaranteed.

Despite all these disadvantages, with some effort, the cost function $\Pi_{elec}$ can capture the influence of losses which are not directly proportional to power flow, but which rely on complicated relationships with speed and torque. Coulomb friction, the main contributor to friction in a realistic system, is an example of such an energy loss. Another important advantage of this approach is that, with a realistic estimation of motor current and voltage, motor limitations can be taken into account. As we will show in Section 3.5, this can have a huge impact on the results of the optimization, especially if the system becomes more complex.

### 2.5.3 Summary

When it comes to choosing the cost function for energy optimizations, there is no definite answer as to what is the best option. If an accurate model of the actuator is available, a cost function based on electrical energy can be expected to give the best results. Otherwise, cost functions based on the motor torque or output power are more practical candidates. The choice for one or the other depends on the relative contribution of the motor (Joule) losses. If the motor needs to deliver relatively high torques – something which is highly influenced by the gear ratio – Joule losses can be expected to dominate over other losses, and a $T_m^2$ cost function will most likely yield the most energy-efficient design. Conversely, if the motor torque is relatively low in comparison to the speed, gearbox and friction losses will be the dominant factor. In this case, a minimization of the mechanical power flow will have the best chance of minimizing the overall energy consumption of the actuator.

### 2.6 Conclusion

There is a considerable amount of literature on model-based optimizations and analyses of actuators for robotics. Obviously, the outcome of these works strongly depends on the way how the actuators are modeled. This is particularly the case if the goal is to minimize the energy consumption. Interestingly, authors often employ very different methods and models for their optimizations.
In this chapter, we have given an overview of different losses that occur in a geared electric motor. This very basic drivetrain can stand alone as an actuator, but also often constitutes the heart of more complex actuators and robotic systems. As a result of the losses, the energy consumption of even such a simple actuator becomes a complex non-linear function of the output torque and speed, with a strong relationship to the power flow through its components and, especially, the direction of this power flow.

A series of experiments on a pendulum setup have revealed severing interesting observations regarding the energy-efficient design of actuators. Although the motor is a very efficient component, its energy efficiency in robotic applications is very poor, because it is rarely used in its most efficient working regions. Furthermore, the high gear reductions that are often required in robotics add to the losses. An efficient selection of motor and gearbox is therefore one of the keys to reducing energy consumption. Drivetrain inertia, in particular, was shown to have a great influence on the energy consumption due to its impact on the internal power flows in the system.

Finally, a surprisingly important component was found to be the drive. Drive electronics consume a fairly constant amount of power, which can become important in low-power applications. Furthermore, if the application requires negative (reverse) power flows, the capability of the drive to regenerate energy influences the optimal working point of the actuator. This indicates that even losses way upstream have a considerable the energy-efficiency of the actuator.

In short, a good understanding of the entire actuation unit, from load to battery, is crucial to reduce its energy consumption. In the next chapters, this knowledge is applied to the analysis of two energy-efficient actuation concepts.
3

Series and Parallel Elastic Actuation

3.1 Introduction

Every structure exhibits some degree of elasticity. In an actuator, the elasticity acts as a low-pass filter for the motor output, limiting the bandwidth of the actuator. For this reason, actuators have traditionally been designed as stiff as possible with the goal of achieving accurate, stable, high-bandwidth position control. In the past decades, however, robotics has seen the emergence of fields such as prosthetics, exoskeletons, social robotics and co-working robots. Here, other requirements like safe human-robot interaction, shock resistance and energy efficiency are more important than accurate position control. These are exactly the requirements stiff actuators struggle with.

To solve the problem, Pratt et al., in their classic 1995 paper [176], decided to turn the “stiffness is better” paradigm around. They argued that series elasticity acts as a filter not only for the actuator output, but also for the load. Series elasticity can therefore be exploited to, for example, protect the gearbox from high forces on the gear teeth. Additionally, Pratt et al. showed that a deliberate introduction of series elasticity can make stable force control easier to achieve. The spring of such a Series Elastic Actuator (SEA) – the name they gave to the concept – can also be used to transform the force control problem into a position control problem, which can improve the accuracy of the force control.

Series compliance can also improve the safety of robots interacting with humans. The elastic element decouples the actuator’s inertia from that of the link, decreasing the robot’s effective inertia and, as a result, the risk of injury upon collision [24, 251, 90].

Consequently, compliant joints have become an integral part of many of today’s state-of-the-art anthropomorphic arms for human-robot co-operation [123, 89] and humanoid robots designed to operate in human environments [216, 215, 181].

Other benefits can be traced back to the ability of the elastic element to store energy. By loading a spring and then releasing all of its energy at once, an elastic actuator can achieve very high speeds for a short period of time. This property can be exploited in explosive tasks such as hammering [76], kicking [77], throwing [31, 30] and jumping [93, 220]. Another important area of application for SEA designs are active ankle prostheses, where series springs are used to decrease the peak power demand of the motor [164, 95], allowing for more compact motors to be used. A spring’s energy storage capability also enables the actuator to be more efficient in cyclic tasks, which is what we will discuss in this chapter.

In recent years, several authors have put forward another way of introducing the elastic element: placing it in parallel with the drivetrain. The so-called Parallel Elastic Actuators (PEA) do not have a decoupled link and motor inertia, and are therefore more comparable to rigid actuators in terms of safety and force controllability. The benefits related to energy storage remain, although they manifest itself in a slightly different way. While a SEA’s compliance can enhance the actuator’s maximum speed, the compliant element of the PEA enables to decrease the torque demand of the motor. Just like for the SEA, this leads to a decrease in peak power and energy consumption as well [212, 73, 240, 144, 226, 114].

An important issue of PEAs is the direct relationship between the torque of the elastic element and the position of the controlled link. For this reason, it is very important to tune the parallel spring to the task-specific requirements, not only by selecting an appropriate spring stiffness, but also by setting the appropriate equilibrium angle [60, 144]. If the task requires versatility from the actuator or if the load does not match the spring characteristic well, the torques delivered by the parallel elastic element may counteract the motion during parts of the task and, as a consequence, increase the actuator’s torque requirement instead of decreasing it [97]. To solve this issue, some authors have included clutches in their PEA designs [92, 174]. These clutched PEAs engage the clutch when the actuator benefits from the parallel spring, and disengage it when the spring would counteract the motion.

This chapter provides an extensive discussion on how series and parallel elastic elements influence the peak powers and energy consumption of actuators. We start by explaining the basic characteristics of the series and parallel configuration in a quasi-static analysis (section 3.2). Next, we move to dynamic motions. In section 3.3, we discuss how springs influence the natural dynamics of the system, and which consequences this has for the actuator’s peak power and energy consumption, both mechanically and electrically. The influence of static loads is analyzed in section 3.4. In section 3.5, we study the usage of springs in a real-world application: an active ankle prosthesis. A final comparison between series and parallel elastic elements is given in section 3.6.
3.2 Quasi-static analysis

In the previous chapter, we provided an extensive discussion of the energy efficiency of rigid actuators consisting of an electric motor and a planetary gearbox. Here, a direct relationship exists between the motor torque $T_m$ and the output torque $T_{load}$

$$T_m = \frac{C}{n} T_{load}$$  \hspace{1cm} (3.1)

as well as between the motor speed $\dot{\theta}_m$ and the output speed $\dot{\theta}$:

$$\dot{\theta}_m = n \dot{\theta}$$  \hspace{1cm} (3.2)

where we assumed that the motor speed is reduced by a gearbox with gear ratio $n$ and efficiency function $C$, as defined in section 2.2.2. When springs are introduced to the system, the relationship between the motor and the output becomes more complex. This opens up possibilities for energy reduction.

3.2.1 Idealized system

Let us first start by analyzing an idealized system, i.e., a completely linear system with no losses. An ideal torsion spring with linear stiffness $k$ delivers a torque $T_{spring}$ proportional to its deflection. This torque is defined by Hooke’s law:

$$T_{spring} = k \Delta \theta$$  \hspace{1cm} (3.3)

The spring can be introduced to the drivetrain in two ways\(^2\): in series or in parallel with the motor. These two topologies are depicted in Fig. 3.1. If a spring with stiffness $k_p$ is placed in parallel with the load with equilibrium angle $\theta_{eq}$ (Fig. 3.1a), we obtain a Parallel Elastic Actuator (PEA). In this case, the torque required from the motor is reduced by the spring:

$$T_{m, PEA} = \frac{1}{n} [T_{load} - k_p (\theta - \theta_{eq})]$$  \hspace{1cm} (3.4)

whereas the output speed remains equal to the reduced motor speed, according to Eq. (3.2).

Another option is to place the spring in between the motor and the load. This arrangement is called a Series Elastic Actuator (SEA). In contrast to the PEA, the motor torque is not affected by the spring, but remains the same as for a rigid actuator (Eq. (3.1)). What changes is the motor speed which, with a series spring of stiffness $k_s$, becomes

$$\dot{\theta}_{m, SEA} = n \left( \dot{\theta} - \frac{T_{load}}{k_s} \right)$$  \hspace{1cm} (3.5)

\(^2\)Technically, other topologies are possible. The spring can, for example, be placed in between housing of the gearbox and the ground [198]. The series and parallel topology shown in Fig. 3.2 are, however, the most common by far.
Figure 3.1: Actuator topologies.
The conclusion from this short and simple analysis is that series elasticity has an impact on the motor speed, while parallel elasticity changes the motor torque.

Now how does this help to reduce an actuator’s energy consumption? Let us assume that the task is such that a linear relationship exists between $T_{load}$ and $\theta$, i.e.,

$$T_{load} = c_1 \theta + c_2$$

According to Eq. (3.4), a PEA’s motor torque $T_m$ can be reduced to zero by tuning the spring stiffness $k_p$ and its equilibrium angle $\theta_{eq}$ to

$$k_p = c_1$$

$$\theta_{eq} = -\frac{c_2}{c_1}$$

Similarly, based on Eq. (3.5), the SEA’s motor speed can be reduced to zero by choosing a series stiffness

$$k_s = c_1$$

Because the mechanical power of the motor is the product of its speed and torque, i.e.,

$$P_{mech} = T_m \dot{\theta}_m$$

it is easy to understand that a reduction of motor torque or speed has a positive influence on an actuator’s power consumption. We can thus conclude that an actuator can be made more energy-efficient by introducing well-tuned elastic elements, provided that the required output torque has a linear relationship with the desired output position.

An important note is that mechanical springs have a stiffness $k>0$. Considering that, according to Eqs. (3.4) and (3.5), the ideal spring stiffness for both PEA and SEA is $c_1$, elastic elements should only be used if $c_1 > 0$. If the task were to compress a mechanical spring with stiffness $k_L$, for example, $c_1 = k_L < 0$ since Eq. (3.6) is defined from a motor perspective. An SEA or PEA would therefore not lead to a reduction in energy consumption for this specific type of load.

### 3.2.2 Impact of motor efficiency and friction

Even if springs can reduce the mechanical power to zero, the previous chapter has taught us that this is not necessarily the optimal solution in terms of the electrical power. This places the above results in a completely different light. Imagine, for example, an SEA of which the spring is tuned perfectly to the load, such that $\dot{\theta}_m = 0$ throughout the entire motion. Because the series spring has no effect on the motor torque, the motor – assuming that the gearbox is 100% efficient – would need to deliver a torque

$$T_m = \frac{1}{n} T_{load}$$
Inserting this into the motor model specified by Eq. (2.7), the motor voltage would be

\[
\begin{align*}
I &= \frac{1}{nk} T_{load} \\
U &= \frac{R}{nk} T_{load}
\end{align*}
\]

resulting in an electrical power consumption of

\[
P_{elec} = \frac{R}{(nk)^2} T_{load}^2
\]

which corresponds to the Joule losses of the actuator. This means that, even if the SEA does not need to provide any mechanical output power, it still needs to deliver energy to compensate for the heat dissipated in the motor windings. Because this loss is proportional to the square of the output torque, these losses can become very high when high torques are required.

A perfectly tuned PEA, on the other hand, would still need to compensate for the speed-dependent losses of the motor. If we model these as a combination of Coulomb and viscous friction, the resulting in an energy consumption would be

\[
P_{elec} = \frac{R}{k_f} \left[ n v_m \dot{\theta} + T_{Cm} \text{sign} (\dot{\theta}) \right]^2 + n^2 v_m \dot{\theta}^2 + n T_{Cm} |\dot{\theta}_m| 
\]

i.e., the sum of the speed-dependent losses and the Joule losses they cause. Again, the electrical power consumption is not zero. Eq. (3.14) does, however, not depend on the output torque. This makes a PEA the more energy efficient alternative if the tasks requires high torques at relatively low speeds.

### 3.3 Natural dynamics and efficiency

The quasi-static analysis from the previous section already provides a basic understanding about the working principles of SEAs and PEAs. However, aspects such as drivetrain inertia, friction and motor/gearbox losses can have a considerable influence on the energy consumption, as discussed in chapter 2. These were not covered in the quasi-static analysis, although recent works have shown their importance for the energy consumption of SEAs of PEAs [19, 21, 64, 184, 106].

In this section, we study the natural dynamics of rigid actuation (RA), PEA, and SEA, as well as its effect on their power/energy characteristics. To investigate mechanical and electrical energy, the dynamics of the whole system comprising load, actuator, kinematics/gear boxes and electronics are considered, according to the methods and conclusions presented in chapter 2. In this regard, it is an extension of the work presented by Beckerle et al., where the relationship between the natural dynamics and power/energy consumption of SEA and PEA was investigated in the mechanical domain [19].

In this section, we first describe the investigated actuator types and their dynamics (section 3.3.1). A power and energy analysis based on simulations is given in Section...
3.3.1 Actuator types and their dynamics

Figure 3.2 presents schematics of the three studied actuator topologies, actuating a one degree of freedom pendulum with a mass \( M \) and a length \( l \) (the distance between the rotation axis and the center of mass). Combined actuator and gearbox inertia is denoted as \( J_m + J_{tr} \). The load inertia is \( J_l \) and angular positions of the pendulum are equal to those at the gearbox output in the RA and PEA cases. Pendulum motion corresponds to the reduced motor motion \( \theta = n^{-1} \theta_m \), where \( \theta \) and \( \theta_m \) are the positions of the output and motor, respectively, and \( n \) is the gear ratio. The frontal view of the pendulum given in Figure 3.2d defines the direction of \( \theta \) as well as its maximum and minimum values \( \pm \theta_{\text{max}} \) equal to the amplitude of the motion \( \theta_a \). Motor torque \( T_m \), as defined in Figure 3.2a-c, is the sum of the torque available at the motor shaft and the torque required to accelerate the rotor inertia \( J_m \).

The stiffness of the parallel elasticity in the PEA (Figure 3.2b) is given by \( k_p \). In a
SEA with series stiffness $k_s$ (Figure 3.2c), inertias $J_{l1}$ and $J_{l2}$ are separated by the elastic element, and as a result, the positions of pendulum $\theta$ and gearbox output $n^{-1} \theta_m$ differ.

### 3.3.1.1 Rigid actuation

For the RA topology given in Figure 3.2a, the system’s equations of motion are given by

$$T_m = (J_m + J_{tr})n\ddot{\theta} + \frac{C}{n} T_{load}$$

(3.15)

where $T_{load}$ is defined as

$$T_{load} = J_l\ddot{\theta} + T_{c,l}\text{sign}(\dot{\theta}) + \nu_l \dot{\theta} + Mgl \sin \theta$$

(3.16)

The first term on the right side of Eq. (3.15) represents the inertial torque due to rotating components in the gearbox and motor. $T_{load}$ represents the torque due to the motion of the pendulum. Essentially, it includes the gravitational and inertial pendulum loads as well as Coulomb and viscous friction, characterized by their respective coefficients $T_{c,l}$ and $\nu_l$. The term is scaled by the gear ratio $n$ and the gearbox efficiency function $C$:

$$C = \begin{cases} 
1/\eta_{tr} & \text{(load driven by motor)} \\
\eta_{tr} & \text{(motor driven by load)} 
\end{cases}$$

(3.17)

For RA and PEA, $T_{load} \dot{\theta}$ designates whether the motor is driving the load or vice versa. If $T_{load} \dot{\theta} \geq 0$, the power through the gearbox is positive and the load is driven by the motor, so $C = 1/\eta_{tr}$. Conversely, if $T_{load} \dot{\theta} < 0$, the motor is driven by the load and $C = \eta_{tr}$.

As discussed in the previous chapter, the natural behavior of actuation systems is crucial for achieving energy efficient operation. At resonance frequency, a system with one degree of freedom requires nearly no torque\(^3\) to perform an oscillating motion, regardless of the desired amplitude. Consequently, an actuator which operates near resonance can potentially be designed to incorporate smaller motors and will consume only a small amount of power.

The resonance frequency is calculated from the equations of motion of the linearized, frictionless system. Rewriting Eq. (3.15) as such, and transforming it to the frequency domain, we find

$$T_m(\theta) = (J_m + J_{tr})n\omega^2 \theta + \frac{1}{n} (J_l \omega^2 \theta + Mgl \theta)$$

(3.18)

The resonance frequencies can then be found by imposing $T_m(\theta) = 0$ and solving for $\omega$. This results in

$$\omega_{rs,RA} = \pm \frac{Mgl}{\sqrt{J_l + n^2(J_m + J_{tr})}}$$

(3.19)

\(^3\)The motor still needs to deliver some torque to compensate for the energy losses.
There is only one resonance frequency, since the system given by Eq. (3.18) is of second order. We can, however, find more resonance frequencies by looking at different subsystems of the actuator. As discussed in section 2.3.3.2, another resonance frequency is of importance in rigid actuators. It is retrieved from the torque on the gearbox shaft, given by Eq. (3.16). After linearizing and removing friction terms, we find

\[ T_{\text{load}}^* = J_l \dot{\theta} + Mgl\theta \]  

(3.20)

which yields the resonance frequency of the link subsystem

\[ \omega_{rl,RA} = \pm \sqrt{\frac{Mgl}{J_l}} \]  

(3.21)

### 3.3.1.2 Parallel Elastic Actuation

If the actuator is equipped with a parallel elastic element, the required motor torque can be calculated with

\[ T_m = (J_m + J_{tr})n\dot{\theta} + \frac{C}{n} [T_{\text{load}} + k_p\theta] \]  

(3.22)

This equation is identical to that of a stiff actuator (Eq. (3.15)), except for the additional spring term \( k_p\theta \), in which \( k_p \) is the spring constant of the parallel spring. The single resonance frequency of this second-order system is:

\[ \omega_{rs,PEA} = \pm \sqrt{\frac{Mgl + k_p}{J_l + n^2(J_m + J_{tr})}} \]  

(3.23)

Similarly, we can associate a resonance frequency with the torque at the gearbox shaft:

\[ \omega_{rl,PEA} = \pm \sqrt{\frac{Mgl + k_p}{J_l}} \]  

(3.24)

These formulas demonstrate that parallel stiffness can be used to increase the pendulum’s natural frequency. Note that, if \( k_p = 0 \), we retrieve the same resonance frequency as that of the stiff system (given by Eq. (3.19)).

### 3.3.1.3 Series Elastic Actuation

Due to the decoupling of motor and gearbox inertia, the equations of motion of the Series Elastic Actuator are more sophisticated than the ones presented previously. The spring introduces a relationship between the output angle and the motor angle which depends on the load torque. Defining the load torque \( T_{\text{load,SEA}} \) of the SEA as

\[ T_{\text{load,SEA}} = J_{l2} \ddot{\theta} + v_{l2} \dot{\theta} + T_{c,l2} \text{sign}(\dot{\theta}) + Mgl \sin \theta \]  

(3.25)
where \( \nu_l \) and \( T_{c,l} \) represent the viscous and Coulomb friction coefficients of output shaft, the relationship between the motor angle \( \theta_m \) and output angle \( \theta \) can be written as

\[
\theta_m = n \left( \frac{T_{\text{load,SEA}}}{k_s} + \theta \right)
\] (3.26)

The equations of motion are given by

\[
\begin{align*}
T_{\text{load,SEA}} - k_s \left( \frac{\theta_m}{n} - \theta \right) &= 0 \\
(nJ_m + nJ_{tr} + J_{l1}) \frac{\dot{\theta}_m}{n} + \frac{C}{n} \left[ k_s \left( \frac{\theta_m}{n} - \theta \right) + T_{c,l1} \text{sgn}(\frac{\theta_m}{n}) + \nu_l \frac{\dot{\theta}_m}{n} \right] &= T_m
\end{align*}
\] (3.27)

Here, \( T_{c,l1} \) and \( \nu_l \) are the Coulomb and viscous friction coefficients of the shaft connected to the gearbox output. The first equation represents the dynamics of the link, the second those of the motor. The gearbox efficiency function \( C \) is still defined as in Eq. (3.17), but here, the case where the motor is driving the load is designated by

\[
\dot{\theta}_m (T_m - (J_m + J_{tr}) \dot{\theta}_m) \geq 0
\]

meaning that the power through the gearbox is positive.

Removing all friction terms and linearizing, we can combine both equations of (3.27) to

\[
(nJ_m + nJ_{tr} + J_{l1}) \frac{\ddot{\theta}_m}{n} + \frac{1}{n} \left( J_{l2} \ddot{\theta} + Mgl \dot{\theta} \right) = T_m
\] (3.28)

Note that the inverse dynamic calculation of motor torque \( T_m \) would require differentiation of (3.26). This function is generally discontinuous because of Coulomb friction, leading to the appearance of Dirac pulses in the first derivative [56]. Because such pulses cannot be dealt with in inverse dynamic calculations, the derivatives of Coulomb friction were neglected in the simulations.

Replacing \( \ddot{\theta}_m \) in Eq. (3.28) by the second derivative of Eq. (3.26) and transforming it to the frequency domain, we find a transfer function

\[
G(\omega) = \frac{\theta}{T_m} = \frac{k_s}{c_4 \omega^4 + c_2 \omega^2 + c_0}
\] (3.29)

with coefficients

\[
\begin{align*}
c_4 &= (nJ_m + nJ_{tr} + J_{l1}) J_{l2} \\
c_2 &= - (nJ_m + nJ_{tr} + J_{l1}) (Mgl + k_s) - \frac{k_s}{n} J_{l2} \\
c_0 &= k_s \frac{1}{n} Mgl
\end{align*}
\] (3.30)

The poles of this fourth-order transfer function correspond to the two resonance frequencies \( \omega_{r1,\text{SEA}} \) and \( \omega_{r2,\text{SEA}} \) of the system. They are given by \( (\lambda = 1, 2) \)

\[
\omega_{r\lambda,\text{SEA}} = \pm \left( \frac{-c_2 \pm \sqrt{c_2^2 - 4c_4c_0}}{2c_4} \right)^{1/2}
\] (3.31)
Alternatively, by rewriting Eq. (3.55) as a function of $\theta_m$ and transforming it to the frequency domain, we find the transfer function

$$H(\omega) = \frac{\theta_m}{T_m} = -\frac{nJ_2 \omega^2 + n(Mgl + ks)}{c_4 \omega^4 + c_2 \omega^2 + c_0}$$

with coefficients $c_4$, $c_2$ and $c_0$ as defined in Eq. (3.30). The zeros of this transfer function,

$$\omega_{n,SEA} = \sqrt{\frac{ks + Mgl}{J_2}}$$

(3.33)

**correspond to the antiresonance frequencies of the system, at which the output can oscillate while the input (motor) is standing still. Since mechanical power is proportional to speed, antiresonance can be expected to be an advantageous operating point, just like resonance.** Note the similarity between Eq. (3.33) and Eq. (3.24), the only difference being the exclusion of the gearbox shaft’s inertia in Eq. (3.33). Since $J_2$ is bigger than $J_1$ in most practical systems, $J_1 \approx J_2$ and thus $\omega_{n,SEA} \approx \omega_{rl,PEA}$ for identical spring stiffnesses $k_s$ and $k_p$.

3.3.1.4 Motor model

To calculate the electrical power consumption, we follow the approach outlined in the previous chapter. The electrical power consumption $P_{elec}$ is calculated as follows:

$$P_{elec} = UI$$

(3.34)

In this equation, the motor voltage $U$ and current $I$ can be calculated by using the DC motor model from section 2.2.3:

$$\begin{cases} I = \frac{T_m + \nu_m \dot{\theta}_m}{k_t} \\ U = L \frac{dI}{dt} + RI + k_b \dot{\theta}_m \end{cases}$$

(3.35)

In this equation, $R$ is the winding resistance and $L$ is the terminal inductance. $k_t$ and $k_b$ represent the torque and speed constants of the motor and have equal values. All these parameters are readily available from the manufacturer’s datasheet. As suggested in section 2.2.3, the motor’s damping coefficient $\nu_m$ is calculated from the no-load speed $\omega_{nl}$ and the no-load current $I_{nl}$ by applying the formula

$$\nu_m = \frac{k_t \cdot I_{nl}}{\omega_{nl}}$$

(3.36)

**Influence on natural system dynamics** The DC motor also introduces electrical dynamics to the system. Before discussing these dynamics, some simplifications can be made to facilitate the analysis. Regarding voltage, the motor’s terminal inductance $L$ is generally several orders of magnitude smaller than its winding resistance $R$, meaning that
3.3 Natural dynamics and efficiency

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gearbox shaft inertia $J_1$</td>
<td>1.31e-4 kgm²</td>
</tr>
<tr>
<td>Output shaft inertia $J_2$</td>
<td>1.57e-1 kgm²</td>
</tr>
<tr>
<td>Mass $M$</td>
<td>1.85 kg</td>
</tr>
<tr>
<td>Distance from rotation axis to COG $l$</td>
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</tr>
<tr>
<td>Coulomb friction coefficient $T_{c,l1}$</td>
<td>0.049 Nm</td>
</tr>
<tr>
<td>Coulomb friction coefficient $T_{c,l2}$</td>
<td>0.064 Nm</td>
</tr>
<tr>
<td>Damping coefficient $\nu_{l1}$</td>
<td>0.044 Nms/rad</td>
</tr>
<tr>
<td>Damping coefficient $\nu_{l2}$</td>
<td>0.079 Nms/rad</td>
</tr>
</tbody>
</table>

**Table 3.1:** Pendulum properties

The $L \frac{dl}{dt}$ term can be neglected unless the application requires extreme variations in current. Furthermore, the term $RI$ is a few orders of magnitude smaller than the back-EMF $k_b \dot{\theta}_m$, except at very low speeds. Consequently, for the majority of the motor’s operating range, $U = k_b \dot{\theta}_m$, the dynamics of motor voltage $U$ are similar to that of motor speed $\dot{\theta}_m$, except for an additional gain and an additional pole at $\omega = 0$. The minimum voltage can therefore be expected to coincide with the antiresonance frequency.

Regarding current, the $\nu_m \dot{\theta}_m$ term in Eq. (3.35) can be significant when low torques are commanded. Throughout the rest of the operating region, however, $T_m$ will dominate. Consequently, the dynamics of motor current ($\dot{\theta}/I_m$) will roughly correspond to that of motor torque ($\dot{\theta}/T_m$), and resonance frequencies will lead to a reduction in motor current.

### 3.3.2 Power and energy analysis

As in the previous chapter and previous works by Vanderborgh et al. [221] and Beckerle et al. [19], a pendulum setup is used to evaluate the performance of the actuator. The actuator, which is identical to the one in chapter 2, can be mechanically modified to a SEA or a PEA. It imposes a sinusoidal trajectory with variable frequency around an equilibrium position $\theta = 0^\circ$, which corresponds to a vertical line. The amplitude of the motion is $30^\circ$, which is significantly larger than in the aforementioned works by Vanderborgh et al. [221] and Beckerle et al. [19], where the amplitude was limited to $10^\circ$. Consequently, the nonlinear equations of motion need to be considered without making a small-angle assumption.

The physical properties of the pendulum, derived from a CAD model, are listed in Table 3.1. For the properties of the motor and gearbox we refer back to Table 2.2. Note that, for RA and PEA, the Coulomb friction coefficients and damping coefficients are lumped together because of the rigid connection between the load shaft and gearbox shaft:

$$
\begin{align*}
T_{c,l} &= T_{c,l1} + T_{c,l2} \\
\nu_l &= \nu_{l1} + \nu_{l2}
\end{align*}
$$

The simulations are based on the equations established in Section 3.3.1. The imposed
trajectory is assumed to be tracked perfectly by both actuators, i.e. the angle $\theta$ can be replaced by the time-dependent function

$$\theta(t) = \theta_0 \sin(\omega t)$$

(3.38)

The amplitude $\theta_0$ is restricted to $30^\circ$ because of limitations on the maximum extension of the springs on the physical setup, something which is especially critical for the PEA. This way, the SEA and the PEA can impose the same motion to the load without reaching the physical limits of the system, such that a fair comparison between both is obtained.

### 3.3.2.1 Mechanical peak power

Figure 3.3 shows mechanical peak power for the PEA (left) and SEA (right), as a function of frequency and spring stiffness. The power is calculated as the product of motor speed $\dot{\theta}_m$ and motor torque $T_m$ (taking into account the inertia at the motor itself).

Apart from the obvious minimum at $\omega = 0$, the PEA only demonstrates one minimum in the peak power plot, occurring at the resonance frequency $\omega_{rs,PEA}$. For the SEA, there are two minima: one at the first resonance frequency $\omega_{r1,SEA}$ (local minimum, less distinct) and one at antiresonance $\omega_{a,SEA}$ (global minimum, very distinct). The second resonance frequency $\omega_{r2,SEA}$ does not lead to a clear minimum, which is in line with the results obtained with another test rig [17]. The plots thus suggest that the optimal operating points in terms of mechanical peak power lie at the SEA’s antiresonance frequency $\omega_{a,SEA}$ and at the PEA’s resonance frequency $\omega_{rs,PEA}$. As seen in Figure 3.3, the PEA requires higher stiffnesses than the SEA in order to operate at its optimal frequency, confirming the results from [19].

For $k_p \to 0$ and $k_s \to \infty$, we find that $\omega_{rs,PEA}$ and $\omega_{r1,SEA}$ both converge to the same resonance frequency of 3.9 rad/s, which corresponds to the rigid system’s resonance frequency $\omega_{rs,RA}$ given by Eq. (3.19). The introduction of a series spring hardly affects this resonance frequency, except at very low stiffnesses. However, such compliant springs may compromise the operation of the SEA due to the large spring extensions required, possibly demanding excessive speeds from the motor. In the PEA, on the other hand, the presence of the spring moves the resonance frequency to higher values. This can be exploited in the design of the PEA to decrease peak power. Note that, the stiffer the parallel spring is, the lower the decrease in mechanical peak power at resonance will be. This is because the mechanical power is the product of torque and speed, of which only the former is decreased at resonance, and the latter increases with frequency. As seen in Figure 3.3, frequency is positively related to stiffness at resonance, and therefore energy consumption increases with stiffness as well. Conversely, in the SEA at antiresonance, the speed is decreased instead of the torque. As a result, the mechanical peak power is largely unaffected by an increase in frequency, and low peak powers can still be achieved at high frequencies.

Finally, comparing the theoretical resonance and antiresonance lines to the actual minima of the plot, one observes that optimal peak power occurs at a slightly lower stiffness than expected for PEA. Conversely, for the SEA, optimal stiffness is slightly higher
Figure 3.3: Mechanical peak power for (a) PEA and (b) SEA, for varying swinging frequencies and spring stiffnesses. Minimal mechanical peak power for the PEA approximately corresponds to the system’s resonance frequency $\omega_{r,\text{PEA}}$. For the SEA, the minimum occurs at the antiresonance frequency $\omega_{a,\text{SEA}}$, but a decrease is also observed at the resonance frequency $\omega_{r,\text{SEA}}$. 
than predicted by the $\omega_a$ line. These deviations are due to the combination of nonlinearities and friction.

### 3.3.2.2 Energy consumption

Essentially, the power is the sum of two contributions $P_{\text{load}}$ and $P_{\text{loss}}$:

$$P = P_{\text{load}} + P_{\text{loss}}$$  \hspace{1cm} (3.39)

where $P_{\text{load}}$ represents the power consumed by the load (i.e. the power consumed by the lossless system) and $P_{\text{loss}}$ the power losses. The latter can be attributed to bearing and gearbox friction (mechanical losses) and the motor’s winding resistance (electrical losses), as discussed in section 2.5.1. Throughout most of the motion, the power consumed by the load will be the dominant factor; as a result, power is strongly related to the motion and the properties of the output link.

By definition, energy is calculated as

$$E = \int P \, dt = \int P_{\text{load}} \, dt + \int P_{\text{loss}} \, dt$$  \hspace{1cm} (3.40)

$$= E_{\text{load}} + E_{\text{loss}}$$

In the specific case studied in this paper, $E_{\text{load}}$ equals zero because the imposed motion is a cyclic motion applied to a conservative force field. Consequently, the total energy consumption $E$ is equal to the system’s energy losses $E_{\text{loss}}$. In contrast to power, energy consumption will therefore be dictated by the energy losses rather than the motion of the load itself. Of course, losses being closely related to torque and speed, energy losses can also be expected to be somehow related to the output link and its motion. Nevertheless, there are some differences between the peak power of SEAs and PEAs and their – mechanical or electrical – energy consumption. Those differences are the subject of this section.

**Mechanical energy consumption** Figure 3.4 shows mechanical energy consumption for the PEA (left) and SEA (right), as a function of frequency and spring stiffness. Mechanical energy is calculated as the integral of the mechanical power. As in section 3.3.2.1, mechanical power is the product of speed and torque at the motor (including the torque due to the acceleration of the motor’s own inertia). Consequently, it is the mechanical energy required to complete one entire pendulum cycle at a specific frequency.

For the PEA, mechanical energy is minimal at link resonance $\omega_{rl,\text{PEA}}$ (i.e. resonance without motor inertias). This is a significant difference with the results in mechanical peak power, where the minimum occurred at motor resonance $\omega_{\text{rs,PEA}}$. At $\omega_{\text{rs,PEA}}$, motor torque is reduced, leading to minimal (electrical) motor losses. However, as explained in
Figure 3.4: Mechanical energy consumption per cycle for (a) PEA and (b) SEA, for varying swinging frequencies and spring stiffnesses. The PEA consumes the least amount of mechanical energy at the link’s resonance frequency $\omega_{rl, PEA}$. For the SEA, minimum energy consumption occurs at antiresonance $\omega_{a, SEA}$, and another decrease in energy consumption is also observed near $\omega_{rl, RA}$. 
section 2.3, gearbox torque is nonzero at this frequency because of the inertia of the motor, which adds an inertial torque. Gearbox losses, which are proportional to the torque going through it, are instead minimized at the resonance frequency of the output link $\omega_{rl, PEA}$. The minimum in mechanical energy consumption, which is dominated by the gearbox losses, therefore lies at $\omega_{rl, PEA}$ instead of $\omega_{rs, PEA}$.

For the SEA, the global minimum still occurs at antiresonance. The second resonance frequency, again, does not lead to a reduction in mechanical energy consumption. A local minimum is present at about 5 rad/s, a slightly higher frequency than the first resonance frequency $\omega_{r1, SEA}$. The reasons for this shift are the same as those for the PEA: for minimal mechanical power consumption, gearbox losses need to be minimized, which is the case at $\omega_{r1, RA}$ instead of $\omega_{r1, SEA}$ (minimal motor losses).

As was the case for mechanical power, we observe that, if the SEA is operated at antiresonance, energy consumption is low regardless of the frequency. For the PEA, higher frequencies demand higher mechanical energy input, because speed is not reduced by the parallel spring. Also note that, as explained in Section 3.3.1.3, $\omega_{rl, PEA}$ and $\omega_{a, SEA}$ are nearly identical. Hence, for minimizing the mechanical energy consumption, there is almost no difference between the optimal spring stiffness of the SEA and PEA.

### Electrical energy consumption

Figure 3.5 shows electrical energy consumption for the PEA (left) and SEA (right), as a function of frequency and spring stiffness. Electrical energy is calculated as the integral of electrical power, given by Eq. (3.34). As predicted in Section 3.3.1.4, the plots are very similar to those for the mechanical energy, but the results are more pronounced due to the addition of the Joule and gearbox losses, which are both very dependent on torque.

For the PEA (left of Figure 3.5), minimum electrical energy consumption occurs in between gearbox resonance $\omega_{rl, PEA}$ (gearbox losses minimized) and motor resonance $\omega_{rs, PEA}$ (Joule losses minimized). Neither of both lines provides a good approximation of the optimum by itself. In this case, the minimum is closer to gearbox resonance, but this observation cannot be generalized to any actuator system. The location of the actual minimum depends entirely on the system’s losses. If most losses are due to the gearbox, the minimum will be closer to $\omega_{rl, PEA}$; if motor losses dominate, the minimum will be closer to $\omega_{rs, PEA}$. Therefore, the optimal stiffness needs to be calculated based on the full equations of the system, including friction factors, gearbox efficiency and a motor model.

For the SEA (right of Figure 3.5), antiresonance can serve as a good approximation to calculate the optimal spring stiffness. There is a clear minimum in the electrical energy consumption, which does not deviate much from the theoretical antiresonance line. Energy consumption is also lowered at a frequency slightly higher than resonance, but the reduction is not as pronounced as at antiresonance.

Again, we notice that the PEA’s electrical energy consumption rises at higher frequencies due to the increased speed required from the motor. The SEA maintains low energies up to higher frequencies. However, unlike in the mechanical power and energy plots, electrical energy of the SEA also increases at higher frequencies. This is due to the proximity of the resonance frequency $\omega_{r2, SEA}$ which reduces motor torque, and the link’s
Figure 3.5: Electrical energy consumption per cycle for (a) PEA and (b) SEA, for varying swinging frequencies and spring stiffnesses. Minimal electrical energy consumption for the PEA occurs somewhere in between the resonance frequencies $\omega_{rs,PEA}$ and $\omega_{rl,PEA}$. The SEA has a distinct minimum at antiresonance $\omega_{a,SEA}$.
resonance frequency $\omega_{rl,RA}$, at which gearbox torque is minimized. As $\omega_{r2,SEA}$, $\omega_{a,SEA}$ and $\omega_{rl,RA}$ approach each other at low stiffnesses, a simultaneous reduction of torque and speed occurs, and energy consumption will drop to rather low values. At higher stiffnesses, the SEA still profits from the decrease in speed, but the torque-reducing influence of $\omega_{r2,SEA}$ and $\omega_{rl,RA}$ is less strong, leading to an increased energy consumption.

In conclusion, minimum mechanical peak power, minimum mechanical energy and minimum electrical energy all occur at different frequencies for the PEA. This makes the calculation of its optimal spring stiffness a difficult task, which is highly dependent on the model being used. For the SEA, the calculation is a lot simpler, since in good approximation all minima can be traced down to the antiresonance frequency. Moreover, in this set of simulations, the SEA at antiresonance outperforms the PEA in terms of peak power and, especially, energy consumption.

3.3.2.3 Comparison with rigid actuation

In this section, we will compare the energy consumption of the PEA and the SEA to that of rigid actuation. The relative difference is calculated as

$$e = \frac{E - E_{RA}}{E_{RA}}$$ (3.41)

where $E$ stands for the energy consumption of the PEA or SEA, depending on which type of actuation is considered for the comparison. The results for different frequencies and stiffnesses are shown in Figure 3.6.

In this case study, elastic elements do not yield energetic advantages below 5 rad/s. For the PEA, the most distinct reduction in energy consumption is focused around $\omega_{rl,PEA}$. The favorable region includes this frequency completely, while $\omega_{rs,PEA}$ does not yield a reduction in energy consumption at frequencies close to $\omega_{rl,RA}$. As expected, antiresonance marks the greatest energy reduction for the SEA. Reduced energy consumption covers a certain area around this frequency. Hence, tuning for antiresonance is advantageous, even if the model does not fit the real system perfectly. The resonances lie outside the beneficial areas for the SEA and should therefore be avoided [17]. In fact, the resonance frequency $\omega_{r2,SEA}$ approximately matches the upper boundary of energetic advantage of the SEA. The lowest frequency at which an energetic advantage is achieved with series springs is $\omega_{rl,RA} = 5.3$ rad/s; for any frequency below this value, RA will perform better. Note that, at $k_p = 0$, the PEA transforms into a system equivalent to RA, hence $e = 0$. Consequently, in contrast to the SEA, there is no practical lower boundary on the spring stiffness of the favorable region for the PEA.

3.3.3 Experimental evaluation

In order to evaluate the conclusions from the power and energy analysis, the electrical energy plots were obtained experimentally from a physical setup matching the properties of the simulated system. In this section, the physical test set-up is described along with the
3.3 Natural dynamics and efficiency

Figure 3.6: Relative difference between the energy consumption of the rigid actuator compared to (a) PEA and (b) SEA. The black lines denote the combinations of frequencies and stiffnesses at which RA and PEA (RA and SEA) perform equally well. The highest relative differences are situated around $\omega_{rl,PEA}$ and $\omega_{rl,SEA}$. At any frequency below $\omega_{rl,RA} = 5.3$ rad/s, RA performs best. Note that RA corresponds to the cases of zero parallel stiffness ($k_p = 0$ Nm/rad in the plot to the left) or infinite series stiffness ($k_s \rightarrow \infty$ in the plot to the right).
corresponding control algorithms. Subsequently, the experimental results are presented, discussed and compared to the simulations.

### 3.3.3.1 Test setup

The tests are performed on the setup presented in section 2.3.2, which was modified to accommodate series and parallel springs in accordance with section 3.3.1. The parameters of the setup are given in Table 3.1 and are identical to those that are used for the simulations in Section 3.3.2. The actuator consists of a 80 W Maxon DCX35L motor and a Maxon GPX42 338:3 planetary gearbox (1), of which the parameters are listed in Table 2.2. The compliant elements are implemented by means of two antagonistic tension springs (4), which are mounted on one of both sides of the pendulum to yield a PEA (Figure 3.7a) or a SEA (Figure 3.7b). Additionally, the springs can be replaced to change the stiffness of the actuator in discrete steps between the experimental trials. The resulting rotational stiffness values can be varied between 2 Nm/rad and 10 Nm/rad for the SEA and between 0.3 Nm/rad and 2.3 Nm/rad for the PEA. By removing the parallel springs and making the series connection rigid, the RA case can be investigated as well.

A torque sensor (3) and an encoder (2) are placed on the gearbox shaft to measure the mechanical energy consumption. The torque sensor is an ETH Messtechnik DRBK torque transducer (range 20 Nm, accuracy 0.5%) while positions are acquired using a US Digital E6 series optical encoder with 2000 counts per turn. To measure the output shaft angle of the SEA, another unit of this encoder type (5) is placed on a measurement shaft, which is connected to the load shaft with a pulley (1:1 ratio). The inertial properties of the torque sensor (136 gcm²) and the encoder wheels (0.073 gcm²) are included in the simulation, despite their insignificance with respect to the total inertia of the system. To assess electrical energy consumption, the voltage at the motor terminals and the motor current are measured. To sense current, an Allegro ACS712 current sensor with a range from -5 A to 5 A, a total output error of 1.5%, and a resistance of 1.428 mΩ is used. All sensors and the resistance of the cables in between the motor and the controller (0.228 Ω) are considered in the system model in order to obtain an appropriate comparison of simulations and experiments.

---

**Figure 3.7:** Setups used for experiments. 1. Motor-gearbox, 2. Gearbox shaft encoder, 3. Torque sensor, 4. Springs, 5. Output shaft encoder
3.3 Natural dynamics and efficiency

Figure 3.8: General control architecture

The sensory data is acquired with a National Instruments sbRIO 9626 board, which is also used to implement the control algorithms. For every measurement, at least ten pendulum periods are recorded after start-up transients receded. The measurement is then decomposed into separate sine periods, which are averaged with respect to one another in order to reduce noise and other non-reproducible effects.

3.3.3.2 Control algorithm

As demonstrated in Section 3.3.1, the equations of motion for the actuated pendulum are nonlinear. For small angles, the equations for the stiff actuator and the PEA can easily be linearized, and a PID controller can deliver a satisfactory control performance. For the SEA, however, the missing collocation due to the elastic coupling has to be taken into account [58], and the resulting fourth-order dynamics can no longer be controlled by a simple PID system. Because of its ability to handle these nonlinearities and collocation issues, the model-based control strategy of feedback linearization [205] provides a suitable solution for this type of problem. This control strategy, which will be employed for all actuator types, results in the controller architecture sketched in Figure 3.8.

The control law for the PEA (and RA) compensates the second-order nonlinear dynamics by applying the motor current

$$I_{m,PEA} = \frac{1}{k_t} \left[ \left( nJ_m + nJ_t + \frac{J_l}{n} \right) y_{PEA} + \frac{1}{n} Mgl \sin(\theta) + \frac{1}{n} k_p \theta \right] \quad (3.42)$$

It consists of a feedforward term that is based on Eq. (3.22) in combination with feedback motion control by the auxiliary input $y_{PEA}$. This is chosen to be

$$y_{PEA} = \dot{\theta}_d + P (\theta_d - \theta) + D (\dot{\theta}_d - \dot{\theta}) \quad (3.43)$$

where $P$ and $D$ are the proportional and differential control parameters [203] and $\dot{\theta}_d$, $\theta_d$, $\theta_d$ are the desired angular accelerations, velocities, and positions of the pendulum. In the experiments, the P and D coefficients are tuned by hand.

To tackle the fourth order dynamics of the SEA presented in Eq. (3.55), the control law is extended to
\\[ I_{m,SEA} = \frac{J_{\text{drive}}}{k_t} \left( J_{l2} y_{\text{SEA}} + M g l (\cos \theta \dot{\theta} - \sin \theta \ddot{\theta}^2) + \ddot{\theta} \right) + \frac{1}{n k_t} (J_{l2} \ddot{\theta} + M g l \sin \theta) \] (3.44)

in which we defined the drive side inertia \( J_{\text{drive}} \) as

\[ J_{\text{drive}} = n J_m + n J_{tr} + J_{l1} \] (3.45)

The auxiliary control input \( y_{\text{SEA}} \) in Eq. (3.44) is

\[ y_{\text{SEA}} = \ddot{\theta}_d + R_0 (\dot{\theta}_d - \dot{\theta}) + R_1 (\dot{\theta}_d - \dot{\theta}) + R_2 (\dot{\theta}_d - \dot{\theta}) + R_3 (\ddot{\theta}_d - \ddot{\theta}) \] (3.46)

as suggested in [205]. In this equation, the feedback control parameters \( R_0, R_1, R_2, R_3 \) are related to the errors on pendulum position, velocity, acceleration, and jerk which define the system state. In both cases, these control parameters are manually tuned for each investigated combination of frequency and stiffness. This way, the best possible tracking for a sound comparison of power and energy characteristics is assured.

### 3.3.3.3 Experimental results

The experimentally obtained energy consumption of PEA and SEA is presented in Figure 3.9. Due to limitations on spring extension and permissible speed of the motor, the frequency range is limited to 7 rad/s for both actuators, and the stiffness is limited to 2.3 Nm/rad for the PEA and 10 Nm/rad for the SEA. Comparing the measured (green dots) and simulated (blue wiregrid) energy profiles shows that the consumption of PEA is slightly overestimated by the models. Yet, the model yields a good prediction of the global actuator behavior. In the SEA case, the experimental results show good accordance with the analytical ones, except for overestimating the consumed energy at combinations of low frequencies and low stiffness values. However, those combinations are not of interest for practical SEAs, because energy consumption is rather high at these operating points.

The results for the PEA (left of Figure 3.9) are generally in accordance with the minimum area found in the simulations, although the experiments do not allow to exactly trace the minimum to the resonance frequencies of either the system or the link due to the limited frequency resolution. Furthermore, the PEA suffers from possible inaccuracies due to the setting of the equilibrium angle, which is of no concern for the SEA. Nevertheless, the experiments confirm that the PEA can be operated in a favorable region by adjusting its stiffness.

The experimental energy profile of the SEA (right of Figure 3.9) shows that operation at antiresonance leads to minimum energy consumption. As observed in the simulation, the region of reduced energy consumption covers a certain area around antiresonance: lowest energy consumption values are 0.45 J for 1.4 Nm/rad and 0.70 J for 2 Nm/rad considering operation at 6 rad/s. For 7 rad/s, minimum values are 1.4 J for 2 Nm/rad and
Figure 3.9: Electrical energy consumed by the (a) PEA and (b) SEA at different frequencies and stiffnesses. Measurements are indicated as green dots, while simulated energies are shown as a blue wiregrid. As predicted in the simulations, for the SEA, a distinct minimum exists around antiresonance. For the PEA, the region of low energy consumption is more spread over the range of frequencies. Note that the experiments for the RA correspond to the experimental results for the PEA at $k_p = 0 \text{ Nm/rad}$.
Table 3.2: Measured electrical energy consumption $E_{elec}$ of RA, SEA and PEA for various frequencies. For SEA and PEA, the lowest value across all stiffnesses is shown. The corresponding spring stiffness is mentioned in brackets.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$E_{elec}$ RA</th>
<th>$E_{elec}$ PEA</th>
<th>$E_{elec}$ SEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 rad/s</td>
<td>2.4 J</td>
<td>2.4 J</td>
<td>3.1 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(10 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>2 rad/s</td>
<td>1.8 J</td>
<td>1.8 J</td>
<td>2.6 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(8 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>3 rad/s</td>
<td>1.4 J</td>
<td>1.4 J</td>
<td>2.2 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(8 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>4 rad/s</td>
<td>1.5 J</td>
<td>1.5 J</td>
<td>1.9 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(8 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>5 rad/s</td>
<td>1.8 J</td>
<td>1.6 J</td>
<td>1.9 J</td>
</tr>
<tr>
<td></td>
<td>(0.3 Nm/rad)</td>
<td>(6 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>6 rad/s</td>
<td>2.0 J</td>
<td>2.0 J</td>
<td>0.45 J</td>
</tr>
<tr>
<td></td>
<td>(1.3 Nm/rad)</td>
<td>(1.4 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>7 rad/s</td>
<td>2.4 J</td>
<td>2.0 J</td>
<td>0.57 J</td>
</tr>
<tr>
<td></td>
<td>(2.3 Nm/rad)</td>
<td>(4 Nm/rad)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2 shows that 5 rad/s is the lowest frequency at which a parallel spring becomes beneficial.

Besides the increasing energy requirement with rising frequency above $\omega_{r1,RA}$, it becomes distinct that the PEA always demands more energy than the RA operated at this specific point. This is consistent with our findings in Section 3.3.2.2. Still, the PEA can be beneficial if stiffness is modified to match varying trajectory frequencies, as can be seen from the results for 6 rad/s (same energy consumption) and 7 rad/s (energy consumption lowered by 20%). In contrast to the PEA, the SEA at antiresonance exhibits significantly
lower energy requirements than the RA operated at resonance, with reductions up to 78% at 6 rad/s. Note that, both for SEA and PEA, the optimal stiffnesses in Table 3.2 depend on the discrete set of available springs for the test setup. Higher reductions can most likely be obtained if the spring stiffness could be matched exactly to the theoretically optimal stiffness. Anyhow, these results give a clear indication that, for a 1-DOF link swinging above its resonance frequency, a properly tuned SEA is superior to the PEA in terms of energy consumption.

3.3.4 Discussion

As mentioned in the introduction of Section 3.3.2.2, the energy consumption of the studied system depends completely on the system’s energy losses. These losses can roughly be classified into three major categories: bearing losses, gearbox losses and motor losses. As explained in section 2.5.1, bearing losses are related to speed, motor losses (mostly) to torque, and gearbox losses to both.

In sections 3.2 and 3.3.2, we established that the optimal operation of PEA and SEA relies on very different principles. In PEA and RA, the resonance frequency can be exploited, reducing the torque on the motor and gearbox. Consequently, motor and gearbox losses are minimized, resulting in a more energy-efficient operation. SEA, on the other hand, relies on the exploitation of antiresonance to decrease motor speed. Smaller motor speeds lead to a reduction in friction losses, while gearbox and motor losses remain unaffected. Since, in the actuator system studied in this paper, the latter are responsible for the majority of the losses, one would expect the usage of a PEA to lead to better overall efficiencies. The simulations, however, reveal some additional considerations which need to be taken into account when comparing SEA to PEA. Firstly, as discussed in Section 3.3.2.2, gearbox and motor losses cannot be minimized simultaneously because they correspond to different resonance frequencies, and optimal stiffness for the PEA is a trade-off between both. Consequently, even at the optimal stiffness, a significant amount of energy will still be lost in the gearbox and motor. Secondly, the dynamics of the SEA demonstrate a very interesting feature: the antiresonance frequency \( \omega_{a,\text{SEA}} \) (minimal motor speed), the resonance frequency \( \omega_{r,\text{SEA}} \) (minimal motor torque) and the link’s resonance frequency \( \omega_{r,\text{RA}} \) (minimal gearbox torque) approach each other for \( k_s \to 0 \). As a result, an SEA operated at antiresonance does not only take advantage of the reduced motor speed, it also enjoys a slight reduction of motor and gearbox torque. While the torque reduction is not as distinct as in the PEA – the PEA’s resonance frequencies \( \omega_{r,\text{PEA}} \) and \( \omega_{l,\text{PEA}} \) being much nearer to each other – the combination of reduced motor speed and motor torque provides an energetic advantage to the SEA which cannot be rivaled by the PEA. In other words, the favorable dynamics of the SEA allow it to outperform the PEA in terms of energy consumption, at least for this case study.

3.3.5 Summary

Consistent with the findings of earlier work [17], we found that SEAs should be tuned to antiresonance, regardless of whether peak power or energy consumption is considered.
For PEAs, the optimal tuning is case-dependent. Minimum mechanical peak power occurs at the resonance frequency $\omega_{rs,PEA}$, whereas minimum mechanical energy consumption corresponds to the resonance frequency $\omega_{rl,PEA}$ of the gearbox subsystem. Minimum electrical energy consumption, finally, is not defined by any of these frequencies, but occurs somewhere in between depending on how the losses are distributed between gearbox and motor. Consequently, finding the energy-optimal stiffness of a PEA requires a detailed system model, whereas for the SEA, the antiresonance frequency already provides an excellent estimate.

Comparing the energy consumption of SEA and PEA to RA, energy can be reduced at frequencies above the link’s resonance frequency $\omega_{rl,RA}$. The highest reductions are situated around the antiresonance frequency for the SEA and around the resonance frequency $\omega_{rl,PEA}$ for the PEA. In a resonance-tuned PEA, however, energy consumption rises with increasing stiffness; consequently, its energy consumption can never be lower than that of the rigid actuator ($k_p = 0$) at resonance. Conversely, the antiresonance-tuned SEA consistently demonstrates lower energy consumption than the rigid system at any frequency above $\omega_{rl,RA}$. The drop is significant, with experimental results indicating gains in energy consumption of up to 78%.

In conclusion, these results show that natural dynamics can be exploited to achieve efficient operation for RA, PEA, and SEA. Beyond the link’s resonance frequency, PEA and SEA allow to decrease the energy consumption with respect to RA. In this respect, the SEA clearly outperforms the PEA thanks to its more favorable dynamics.

### 3.4 Influence of static loads

In the previous section, we compared the PEA and SEA by imposing a sinusoidal trajectory, symmetrical around a vertical line $\theta_o = 0$. In this section, an offset will be added to the sinusoidal trajectories:

$$\theta = \theta_a \sin(\omega t) + \theta_o$$

(3.47)

This results in an additional static torque $Mgl\theta_o$ that will need to be provided by the motor:

$$T_{load} \approx J_1\ddot{\theta} + Mgl\theta$$

$$\approx (Mgl - J_1\omega^2) \theta_o \sin(\omega t) + Mgl\theta_o$$

(3.48)

The goal of this section, which is based on the journal paper Beckerle et al. (2017) Series and Parallel Elastic Actuation: Influence of Operating Positions on Design and Control. IEEE/ASME Transactions on Mechatronics, 22, pp. 521-529 [18], is to evaluate how well the SEA and PEA concept can deal with this static torque.
Figure 3.10: Frontal view of the pendulum with definition of angles.
3.4.1 Dynamic modeling

3.4.1.1 Parallel Elastic Actuation (PEA)

The frontal view of the pendulum is given in Figure 3.10. It presents the direction of $\theta$, the operating position offset $\theta_o$ with its maximum and minimum values $\theta_{max}$, and the static equilibrium angle $\theta_{eq}$. This is the angle to which the pendulum returns if no external load is applied to the PEA. Literature indicates that the equilibrium angle of the parallel spring can be an important design parameter with respect to energy efficiency [60, 144]. Including this angle in the nonlinear dynamics model from section 3.3.1.2, the required motor torque can be calculated by

$$T_m = n (J_m + J_{tr}) \ddot{\theta} + \frac{C}{n} (T_{load} + k_p (\theta - \theta_{eq}))$$

(3.49)

in which $T_{load}$ represents the load torque in the elastic element given by

$$T_{load} = J_l \ddot{\theta} + T_c \text{sign} (\dot{\theta}) + \nu \dot{\theta} + Mgl \sin \theta$$

(3.50)

Eq. (3.49) corresponds to Eq. (3.22) for $\theta_{eq}$, as required. Linearizing Eq. (3.49) using a Taylor series to analyze the natural dynamics at a certain operating position offset $\theta_o$ yields

$$\Delta T_m = n (J_m + J_{tr}) \Delta \ddot{\theta} + \frac{C}{n} (J_l \Delta \dot{\theta} + \nu \Delta \dot{\theta} + Mgl \cos \theta_o \Delta \theta + k_p \Delta \theta)$$

(3.51)

which describes small motions $\Delta \theta = \theta - \theta_o$ of the pendulum around this position. Note that this the equation corresponds to (3.22) if $\theta_o = 0$. Eq. (3.51) shows that the offset angle $\theta_o$ does not affect this behavior, nor does Coulomb friction, which only influences the amplitude.

Neglecting friction and gearbox losses, the single resonance frequency of this second-order system is

$$\omega_{rs,PEA} = \pm \sqrt{\frac{k_p + Mgl \cos \theta_o}{J_l + n^2 (J_m + J_{tr})}}$$

(3.52)

Comparing this to the model given in section 3.3.1.2 indicates that the operating position offset $\theta_o$ does affect the dynamic behavior. A similar influence is found when calculating the resonance frequency of the link only with the torque at the gearbox shaft:

$$\omega_{rl,PEA} = \pm \sqrt{\frac{k_p + Mgl \cos \theta_o}{J_l}}$$

(3.53)

The resonance frequency of the link is an important characteristic frequency because, as discussed in Section 3.3, the mechanical energy consumption is minimal at this frequency for offset-free sinusoidal motions.
Analyzing the steady state behavior of the PEA can shed light on the influence of the offset angle \( \theta_o \). In terms of energy consumption, the favorable equilibrium angle \( \theta_{eq} \) for this specific position offset is found if no torque is required to hold the static position, i.e., \( T_m(\theta_o) = 0 \). It can be calculated by substituting the operating position offset \( \theta_o \) to the static part of Eq. (3.49) [205]:

\[
\theta_{eq} = \theta_o + \frac{Mgl \sin \theta_o}{k_p} \quad (3.54)
\]

Here, modeling gives an important design indication since this favorable equilibrium angle \( \theta_{eq} \) is not equal to the operating position offset \( \theta_o \). Still, it can be calculated analytically without simplifying the nonlinear dynamics of the system. Functionally, this mechanical implementation corresponds to a feed-forward compensation of the gravitational effect. This brings the system to the offset position \( \theta_o \) in the steady state.

### 3.4.1.2 Series Elastic Actuation (SEA)

Because of the relation between link and motor position (3.26), an equilibrium angle as in the PEA case cannot be defined. Therefore, the equations of motion from section 3.3.1.3 remain unchanged.

Linearizing and combining Eq. (3.27) while neglecting friction yields

\[
\left[ n(J_m + J_{tr}) + J_{l1}\right] \frac{\Delta \ddot{\theta}_m}{n} + \frac{1}{n} \left(J_{l2} \Delta \dot{\theta} + Mgl \cos \theta_o \Delta \theta\right) = \Delta T_m \quad (3.55)
\]

The natural dynamics are analyzed based on the transfer function found by rewriting Eq. (3.55) as a function of \( \theta_m \):

\[
H(\omega) = \frac{\Delta \theta_m}{\Delta T_m} = \frac{c_{n2} \omega^2 + c_{n0}}{c_{d4} \omega^4 + c_{d2} \omega^2 + c_{d0}} \quad (3.56)
\]

with the coefficients

\[
\begin{align*}
    c_{n2} &= -nJ_{l2} \\
    c_{n0} &= n(Mgl \cos \theta_o + k_s) \\
    c_{d4} &= (nJ_m + nJ_{tr} + J_{l1})J_{l2} \\
    c_{d2} &= -(nJ_m + nJ_{tr} + J_{l1})(Mgl + k_s) - \frac{k_s}{n}J_{l2} \\
    c_{d0} &= k_s \frac{1}{n}Mgl 
\end{align*} \quad (3.57)
\]

The transfer function \( H(\omega) \) is identical to the one in section (3.3.1.3), except for the coefficient \( c_{n0} \), which now contains the operating position offset \( \theta_o \). The zeros of this transfer function determine the antiresonance frequency of the system, now given by

\[
\omega_{a,SEA} = \sqrt{\frac{k_s + Mgl \cos \theta_o}{J_{l2}}} \quad (3.58)
\]
The two resonance frequencies of the system are calculated by $\lambda = 1, 2$:

$$
\omega_{r\lambda, \text{SEA}} = \pm \left( -\frac{c_d^2}{2c_d^2} \pm \sqrt{\frac{c_d^2 - 4c_d^2d_0}{2c_d^4}} \right)^{1/2}
$$

Compared to the natural frequencies from section (3.3.1.3), we see that the antiresonance frequency exhibits a dependency on the operating position offset $\theta_o$, while the resonance frequencies $\omega_{r\lambda, \text{SEA}}$ remain unchanged.

### 3.4.2 Power and energy analysis

To analyze the influence of static torques on the pendulum’s energy consumption, we will impose the sinusoidal trajectory with offset defined in Eq. (3.47). The amplitude $\theta_a$ of this trajectory is 15° and the offset $\theta_o$ is varied between 0° to 25°. The small-angle approximation is not applied, but the full nonlinear equations of motion are considered in the simulations. The results from the simulations are obtained from inverse dynamics, assuming perfect tracking of the trajectory at the output. The physical properties of the setup are the same as in section 3.3; they are listed in Tables 3.1 (pendulum) and 2.2 (motor).

#### 3.4.2.1 Parallel Elastic Actuation

Figure 3.11 presents the results obtained by inverse dynamics analysis of mechanical peak power as well as mechanical and electrical energy consumption of PEA. The relations of those quantities to the natural dynamics and equilibrium angle are indicated by the grey, black, and white lines which represent system resonance, link resonance, and the offset to which the system is tuned. Globally, mechanical peak power increases with rising frequency and is reduced to about 0.6 W and 2.8 W at the resonance frequencies of the system $\omega_{rs, \text{PEA}}$ (grey line) and the link $\omega_{rl, \text{PEA}}$ (black line), respectively. The equilibrium angle is set to the favorable value of 17.3° for operation at 10° which is indicated by the white line. For operation around this favorable value, over a large range of frequencies, a significant reduction of peak power is observed. Furthermore, a distinct minimum is observed when operating at the favorable offset slightly above the system resonance frequency $\omega_{rs, \text{PEA}}$ since the torque of the motor is minimal at this point.

The results for mechanical energy consumption show a very distinct reduction when operating at the favorable equilibrium angle as well. In contrast to mechanical peak power, a minimum occurs near the intersection of the lines representing operation tuned with respect to the offset and the link resonance frequency $\omega_{rl, \text{PEA}}$. The latter is due to the dominance of gearbox losses when it comes to mechanical energy, as explained in section 3.3.2.2. The optimum is observed at a frequency of 7.9 rad/s, slightly lower than the theoretical resonance frequency of 8.1 rad/s. This is due to friction losses proportional to speed, which tend to lower the optimal frequency (see section 3.3.2.2).
Figure 3.11: Mechanical peak power, mechanical, and electrical energy consumption obtained by inverse dynamics calculations of PEA with $k_p = 4.5 \text{ Nm/rad}$. Grey, black, and white lines represent system resonance, link resonance, and the offset to which the system is tuned.
Figure 3.12: Mechanical peak power, mechanical, and electrical energy consumption obtained by inverse dynamics calculations of SEA with $k_s = 4\text{Nm/ rad}$. Grey, black, and white lines represent first resonance and second resonance as well as antiresonance.
The minimal electrical energy consumption is located around the operating position offset $\theta_o$ for which the system is tuned. Energy consumption is generally high when not operating at the favorable angle. In this case, the actuator needs to deliver a higher torque, leading to an increase in Joule losses. In terms of frequency, the optimum is found at $7.0\text{rad/s}$. This is in between the system and link resonances $\omega_{rs,PEA} = 6.0\text{rad/s}$ and $\omega_{rs,PEA} = 8.1\text{rad/s}$ which, as discussed in section 3.3.2.2, is typical for PEA. Compared to the mechanical energy consumption, the minimum of the electrical energy is even more pronounced. The area of low energy consumption due to adjustment of the equilibrium angle $\theta_{eq}$ covers a wide range of frequencies. Hence, this parameter seems to be an important tool to cancel out the static torque caused by the operating position offset. The tuning of the PEA can further be extended by changing the stiffness of the elastic element, as studied in the previous section.

3.4.2.2 Series Elastic Actuation

The mechanical peak power as well as the mechanical and electrical energy consumption found for SEA are presented in Figure 3.12. A distinct minimum of the mechanical peak power is found at antiresonance frequency $\omega_{a,SEA}$ (white line) that is warped by the influence of the operating position offset. For higher offsets, the analytically predicted curvature of antiresonance is more distinct than that of the power minimum in the simulations (between 7.3 and 7.1 rad/s). This is caused by the relatively high value of the offset compared to the amplitude of the movement and the fact that the small angle approximation loses accuracy. However, the minimum can roughly be predicted analytically using linear models. Generally, mechanical power is rather low if $\theta_o = 0\text{rad}$, which corresponds to static equilibrium for the pendulum. At non-zero offsets, an additional static torque is required. The SEA, unlike the PEA, cannot compensate for this static torque by means of tuning the equilibrium angle. As a result, its energy consumption increases at non-zero offsets. A less distinct minimum is found around zero offset and $3.7\text{rad/s}$ which is close to the first resonance $\omega_{r1,SEA}$ (grey line). The second resonance $\omega_{r2,SEA}$ (black line) does not lead to any observable minimum. Consequently, power requirements rise monotonically with increasing frequency beyond antiresonance (see section 3.3.2.2)

Mechanical energy consumption shows a minimum at antiresonance $\omega_{a,SEA}$ which is even more pronounced than in mechanical peak power. It resembles the analytically determined lines almost perfectly. As in the previous section, no minima are observed at first resonance $\omega_{r1,SEA}$ and second resonance $\omega_{r2,SEA}$ since the gear box speed is minimal at antiresonance while gear box torque is minimal close to the link resonance of the rigid system (which corresponds to $\omega_{r1,PEA}$ for $k_p = 0\text{Nm/\text{rad}}$). As for mechanical power, reduced energy consumption is found around zero offset because the SEA cannot compensate for a static torque.

The electrical energy consumption shows the same trends as the mechanical energy. Energy consumption is minimal around antiresonance $\omega_{a,SEA}$ and at low offsets $\theta_o$. Comparing Figure 3.11 and Figure 3.12 shows that the energy requirements of the SEA at antiresonance are still lower than those of the PEA within the investigated offset range
CHAPTER 3. SERIES AND PARALLEL ELASTIC ACTUATION

Even if the latter is tuned with respect to the offset, yet, the minimum energy consumption area of the offset-adjusted PEA covers a wider frequency range. For SEA, energy consumption is more sensitive to frequency variation. This might be an issue for certain applications, to the point where a variable stiffness mechanism would be required. In contrast, the PEA might not require stiffness variation for tasks with a fixed offset and a limited frequency range. However, both effects depend on the value of the offset which is investigated in detail further ahead.

### 3.4.2.3 Effects of arbitrary offsets

As shown above, antiresonance-adjusted SEA can outperform PEA in terms of electrical energy consumption in the investigated offset range, even if the PEA is tuned by setting the equilibrium angle to the optimal value. To find out whether this is a general effect or the advantage of SEA disappears for larger offsets, energy consumption is calculated for offsets of 0°, 15°, 30°, and 45°. The springs of the PEA and SEA are optimally designed, i.e., stiffness is tuned to the system resonance $\omega_{rs,PEA}$ (PEA) or antiresonance $\omega_{a,SEA}$ (SEA), and the equilibrium angle of PEA is set according to Eq. (3.54). The curves of the minimum electrical energy consumptions depending on the offset are fitted using second order polynomials and given in Figure 3.13. The results indicate that SEA performs better for low offsets, whereas PEA is beneficial for trajectories with offsets higher than 36°. This is due to the static gravitational torque that can be compensated by changing the equilibrium angle of the spring. It must be noted that PEA consumption is slightly overestimated since its real optimum would be found below system resonance as described in Section 3.4.2.1. Nevertheless, the advantages of PEA at higher offsets are obvious.
3.4.3 Experimental evaluation

To evaluate the insights regarding power and energy, electrical energy consumption is experimentally validated with the setup specified in Section 3.3.3.1. The experimentally obtained electrical energy consumption data are presented in Fig. 3.14a for the PEA and in Fig. 3.14b the SEA. For PEA, the experimental results (green circles) are in good accordance with those from simulation (blue grid). The experiments confirm that lowest energy consumptions occur at an offset of $\theta_o = 10^\circ$ for which the actuator was tuned. Global minima of 0.89J ($\omega = 5\text{rad/s}$, $\theta_o = 10^\circ$) and 0.86J ($\omega = 6\text{rad/s}$, $\theta_o = 15^\circ$) are found in the experiment. If measurements would be taken continuously and not in a discrete grid, the actual optimum would have been found between those frequencies as discussed in Section 3.4.2.1, i.e., slightly below the intersection of the optimal offset and system resonance $\omega_{rs, PEA} = 5.6\text{rad/s}$. Obviously, energy consumption is on a rather low level in the proximity of the measured minima and thus their accuracy and comparison is affected by experimental deviations. Still, the experiment demonstrates that the analytic expression (3.54) can be used to adapt the equilibrium angle of a PEA to the trajectory offset. During the trials at 7 rad/s, one of the cables was observed to sag due to the highly dynamic loads. For this reason, the results at this frequency are not entirely reliable which may explain their deviation from the calculated values.

Considering the results for the SEA shown in Fig. (3.14)b, the measurements match the simulations similarly well as for PEA. An area of low energy consumption is found at offsets of $\theta_o = 0^\circ$ confirming the simulation results. The global energy consumption minimum of 0.25J is found at 7 rad/s and 0°. The frequency of 7 rad/s roughly corresponds to the theoretical antiresonance frequency $\omega_{a, SEA} = 7.4\text{rad/s}$, which is the energy-optimal operating point for the SEA.

Fig. (3.14)a and Fig. (3.14)b demonstrate that the minimum of the SEA (0.24J) is lower than that of the PEA (0.9J). This confirms the analytical finding of lower energy consumption in an antiresonance-tuned SEA. However, this advantage of the SEA should disappear at higher offsets as shown in the simulation in Section 3.4.2.

3.4.4 Discussion

In section 3.3, we established that a SEA was considerably more energy-efficient than a PEA in performing a sinusoidal motion with zero offset. When a static component is added to the load by adding an offset, the advantages of SEAs start to disappear. Conversely, a PEA can compensate the static torque by tuning its equilibrium angle. In our simulations and experiments, a PEA with resonance and offset-tuning showed a roughly constant energy consumption for offsets between 0° and 45° while the requirements of an antiresonance-tuned SEA clearly increased with the offset value.

While these simulations and experiments correspond to a simple case study, the results regarding the impact of static loads are highly relevant for design and control of elastic actuators. They provide a guideline for the selection of an elastic actuator topology with respect to the fraction of static loads. Furthermore, they give insight into the
Figure 3.14: Electrical energy consumption of (a) PEA and (b) SEA: comparison of analytical calculations (blue grid) and experiments (green circles).
3.5 Elastic elements for an active ankle prosthesis

In the previous sections, we have analyzed the performance of series and parallel elastic elements on a pendulum setup. On such a setup, the sinusoidal motions we have applied so far require torque from the motor which is fairly linear (sinusoidal) with its position. In section 3.2, we demonstrated that this type of load can, theoretically, reduce the mechanical motor power requirement to zero. Additionally, the only external force on the pendulum is gravity, which can be derived from a conservative force field. This means that, aside from dissipation in the drivetrain, there is no net energy gain or loss over a closed cycle. A spring – which is nothing but an energy buffer – is able to store and release energy over the course of a cycle, but if the cycle is closed, it always has to return to its initial state. As a result, springs can never inject or absorb a net amount of energy over a closed cycle. They are, in other words, energy-neutral, just like the task itself.

Based on these two observations, we can conclude that the case studies so far have been a perfect match for SEA and PEA. But what happens if the actuator is required to provide a more random, nonlinear trajectory? And what if the task is no longer energy-neutral, but the actuator needs to inject or absorb energy?

To answer these questions, we will analyze a realistic application where series and parallel elasticity are often applied: an active ankle prosthesis. More specifically, we will discuss how the operating range of the selected motor, the gear ratio, the drivetrain inertia and the springs influence one another. We will do this by optimizing the gear ratio and parallel/series elasticity of an actuated prosthesis, taking into account the constraints of the selected motor and, as in the rest of this work, its energy losses.

The section is based on the journal paper Verstraten et al. (2017) Optimizing the power and energy consumption of powered prosthetic ankles with series and parallel elasticity. Mechanism and Machine Theory, 116, pp. 419 - 432 [231].

3.5.1 Active ankle prostheses

Conventional prosthetic ankles are simple devices, enabling amputees to perform basic tasks. In recent years, they have evolved to advanced powered prostheses, approximating as closely as possible the biomechanical behavior of a healthy ankle. The aim is to reduce the problems associated with lower limb loss and amputees using the rest of their body to compensate for this loss. Amputees tend to walk slower and require a larger amount of energy to walk than able-bodied persons [241]. They also show asymmetric gait and joint pains which can be attributed to compensatory behavior at the sound limb [155]. Joint motion, torque and power generation increase at the healthy ankle of amputees when walking [155], effects which lead to a higher susceptibility to develop osteoarthritis in the
contralateral limb [206]. By adding active elements in ankle prostheses and being able to provide the same amount of energy of a healthy limb, the metabolic energy of walking can be reduced, as has been shown by Au et al. [12] for the powered ankle-foot prosthesis. Consequently, many active prostheses have been developed in recent years [42].

When looking at the compensatory strategies however, studies have shown the active prosthesis may not lead to a reduction, on the contrary. Ferris et al. suggest that this might be linked to the increased ankle power [68]. Adding active elements to prostheses also increases the weight, which is known to increase the energy cost and asymmetry of the gait cycle [138]. It is therefore necessary to reduce the weight of the actuation unit, which is possible by using series and parallel elasticity for power amplification [164] and energy storage. If the prosthesis were actuated by a geared motor, it would need to deliver a peak output power of about 250 W for a 75 kg person [244]. This requirement can be reduced by adding series elasticity, as applied in the SPARKy prosthesis [95] and the CYBERLEGs prosthesis [71]. With a combination of series and parallel elasticity, the peak torque can be reduced in addition to the peak power, like in the Powered Ankle Prosthesis [12]. Simulations show that series elastic actuation has the potential to reduce the ankle peak power by almost 80%, while parallel elastic actuation can reduce peak power by 66% and RMS power by 50% [240].

But what is the optimal combination of parallel and series elasticity in an actuated prosthetic ankle? This is a difficult question, because the optimal stiffness of the series and parallel elastic element are strongly linked, not only to each other, but also to the gear ratio [11]. An extensive analysis was performed by Grimmer et al., who optimized the springs for energy consumption and peak power [86] for various walking and running speeds. They found that parallel springs can be combined with series springs to reduce peak powers, but series springs alone are better for energy reduction. Eslamy et al. presented a similar analysis which also included unidirectional parallel springs [60]. They concluded that a configuration with series spring and unidirectional parallel spring can further decrease the energy demand. However, the optimization in these two works was based on mechanical energy consumption and mechanical peak power, measured on the motor shaft. It therefore disregards motor limitations, drivetrain dynamics and electrical losses. As demonstrated in section 3.3, neglecting these effects can lead to suboptimal results in terms of electrical energy consumption. Motor inertia, for example, can have a significant impact on SEAs for prosthetic limbs [95] and, more specifically, on their optimized stiffness [21]. Farah et al. also observed the importance of the drivetrain in their simulations of the open and closed loop response of an elastically actuated prosthesis [64]. An evaluation of the drivetrain characteristics is therefore gradually becoming an integral part of the design process of powered prosthetic feet [239][78].

3.5.2 Methods

In this section, we study the optimal design of Series Elastic Actuator (SEA) equipped with a unidirectional parallel spring. This actuator concept, sketched in Figure 3.15, is similar MIT’s Powered Ankle-Foot Prosthesis [11] and the one studied by Eslamy et al. [60]. The difference is that, in these works, linear compression springs are used.
Consequently, the spring stiffness also depends on the choice of the lever arm. We used torsion springs in order to remove this dependency. For the analysis presented here, we will work with a 150W RE40 Maxon motor. Our calculations showed that this is the smallest suitable motor from Maxon’s brushed DC motor range. The same motor was used in MIT’s Powered Ankle-Foot prosthesis [12] and ASU’s SPARKy 1 [95].

Using inverse dynamics, of which the equations are presented in section 3.5.2.1, we optimize the stiffnesses of the springs, as well as the equilibrium angle of the parallel spring. The optimization is performed with a parameter sweep, such that a global minimum is guaranteed. The series spring stiffness, parallel spring stiffness and equilibrium angle are varied within a range of 50 to 2000 Nm/rad, 0 to 1000 Nm/rad and -25 to 0 degrees, respectively, with steps of 10 Nm/rad, 10 Nm/rad and 0.5 degrees, respectively. Power and energy consumption are calculated from the inverse dynamics for each combination of series spring stiffness, parallel spring stiffnesses and equilibrium angle. We also impose several constraints based on mechanical and electrical limitations on the drivetrain. Any results which do not satisfy these constraints, specified in section 3.5.2.2, are discarded. The optimal results with respect to peak power and energy, both mechanical and electrical, are retained. These optimization objectives are defined mathematically in section 3.5.2.3.

We assume that the actuator is able to perfectly track the natural trajectory and torque of a sound human ankle. Ankle data was taken from Winter [244], for an able-bodied person of 75 kg walking at a natural cadence (105 steps per minute). The discrete dataset was carefully filtered in order to get smooth first and second order derivatives of the ankle torque and angle.

3.5.2.1 Equations

The torque on the gearbox \( T_l \) can be calculated as the sum of the required output torque \( T \) and the parallel spring torque \( T_s \):

\[
T_l = T + T_s \tag{3.59}
\]

The parallel spring (stiffness \( k_p \)) is unidirectional, i.e. it is only engaged when the ankle reaches a certain equilibrium angle \( \theta_{eq} \). The torque it provides, \( T_s \), can be written as

\[
T_s = \begin{cases} 
    k_p (\theta - \theta_{eq}) & (\theta \geq \theta_{eq}) \\
    0 & (\theta < \theta_{eq})
\end{cases} \tag{3.60}
\]

The ankle angle \( \theta \) and the equilibrium angle \( \theta_{eq} \) are defined in Fig. 3.15. As mentioned earlier, we assume that the required ankle torque \( T \) and ankle position \( \theta \) perfectly match the biological ankle data from Winter for an able-bodied person of 75 kg [244].

Equation (3.60), however, creates a problem at \( \theta = \theta_{eq} \), because it is not differentiable at this angle. Instead of this simple and intuitive equation, we therefore use the more numerically favorable

\[
T_s = \frac{1}{2} k_p \left( \theta - \theta_{eq} + \sqrt{s^2 + (\theta - \theta_{eq})^2} \right) \tag{3.61}
\]
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Figure 3.15: Schematic of the actuator with angle definitions. The actuator consists of a Series Elastic Actuator (red) and a unidirectional parallel spring (orange), which is engaged when $\theta \geq \theta_{eq}$. The ankle angle $\theta$ is zero when the foot is perpendicular to the leg (denoted by a dashed line). A plantarflexed ankle, as depicted in the figure, corresponds to a negative value of $\theta$. Note that, for clarity of the presentation, the elastic elements are drawn as compression springs in the figure, although they are modeled as torsional springs in the work.
### Table 3.3: Motor and gearbox parameters for 150W RE40 Maxon motor and GP52 gearbox. Gear ratios below $n = 165$ did not produce any feasible results in our optimizations. Gearbox efficiency varies with the gear ratio, but above $n = 156$, its value remains constant in the data sheet. This justifies the constant value for gearbox efficiency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque constant $k_t$</td>
<td>30.2 mNm/A</td>
</tr>
<tr>
<td>Speed constant $k_b$</td>
<td>3.155 mV/rpm</td>
</tr>
<tr>
<td>Friction coefficient $\nu_m$</td>
<td>5.21e-6 Nms/rad</td>
</tr>
<tr>
<td>Terminal resistance $R$</td>
<td>0.299 Ω</td>
</tr>
<tr>
<td>Terminal inductance $L$</td>
<td>0.0823 mH</td>
</tr>
<tr>
<td>Motor inertia $J_m$</td>
<td>142 gcm²</td>
</tr>
<tr>
<td>Gearbox inertia $J_{tr}$</td>
<td>9.4 gcm²</td>
</tr>
<tr>
<td>Gearbox efficiency $\eta_{tr}$</td>
<td>68 %</td>
</tr>
</tbody>
</table>

This function is differentiable for any $\theta$. The factor $s$ determines how smooth the engagement of the parallel spring will be. The lower this value, the closer Eq. (3.61) resembles Eq. (3.60). We chose $s = 0.01$ rad.

The torque on the motor shaft $T_m$ is given by

$$ T_m = \frac{C}{n} T_l $$

(3.62)

Here, $n$ is the gear reduction ratio, and $C$ is the gearbox efficiency function given by Eq. (3.17). Motor speed $\dot{\theta}_m$ is influenced by the stiffness of the series spring $k_s$, and is given by

$$ \dot{\theta}_m = n \cdot \left( \frac{T_l}{k_s} + \dot{\theta} \right) $$

(3.63)

Finally, the electrical power consumption $P_{elec}$ is the product of motor current ($I$) and voltage ($U$). These are obtained by applying the motor model

$$ \begin{cases} I = \frac{1}{k_t} \left( (J_m + J_{tr}) \dot{\theta}_m + T_m + \nu_m \dot{\theta}_m \right) \\ U = LI + RI + k_b \dot{\theta}_m \end{cases} $$

(3.64)

which requires knowledge of several motor and gearbox parameters. These are defined in Table 3.3, along with their values for the motor and gearbox used in our optimization. Notice the additional contributions to the torque due to motor and gearbox inertia $(J_m + J_{tr}) \dot{\theta}_m$ and viscous friction $\nu_m \dot{\theta}_m$. The viscous friction coefficient is derived from the motor’s no-load speed and no-load current, based on Eq. (3.36).

#### 3.5.2.2 Constraints

Most optimizations only consider torque and speed at the gearbox shaft. In reality, however, the output torque and speed are limited by mechanical and electrical constraints. A good controller design should therefore ensure that the motor’s speed and torque saturate whenever it reaches its limits, preventing failure of one or more components. The trade-off is a decrease in dynamic performance, since the actuator will not be able to provide...
the required torque or speed to follow the desired output trajectory. Therefore, setting appropriate constraints in the optimization is equivalent to setting a performance threshold for the actuator.

Here, we will only consider constraints related to the motor, although other components in the drivetrain may impose additional constraints. The Maxon GP52 gearbox, for example, can only handle 30 Nm continuously and 45 Nm intermittently (i.e. for a duration of one second), according to the manufacturer’s datasheet. In order to not reduce the range of potential solutions too much, these constraints were omitted from the optimization. From our practical experience, we know that the proposed maximum values are very conservative, and may be exceeded at the potential cost of reduced lifetime.

The constraints applied in our optimization are listed below.

**Max. motor speed (mechanical)** While motor speed can be pushed up by increasing the supply voltage, the lifetime of the motor and gearbox bearings puts a practical upper limit $\dot{\theta}_{\text{max}}$ on the motor speed:

$$|\dot{\theta}_m| < \dot{\theta}_{\text{max}}$$

(3.65)

The speed limit $\dot{\theta}_{\text{max}}$ can be obtained from the motor and gearbox catalogs. This restriction can be considered a *soft restriction*, because motor speed can easily be limited by the controller. Even if the maximum speed is occasionally exceeded by a small amount, this should not directly lead to failure, but at worst to a slightly reduced lifetime. The main issue is the actuator’s reduced capability of following the desired output speed. It is up to the designer to decide whether this could have an adverse effect on the prosthesis’ performance. In this analysis, we set $\dot{\theta}_{\text{max}}$ to the maximum motor speed specified in the data sheet, 12 000 rpm.

**Max. motor torque (thermal - mechanical)** Overheating of the motor can occur due to high RMS currents (long-term heating) or high peak currents (short term). Motor manufacturers typically specify a maximum continuous torque $T_{m,\text{max},\text{cont}}$, below which overheating should not occur. This leads to the following constraint for long-term heating

$$[k_t I]_{\text{RMS}} < T_{m,\text{max},\text{cont}}$$

(3.66)

We consider the long-term heating constraint to be a *hard restriction*: if Eq. (3.66) is not satisfied, it indicates that the actuator is not properly designed for the required loads. The maximum continuous torque can be found in the motor datasheet ($T_{m,\text{max},\text{cont}} = 177 \text{ mNm}$).

Short-term heating is more difficult to handle. A detailed calculation would rely on parameters such as the motor’s thermal resistance and thermal time constant, as well as the ambient temperature. Very often, however, the motor’s peak torque and current will be determined by the strength of the gearbox or the maximum current output of the controller, which can be characterized by a single torque value. Instead of the thermal calculation,
we therefore define a maximum torque $T_{m,\text{max,int}}$ which satisfies these limitations, and write the following constraint for peak torque:

$$|k_t I| < T_{m,\text{max,int}}$$ (3.67)

Whether the peak torque constraint should be considered a hard or soft restriction, depends on the case. Exceeding the controller’s maximum current output, for example, can be tolerated in inverse dynamic optimizations, since the current will in practice be limited by the controller, only leading to a reduced performance of the actuator. The torque should, however, not cause mechanical failure of any components. In the optimizations, we used the controller’s maximum output current of 30 A as the most restrictive case for peak torque, so $T_{m,\text{max,int}} = 30 A \cdot k_t = 0.9 \text{Nm}$ for the 150 W Maxon motor considered in this work. As explained earlier, the limitation on the gearbox output torque (45 Nm) is not taken into consideration for practical reasons.

**Max. motor torque (due to available voltage)** The higher the voltage available from the power supply, the higher the torque that can be delivered at a certain speed. If $U_{\text{max}}$ is the maximum available motor voltage, the relationship between torque, speed and voltage becomes

$$\left| T_m + (J_m + J_{tr}) \dot{\theta}_m + \left( \frac{v + k_b k_t}{R} \right) \dot{\theta}_m \right| < U_{\text{max}} \frac{k_T}{R}$$ (3.68)

This is, again, a soft constraint: if the power source cannot supply the required voltage, this will simply result in a decreased output torque. In our optimizations, we assumed $U_{\text{max}} = 48 \text{V}$.

### 3.5.2.3 Optimization objectives

We consider four relevant optimization objectives, all very common in literature:

**MPP: minimal mechanical peak power** Mechanical peak power is the highest value of $|P_{\text{mech}}|$ during a gait cycle. The mechanical power $P_{\text{mech}}$ is calculated from the torque and speed at the output of the gearbox,

$$P_{\text{mech}} = \frac{1}{n} T_l \dot{\theta}_m$$ (3.69)

$P_{\text{mech}}$ is thus affected by the springs, but not by gearbox losses and drivetrain inertia.

**MEC: minimal mechanical energy consumption** In most works, the mechanical energy consumption is calculated as the integral of the absolute value of the mechanical power $P_{\text{mech}}$:

$$E_{\text{mech,abs}} = \int |P_{\text{mech}}| dt$$ (3.70)
By definition, however, energy should be calculated without absolute value, and (3.70) therefore does not represent the actual mechanical energy. As explained in section 2.5, the concept behind this cost function is that power losses are proportional to the power flow through the components. It is assumed that, by reducing the power flowing in and out of the actuator, the energy losses will be minimized. This method does, however, not account for differences in loss mechanisms between the individual components of the actuator, which could potentially shift the actual minimum.

**EPP: minimal electrical peak power**   Electrical peak power is the highest value of $|P_{elec}|$ during a gait cycle. The electrical power $P_{elec}$ can be calculated by multiplying the motor current $I$ and voltage $U$:

$$P_{elec} = UI$$

(3.71)

**EEC: minimal electrical energy consumption**   Electrical energy consumption is calculated as

$$E_{elec} = \int P_{elec} dt$$

(3.72)

This energy corresponds to the energy available at the motor terminals.

### 3.5.3 Optimization of spring stiffness

In this section, we present a thorough analysis of the optimization and its results. We discuss how different optimization criteria influence the optimal series and parallel spring stiffness. Analyses of power flows and the power loss profile are presented to provide a better understanding of the results. In this section, a fixed gear ratio of $n = 250$, the energy-optimal choice, is assumed. The influence of the gear ratio on the results will be evaluated in section 3.5.4.1.

#### 3.5.3.1 Optimization results

The optimization results are summarized in Table 3.4. For comparison, the powers and energies of the rigid actuator, i.e. the same geared DC motor without springs, are also shown. These values are hypothetical since, unlike for the actuator designs with series and parallel compliance, the rigid actuator would not be able to satisfy the optimization constraints. It can only follow the imposed ankle trajectory if it is overpowered, which is only possible for a limited amount of time. This demonstrates how the capabilities of an actuator can be augmented by adding series and/or parallel springs. Furthermore, the SEA with unidirectional parallel spring yields much lower energy consumption and peak powers than a rigid actuator. Electrical energy, for example, can be reduced by almost 40%, and electrical peak power by no less than 63%.

In general, electrical peak powers are typically 2-3 times higher than the mechanical ones. This indicates that a great deal of power is lost in the drivetrain. Nevertheless,
Table 3.4: Optimization of an SEA (series spring stiffness $k_s$) with unidirectional parallel spring (stiffness $k_p$, equilibrium angle $\theta_{eq}$) for an active ankle prosthesis. The prosthesis is designed to mimic the ankle of an able-bodied person weighing 75 kg, at natural walking speed. Results are presented for four different optimization criteria: mechanical peak power (MPP), electrical peak power (EPP), mechanical energy consumption (MEC) and electrical energy consumption (EEC). Peak powers and energy consumption of a rigid actuator (i.e. no springs) is shown for reference.
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Figure 3.16: Torque-angle plot of the ankle, with optimized unidirectional parallel springs for electrical peak power (red) and electrical energy (blue). At angles below the equilibrium angle $\theta_{eq}$, the parallel spring does not contribute to the output torque. At angles above $\theta_{eq}$, torque increases linearly with increased dorsiflexion. Optimizing for electrical energy consumption (EEC) yields a higher spring stiffness than an optimization for electrical peak power (EPP), indicated by the steeper slope in the EEC case. Higher stiffness is accompanied by a higher $\theta_{eq}$, in order to keep the spring stiffness profile closer to the angle-torque characteristic of the ankle.

whether the optimization is performed based on the mechanical or electrical properties of the system does not make much of a difference. Results mainly depend on whether the objective is peak power or energy reduction. Energy reduction requires the series spring to be as stiff as possible, whereas some series compliance is desired in order to minimize the peak power. In the latter case, the parallel spring should be relatively compliant and tuned to very negative equilibrium angles. This is visualized in a torque-angle plot (Fig. 3.16). For EPP, the spring is engaged from mid-swing to late pre-swing for EPP\(^4\). Conversely, when energy consumption is minimized (EEC), the prosthesis should be equipped with a stiff parallel spring with an equilibrium angle close to zero. This way, the spring will only be active during stance and the early pre-swing phase.

How well do these results match with tests on actual prototypes? Little experimental

\(^4\)Mid-swing and pre-swing are part of the swing phase of the gait cycle. For definitions, we refer to [170].
data is reported in literature, but there is one comparable work by Sup et al. [212]. The actuation concept in this analysis is the same as the one suggested by our MEC and EEC optimizations, i.e. a rigid actuator with unidirectional parallel spring. Sup et al. reported an average electrical power of 45 W, which corresponds to an electrical energy consumption of approximately 31 J. This value is very comparable to our optimal value of 35.2 J. The small difference with our results can easily be explained by the different drivetrain designs.

3.5.3.2 Power flow analysis

The power balance for the ankle prosthesis can be defined as

\[ P_{bio} + P_{inertia} + P_{loss} = P_p + P_s + P_{elec} \]  (3.73)

where \( P_{bio} \) stands for the biological (output) power requirement, \( P_{elec} \) for the electrical power, \( P_p \) for the power of the parallel spring, \( P_s \) for the power of the series spring, and \( P_{inertia} \) for the power delivered by the inertia of the gearbox and motor. These power flows, resulting from the electrical peak power (EPP) and electrical energy (EEC) optimizations, are plotted in Fig. 3.17. From this figure, we can learn several things: the importance of drivetrain inertia in the swing phase\(^5\), the influence of optimization objectives on the equilibrium angle of the parallel spring, and the emergence of power peaks due to the interaction between the series spring and the unidirectional parallel spring. These topics are discussed below.

The importance of drivetrain inertia during swing

Because of the relatively low inertia of the foot, the biological power during swing phase is close to zero. The motor, however, still consumes a significant amount of power in this phase, especially when optimized for electrical peak power. Most of it is due to its own reflected inertia, which tends to be bigger than the inertia of the foot due to the high gear ratios typical of prosthetics. This phenomenon has also been observed in electrical measurements on the SPARKy 1 prosthesis [95] and the CYBERLEGs prosthesis [78]. Note that the inertial torque is not included in the optimizations based on mechanical power and energy. Consequently, these types of optimization are likely to be suboptimal in case of high accelerations and high drivetrain inertia.

Equilibrium angle of the parallel spring

Looking at the parallel spring, we observe that it provides a power to the output which roughly follows the required biological output power. The main difference between the electrical peak power (EPP) and the electrical energy (EEC) optimization is the equilibrium angle of the parallel spring. As shown previously in Fig. 3.16, this equilibrium angle is a lot more negative in the EPP optimization (\( \theta_{eq} = -8.5^\circ \)) than in the EEC optimization (\( \theta_{eq} = -1^\circ \)). This manifests itself in two power bumps at mid-stance (73-83% gait cycle) and terminal swing (88-97%) for the EPP objective, but not for EEC, where the parallel spring is disengaged. In an electrical energy

\(^5\)The swing phase is the part of the gait cycle during which the foot is off the ground.
Figure 3.17: Power flows optimized for electrical peak power (a) and the electrical energy (b). The biological power profile $P_{bio}$ (blue) and the electrical power profile $P_{elec}$ (green) are plotted along with three power flows responsible for the most significant power shifts: the parallel spring ($P_p$, red), the series spring ($P_s$, yellow) and the inertia of the gearbox and motor ($P_{inertia}$, purple). Positive powers indicate that power is injected into the environment, i.e. all powers are defined from a motor perspective. The high inertial power peak is caused by the interaction between series and parallel spring, when the latter is engaged. This is reflected in the electrical power as well.
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optimization, such bumps are to be avoided, since motors are generally inefficient at delivering low powers. Conversely, these bumps are completely irrelevant for the peak power optimization as long as they are not higher than the peak power during the stance phase. For this reason, the optimization engages the parallel spring at a more negative angle in order to further reduce the peak power during stance, even though this is unfavorable during the swing phase. Also notice that, despite the distinct difference in equilibrium angles, the power profile of the parallel spring in stance is still quite similar for both objectives. This is achieved by lowering the spring stiffness for more negative equilibrium angles. In Section 3.5.4.1, we will show that this is a general trend.

Power peaks due to interacting springs

Another important issue is the strong power peak at around 55% gait cycle, which is caused to the interaction between the unidirectional spring and the series spring. The engagement of the unidirectional parallel spring is modeled by Eq. (3.61). The second derivative of this equation, $\ddot{T}_s$, resembles a pulse at $\theta = \theta_{eq}$. Since the acceleration of the motor is given by

\[ \ddot{\theta}_m = n \cdot \left( \frac{\ddot{T}}{k_s} + \dot{\theta} \right) \]  

(3.74)

maintaining a continuous output speed at the moment of engagement requires a high acceleration from the motor, and thus a powerful burst of torque. In Fig. 3.17, this presents itself as a strong peak in $P_{inertia}$ and in the electrical motor power $P_{elec}$ at 57% gait cycle (EPP) and 54% (EEC). Furthermore, according to Eq. (3.74), the power peak should increase as the series spring gets more compliant. The optimization therefore yields high series stiffness values to minimize its influence on the actuator dynamics. Indeed, in Fig. 3.17, the power attributed to the series spring is nearly zero for both objectives.

3.5.3.3 Power loss analysis

Figure 3.18 gives a clearer view on how the springs affect the power losses. The EPP objective yields power losses which do not exceed 75 W, while the EEC objective, at its peak, causes almost 170 W to be lost between 40-60% gait cycle. Nevertheless, the power losses of the EEC objective are very low over the rest of the gait cycle. In the EPP power profile, multiple peaks of approximately 40 W appear, which add up to a greater overall energy loss than the EEC objective. Two of these peaks, which are not present in the EEC optimization, indeed occur in the swing phase, as indicated in our previous discussion. This confirms that preventing their appearance is one of the keys to reducing energy consumption.

3.5.4 Influence of drivetrain

In this section, we discuss the influence of the selected motor and gearbox on the optimal stiffnesses and the resulting peak powers and energy consumption. First, the gear ratio is varied to study its effect on the selection of springs and the resulting power and energy
Figure 3.18: Power losses over a gait cycle. The blue line corresponds to the electrical peak power (EPP) optimization, the red line to the electrical energy (EEC) optimization. Despite the peak loss of almost 170 W, the latter is the more efficient solution because the average power loss is smaller.
consumption (subsection 3.5.4.1). Next, we repeat the analysis with drivetrain inertia reduced by 50%, in order to evaluate its influence on the results.

### 3.5.4.1 Influence of gear ratio

An important consideration in any actuator design is the choice of the gearbox. High gear ratios can be used to match high-speed motors to the required output characteristics, but in general, they come at poor efficiencies. Furthermore, they increase the reflected inertia of the motor, which can be a problem for a high-speed application such as prosthetic ankles. Well-chosen series and parallel springs may provide a solution, because they allow to reduce the speed and/or torque required from the motor, such that the gear ratio can be decreased. Below, we will discuss the influence of the gearbox on the feasible stiffness combinations as well as the resulting power and energy consumption.

**Influence on motor constraints** Figure 3.19 shows which constraints are violated for various combinations of series and parallel stiffness. The gear ratio was fixed to \( n = 180 \) (Fig. 3.19a) and \( n = 360 \) (Fig. 3.19b), two values on either side of the range of gear ratios that lead to feasible solutions. The figures give insight into how the gear ratio influences the choice between parallel and series springs. Two main conclusions can be drawn. First, with high gear ratios, more compliant parallel springs can be used, but the constraint on motor speed is more easily exceeded. Conversely, low gear ratios make the design more likely to exceed the peak torque, such that high accelerations cannot be achieved. In both cases, series springs are not allowed to be very compliant. Second, optimizations that do not take motor constraints into account easily lead to designs that violate these constraints.

As explained in section (3.5.2.2), the most important motor limitation is the maximum continuous torque. Figure 3.19 shows that most spring combinations fail to respect this constraint. Parallel springs can be helpful in this regard, because well-tuned parallel springs allow to decrease the torque on the motor. Indeed, within a certain range of parallel stiffnesses, several combinations of series and parallel springs appear which lead to a feasible design. For high gear ratios (\( n = 360 \), Fig. 3.19b), the range of parallel spring stiffnesses is quite wide, spanning from 900 Nm/rad down to low values of 200 Nm/rad. Compliant parallel springs can be used because the torque output of the geared DC motor is increased by the gearbox, reducing the need for high torques from the parallel spring. The disadvantage is that the gearbox also reduces the attainable output speed, leading to the appearance of a large area where the motor’s maximum speed is exceeded. Speed is no longer a problem at low gear ratios (\( n = 180 \), Fig. 3.19a), but the geared motor’s output torque is decreased, meaning that the motor’s maximum torque (or current) is reached more easily. This is especially a problem at low series stiffnesses, due to the peak in motor acceleration that appears whenever the parallel spring is engaged or disengaged. As explained in section 3.5.3.2, motor acceleration increases when more compliant series springs are used, causing higher peak torques. This explains why, in Fig. 3.19a, the maximum peak torque is exceeded for series stiffnesses between approximately 500-700 Nm/rad.
Figure 3.19: Feasibility of the motor and for gear ratios of (a) $n = 180$ (b) $n = 360$. The equilibrium angle $\theta_{eq}$ is tuned to the optimal value for every combination of series and parallel stiffness. The optimal stiffness combinations found by minimizing mechanical peak power (red) or mechanical energy consumption (green), not taking into account the motor constraints, are indicated as dots. For both gear ratios, these stiffness combinations fail to respect the maximum continuous torque that can be delivered by the motor.
Figure 3.20: Influence of gear ratio on spring selection for different optimization objectives. The optimal series spring stiffness $k_s$ (top), parallel spring stiffness $k_p$ (middle) and equilibrium angle $\theta_{eq}$ (bottom) are plotted as a function of the different gear ratios $n$. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple).

Also plotted on Fig. 3.19 are the optimal stiffness combinations found by minimizing mechanical peak power (red) or mechanical energy consumption (green), not taking into account the motor constraints. The results are consistent with those from Grimmer et al. [86] for bidirectional springs: MEC optimization yields an SEA without parallel spring, whereas MPP optimization yields a combination of series and parallel elasticity. Interestingly, neither of both results fit within the motor’s operating range for the two gear ratios presented above. Twice, the violated constraint is the continuous motor torque. This could have detrimental consequences: if these optimized stiffnesses would be used in combination with the RE40 motor, the windings would burn up within a number of cycles. The only solution is to select a more powerful motor, leading to a heavier and bulkier design, or to settle for decreased actuator performance by lowering the demanded output torques or speeds. By already selecting a motor in the optimization phase and setting appropriate constraints, such problems can be avoided.

Influence on optimal spring stiffness Figure 3.20 shows how the optimal spring stiffnesses $k_s$ and $k_p$ and the optimal equilibrium angle $\theta_{eq}$ evolve with the chosen gear ratio.
This figure confirms the strong influence of optimization constraints on the optimized spring stiffnesses. We notice a strong preference for stiff series springs for all objectives except MPP. The optimal stiffness of the parallel spring depends on whether peak power or energy consumption is minimized. In the former case, compliant parallel springs can be used, while the latter objective requires stiff springs.

In the previous section, we already demonstrated the strong effect of the optimization constraints on the optimal solution for two extreme values of the gear ratio. Figure 3.20 shows that this can be extended to the entire range of gear ratios. Gear ratio does not affect the MPP or MEC objectives, and therefore one might expect a single value for the optimized springs stiffnesses and equilibrium angle. Yet, the optimized springs show a clear dependence on the chosen gearbox due to the constraints. This confirms limitations of the drivetrain and controller as a dominant factor in the design.

In general, the series spring is required to be relatively stiff in order to prevent power peaks. In fact, all optimization objectives except MPP demand the series spring to be as stiff as possible for most gear ratios. This is in contrast with the parallel spring, especially for the energy optimization objectives MEC and EEC. Here, the parallel spring’s stiffness is fairly constant, at a relatively high value of approximately 600 Nm/rad. For the peak power objectives EPP and MPP, the parallel spring is only required to be this stiff at the lowest gear ratios. As the gear ratio increases, the geared DC motor is able to deliver more torque to the output, and the need for the parallel spring decreases. This is reflected in the gradually decreasing parallel stiffness values.

Finally, the optimal equilibrium angle $\theta_{eq}$ (bottom of Fig. 3.20) shows a trend which is nearly identical to that of parallel stiffness. High parallel stiffnesses $k_p$ come with small $\theta_{eq}$ while low $k_p$ comes with higher $\theta_{eq}$. As explained in section 3.5.3.2, this is required for the parallel spring to deliver the optimal amount of power during stance without causing power peaks during swing.

**Influence on optimization objectives** Figure 3.21 demonstrates the effect of different optimization objectives on the optimal gear ratio.

Achieving low mechanical peak power demands sufficiently high gear ratios. The gradual decrease in MPP as the gear ratio increases to 280 reflects the movement towards a broader operating region of the motor. Between $n = 280$ and $n = 320$, the selected spring stiffnesses remain roughly the same (see previous section), and the mechanical peak power converges to its optimal value. Conversely, electrical peak power exhibits a minimum at a lower gear ratio, approximately at $n = 260$. This is explained by the growing torque due to the motor’s reflected inertia, which increases with gear ratio. This inertial torque, which does not affect mechanical peak power, leads to an increase in electrical peak power at high gear ratios.

When it comes to electrical energy consumption, there is no clear difference between the MEC and EEC objective. This is consistent with our findings in previous sections. In section 3.5.3, we drew the same conclusion for a fixed gear ratio of $n = 250$, and in section 3.5.4.1, the optimized spring stiffnesses were similar for both objectives. If the springs are optimized for peak power instead of energy consumption, the electrical energy
Figure 3.21: Influence of choice of gear ratio on mechanical and electrical peak power and energy consumption. The values in this graph correspond to the optimal settings of spring stiffness and equilibrium angle. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple). The mechanical energy consumption plot shows the net mechanical energy input required for walking (16.5 J). It is independent of the gear ratio and optimization objective, since springs are only a medium of energy storage; they do not have the ability to dissipate or generate energy over an energy cycle.
consumption will be much higher. This also proves that both objectives are not necessarily related, as previously remarked by Grimmer et al. [86].

3.5.4.2 Influence of actuator inertia

In section 2.3.3.2, we presented an extensive discussion on how the inertia of the motor and gearbox affects the choice of gear reduction. We derived that, with a pendulum as a load, the optimal gear ratio of the actuator is found by matching the load inertia $J_{\text{load}}$ to the reflected drivetrain inertia $n^2 (J_m + J_{tr})$. The resulting gear ratio,

$$n = \sqrt{\frac{J_{\text{load}}}{J_m + J_{tr}}}$$

(3.75)

maximizes the actuator’s acceleration capability [165] and energy efficiency [104]. This principle, called “inertia matching”, implies that the optimal drivetrain design is defined as soon as the load is known.

Unfortunately, a simple analytical solution as Eq. (3.75) does not exist for prostheses. Here, the load is far from purely inertial, since most of the torque on the ankle joint results from interaction with the environment. The only way to study the effect of inertia, is to repeat the optimization with a different drivetrain inertia. In this section, we reprise the analysis presented in sections 3.5.4.1 and 3.5.4.1 with $J_m$ and $J_{tr}$ reduced by 50%.

**Influence on optimization objectives** Figure 3.22 presents the peak powers and energy consumptions as a function of gear ratio, with reduced drivetrain inertia. Comparing these results to the ones obtained with full actuator inertia (Figure 3.21), the minimal mechanical peak power remains roughly the same (100 W). This is logical: mechanical output power does not depend on drivetrain inertia; it only affects the MPP optimization results through its impact on the constraints. In contrast, electrical peak power is strongly influenced by the decreased inertial torque. At the optimal configurations, it is reduced from 234 W to 113 W for the optimal value. The impact of decreased inertia is not as strong in terms of electrical energy consumption (decrease from 35 J to 32 J). Recall that inertia itself is an energy-neutral element, i.e. it does not dissipate energy. The reduced energy losses are a secondary effect of the decrease in inertial torque, which results in less Joule losses.

**Influence on optimized spring stiffnesses** In Figure 3.23, the optimized spring stiffnesses are plotted as a function of gear ratio with drivetrain inertia reduced by 50%. A major difference with the results in Figure 3.20 is the increased usage of the series spring, especially for the MPP and EPP objectives. The parallel spring design remains roughly the same for the MEC and EEC objectives. The MPP and EPP objectives, however, yield very different parallel spring designs. With reduced inertia, the equilibrium angle is lowered to the point where the spring is engaged throughout the entire gait cycle. In agreement with our results from the previous paragraphs, the parallel spring stiffness is lowered accordingly.
Figure 3.22: Influence of choice of gear ratio on mechanical and electrical peak power and energy consumption, when actuator inertia is reduced by 50%. The values in this graph correspond to the optimal settings of spring stiffness and equilibrium angle. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple).
Figure 3.23: Influence of gear ratio on spring selection for different optimization objectives, when actuator inertia is reduced by 50%. The optimal series spring stiffness $k_s$ (top), parallel spring stiffness $k_p$ (middle) and equilibrium angle $\theta_{eq}$ (bottom) are plotted as a function of the different gear ratios $n$. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple).
Influence on optimal gear ratio  Equation (3.75) predicts an increase in gear ratio when the drivetrain inertia is reduced. Indeed: the range of gear ratios has clearly shifted to higher values in figures 3.22 and 3.23. The same applies to the optimal gear ratios. Minimal electrical power consumption, for example, was previously achieved for \( n = 250 \). With half of the drivetrain inertia, the optimal gear ratio has shifted to \( n = 310 \). This can also be interpreted in terms of reflected inertias. While drivetrain inertia was decreased by 50\%, the reflected inertia only decreased by 23\% percent. This demonstrates that, as predicted by the inertia matching principle, a change in motor inertia will be opposed by a change in gear ratio.

Note that the efficiency of gearboxes decreases with their gear ratio. This may cancel out the energetic benefits gained from the decreased inertial torque.

3.5.5 Implications for the design of prosthetic ankles

The results presented in this section teach us several important things regarding the design and optimization of prosthetic ankles with series and parallel elements.

A first aspect that was discussed, is the difference between electrical and mechanical power flows. Based on the biological power profile, nearly no power would be required during the swing phase. Nevertheless, accelerations in the swing phase - as well as in late stance - are high. This leads to surprisingly high power peaks in these phases, since the motor needs to accelerate its own inertia as well as that of the gearbox. Failing to provide the required accelerations may, however, compromise ground clearance. This must be carefully assessed, because ground clearance prevents the user from stumbling and falling, the most important task during swing \[22\]. It is therefore recommendable to take the inertia of the drivetrain into consideration in a design phase.

Another goal was to evaluate the effect of different objectives on the optimization. As one may expect, peak power-based and energy consumption-based optimizations lead to very different results, because the former only applies to a single point in the gait cycle, whereas the latter takes the entire cycle into account. Whether the optimization is performed in the mechanical or electrical domain makes less of a difference. The common assumption that (electrical) energy consumption can be minimized by minimizing the (mechanical) power flow in the drivetrain, as specified by Eq. (3.70), thus seems to be valid in this specific case. This is a valuable insight because, as discussed in section 2.5.2, the latter cost function is much easier to apply.

Regarding the selection of springs, our results demonstrated a general trend of favoring designs with only parallel elasticity. On a superficial level, this result confirms that of Eslamy et al. \[60\]. Nevertheless, the approach followed here is very different from that in Eslamy et al. Our analysis has demonstrated that the optimal design strongly depends on the choice of the motor – something which was not taking into account in the aforementioned work. Resulting from the interaction between the series spring and the unidirectional parallel spring, designs combining series and parallel elasticity were found

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Ground clearance, also referred to as “foot clearance” or “toe clearance”, is the minimum vertical distance between the ground and the lowest point of the foot during the swing phase of gait.
to require torques beyond the motor’s limits. Such solutions were discarded in our optimization, which is the reason why our optimized series springs tend to be stiff. Not taking motor limits into account in the design phase, like in the approach followed by Eslamy et al., may lead to selecting a larger motor than strictly necessary. A more effective strategy would be to choose the motor first, and then optimize the springs within the operating range of the selected motor. Of course, this may require several iterations in order to find the smallest possible motor.

Finally, the choice of gearbox mainly depends on the required output torques and speed, but the optimal gear ratio is also influenced by the inertia of the drivetrain. Choosing a correct gear ratio is important, because it will determine the range of stiffnesses available for a certain motor. Springs can therefore only be exploited if a matching gear ratio is chosen. Gearboxes, however, have several disadvantages. Firstly, they increase the reflected inertia of the drive. As mentioned earlier, prosthetic ankles require considerable accelerations, which can lead to high inertial torques caused by the inertia of the motor and gearbox. Secondly, gearbox efficiency decreases with increasing gear ratio. This is particularly relevant when the inertia of the drivetrain is low, because, as our results have shown, low-inertia drivetrains will require higher gear ratios. In that case, the most energy efficient design could well be one with suboptimal springs in combination with a smaller, more efficient gearbox. The gear ratio should therefore just as much be considered a part of the optimization as the springs themselves.

### 3.6 Conclusion

In this chapter, we have analyzed how series and elastic elements can reduce the peak power and energy consumption of an actuator. This topic has been the subject of several works already, although most of these works focus on one specific application or task. Moreover, the analysis is often confined to the mechanical peak power and energy consumption. The aim of this chapter was to extend the discussion to the electrical domain and to analyze the influence of various loads.

It is widely known that series elastic elements can be used to decrease the motor speed, while parallel elastic elements reduce the torque required from the motor. As we have shown in section 3.2, this applies when the external load exhibits a reasonable degree of linearity with respect to the output angle. The human ankle and knee joint exhibit such behavior during walking, which is why elastic elements are very common in ankle and knee prostheses and exoskeletons. Several works have proven the great potential of these elastic elements for the reduction of peak power and energy reduction.

An interesting aspect of human walking is that it consists of two distinct phases: a stance phase, dominated by the force exerted by the body weight, and a swing phase, which is governed by the inertia of the links. While the stance phase is described fairly well by its torque-angle relationship, inertial effects make the swing phase a lot more complicated to study. In spite of the low mechanical powers required during the swing phase, these effects can still cause high peaks in electrical power. This observation illustrates the need for a more detailed study on elastic elements applied to inertial loads.
An extensive analysis on a pendulum – an inertial load in a gravitational field, just like the human leg in swing – added some new insights to the “parallel elements for torque reduction, series elements for speed reduction” paradigm. At a first glance, this principle is confirmed by the fact that optimally designed parallel springs exploit a system’s resonance frequency while series springs work optimally at antiresonance. An SEA in highly dynamic systems, however, is also able to achieve a torque reduction through a reduction of the acceleration of the motor. Moreover, because the torque experienced by the motor and the gearbox is different, the stiffness selection of the PEA will be a trade-off between a reduction of gearbox losses and a reduction of motor losses, while the SEA can achieve both at once through a reduction of the speed. As a result, the SEA was found to be much more efficient in exciting oscillations on a pendulum.

A PEA, on the other hand, has the advantage that it can compensate for a static torque by a correct tuning of the equilibrium angle. In applications which require high static torques, an SEA will suffer high Joule losses and, consequently, be very inefficient. As explained in section 3.4, a PEA gains the upper hand over the SEA in such cases.

The discussion above illustrates how strongly the load profile affects the efficiency of both topologies. Additionally, series and spring elements both have a very different impact on the motor selection. In section 3.5, the optimal spring configuration for an active ankle prosthesis was shown to be inextricably tied to the selected motor and gearbox. While decoupling the design of spring and drivetrain facilitates the design process, our results indicate that there is a great potential for improvement in terms of energy efficiency and compactness when both are optimized simultaneously.

On a final note, we would like to emphasize the great difference between SEA and PEA in terms of dynamics. Many of the advantages that served as a motivation for the development of SEA – safety in human-robot interaction, improved force control, shock tolerance – do not apply to the PEA architecture. On the other hand, the PEA still allows for accurate position control because its bandwidth is not affected by the elastic element, unlike in SEA. For the majority of applications, these considerations will most likely determine the choice between both topologies. Whenever energy consumption is the main concern, however, the conclusions from this chapter can serve as a guideline for anyone who wants to minimize the true energy consumption of an actuator by introducing elastic elements.
Chapter 4

Redundant actuation

4.1 Introduction

Electric motors offer very high efficiencies at their nominal working point. A motor’s efficiency, however, varies strongly with speed and torque \( [217, 232] \). This is an issue in robotics, where actuators are required to operate at a wide range of working points. As a result, their motors tend to be used very inefficiently. In the MIT Cheetah robot, for example, up to 68% of the total energy was consumed by heating of the motor windings \( [199] \). Designs focusing on the reduction of motor losses can significantly improve the overall efficiency of a device. With such an approach, Brown and Ulsoy managed to decrease the energy consumption of a passive-assist device by 25%. No less than 90% of the energy savings was attributed to a more efficient use of the motor \( [34] \).

A well-chosen transmission can help by mapping the expected working points as closely as possible to energy-efficient part of the motor’s operating range. Nevertheless, transmissions with fixed reductions can only do so much when the working points are widely spread over the operating range. This is a typical problem of robots interacting with their environment. Tasks such as turning knobs or manipulating high payloads often require high torques from the robot’s actuators at low output speeds. As a consequence, the actuators will consist of strong motors with high reductions. When the payload is removed and the manipulator is brought back to its initial position, however, the requirement changes to delivering high speeds at low torques. The speed with which the arm can move will then be limited by the high reflected inertia resulting from the high-torque design \( [14] \).

The conflicting torque-speed requirements resulting from a loaded and a no-load phase are also a problem in legged robotics. During the stance phase, the leg needs to carry

the robot’s weight, requiring high motor torques. When the foot is lifted from the ground (i.e. during the swing phase of gait), the required motor torque drops considerably since the leg is now only moving its own inertia, but the motors need to work at higher speeds. This has serious consequences for the sizing and energy efficiency of the actuators. A human ankle, for example, requires hardly any power during the swing phase of walking gait, but when a motor performs the same motion, it displays considerable peak powers in this phase, even if springs are used to alleviate the requirements [95, 231].

A potential solution to these problems is to use variable transmissions [211] or, more generally, to actuate a joint with not one, but two motors. The second motor then creates an additional, redundant, degree of freedom, which can be exploited to distribute the power requirements among both motors in the most energy-efficient way. As we will show in this chapter, with a proper design and proper control, the overall quasi-static efficiency of such an actuator can be better than that of a single motor with gear reducer. In dynamic tasks, however, the inertia of the drivetrain has a strong influence on the power flow through the motor and may lead to high Joule losses [173]. This issue is very relevant for redundant actuators, considering that it consists of not one, but at least two motors. How this affects their energy consumption and the ideal power distribution between the motors, is another question we will try to answer here.

We start this chapter by giving a concise state-of-the-art of redundant actuator concepts (section 4.2). Next, in section 4.3, we present the concept of a kinematically redundant actuator based on a planetary differential, along with the equations that describe such an actuator. We then provide a theoretical analysis of the actuator for constant torques and speeds (section 4.4) and for dynamic loads (section 4.5), complemented with experimental results (section 4.6). Finally, in section 4.7, we discuss the potential of the dual-motor actuator architecture with a special focus on the differences between static and dynamic loads.

### 4.2 Types of redundant actuators

Redundancy on robot-level is a well-studied topic, with a vast array of literature on over actuated robotic arms. Redundancy on joint-level has received little attention in comparison. In this section, we provide an overview of existing concepts for redundant actuators. We divide them into four classes: statically redundant actuators, kinematically redundant actuators, Variable Stiffness Actuators and actuators with Variable Transmissions.

#### 4.2.1 Statically redundant actuators

In a *statically redundant actuator*, an infinite number of input motor torques result in the same output torque [150]. This can be achieved in a very simple way, by coupling multiple motors to the same driveshaft. In such an arrangement, the output torque is the sum of both motor-gearbox torques. The torque that can be delivered by a motor increases with its mass to the power 1.25 [91]. Consequently, using one large motor will generally lead to a more compact and lightweight design than two smaller motors on the same driveshaft.
Despite their unfavorable scaling with mass, statically redundant actuators may have benefits in terms of control performance and energy consumption. In the Parallel-Coupled Micro-Macro actuator concept proposed by Morell et al. [149], a small motor is coupled directly to the load, in addition to a larger motor coupled through a compliant transmission. This actuator exhibited a good force control bandwidth and excellent force fidelity. Peak impact force, force distortion and backdrivability were also found to be better than a single-motor actuator. When combined with springs and locking mechanisms, statically redundant actuators can also offer considerable benefits in terms of energy consumption. An example is the +SPEA actuator proposed by Mathijssen et al., which consists of four motors with series springs, connected to a single output shaft. Controllable brakes allow the motors to be locked, such that springs in series with the motors act as parallel springs on the output shaft. In a blocked output experiment, this actuator managed a fourfold decrease in energy consumption compared to a single-motor alternative [134].

4.2.2 Kinematically redundant actuators

An actuator is kinematically redundant if its output speed is not uniquely determined by the speeds of the input motors [150]. One way of achieving this is to couple two motors to a single output through a differential. In this case, the output speed $\dot{\theta}_{out}$ is a linear combination of the input speeds $\dot{\theta}_1$ and $\dot{\theta}_2$:

$$\dot{\theta}_{out} = R_1 \dot{\theta}_1 + R_2 \dot{\theta}_2$$  \hspace{1cm} (4.1)

with coefficients $R_1$ and $R_2$ depending on the design of the differential. The static output torque, however, is divided over both inputs in a fixed ratio, determined by design.

In principle, any type of differential can be chosen to couple both motors. Differential mechanisms that have been proposed include a bevel gear differential [100, 72], a two-stage planetary gear differential [66, 75] and a differential based on harmonic drives [214, 245]. The most common concept by far is one that employs a planetary differential. Here, the output and the two inputs of the actuator can be assigned to any of the three output shafts of the differential. Assuming that the motors are grounded, this can be done in three possible ways. If one of the motors is allowed to be mounted to a movable component, however, 12 more actuator topologies can be created [14].

To the authors’ knowledge, the first roboticists to suggest the use of a kinematically redundant actuator were Ontañon-Ruiz et al., with the aim of reducing stiction [159]. Here, the carrier and the sun of the planetary differential were used as inputs, and the load was coupled to the output. Several important theoretical contributions about this specific configuration were made by Rabindran and Tesar. In their analysis of power flows and efficiency [179], they concluded that the efficiency of this type of actuator decreases when the reduction ratio of the planetary differential increases. Furthermore, they found that the reflected inertia of the actuator has no upper bound, while it can never be lower than the inertias of both input actuators. Another study [178] discussed the cross-coupling in the inertia matrix. The authors argued that the coupling should be minimized by design, because it causes the actuators to fight each other’s acceleration. In addition to these
theoretical works, Rabindran and Tesar also showed the potential of the concept in terms of safe response to collisions [180].

Another configuration places the load on the carrier, and uses the sun and ring as inputs. This topology was used for the first time by Kim et al., where one motor was used to control the output position and the other to control the stiffness [115, 116]. Later, Lee and Choi [122] explored the possibility of shaping the actuator’s operating range more favorably than that of a regular motor. In their concept, worm gears were used to prevent the motors from being backdriven. This makes their concept unsuitable for applications in robotics, where backdrivability is generally desired. More recently, Girard and Asada [81] presented a similar actuator where the worm gears were replaced by controllable brakes. For this actuator, they developed several control strategies to divide the required power over the motors and to deal with the holding brakes [82, 83].

Much work has also been done on impedance control [152]. A problem with these actuators in impedance control is that, in order to obtain high bandwidths, motors are required that are able to deliver high torques while having low inertias. In order to meet these contradicting requirements, Nagai et al. proposed to equip one of both motors with a parallel spring [151]. Nevertheless, the usage of springs in kinematically redundant actuators is quite rare, with the exception of variable stiffness actuators.

### 4.2.3 Variable Stiffness Actuators

Variable Stiffness Actuators (VSAs) are a variant of Series Elastic Actuators, where the stiffness can be varied. In case of a rotational actuator, they can be represented by following idealized equation:

\[
\theta_{\text{out}} = \theta_1 + \frac{T_{\text{out}}}{k(\theta_2)}
\]  

(4.2)

where \(\theta_{\text{out}}\) and \(T_{\text{out}}\) represent the output position and torque, and \(\theta_1\) and \(\theta_2\) are the input positions of the primary and secondary motor. Equation (4.2) shows that variable stiffness \(k(\theta_2)\) changes the kinematic relationship between the primary motor and the output. The variable stiffness \(k(\theta_2)\) is thus the redundant degree of freedom. Several mechanisms exist for the variation of stiffness [219]. Although exceptions exist, the vast majority requires a secondary motor for the regulation of the stiffness [88].

VSAs can be classified as kinematically redundant actuators if the output torque uniquely defines the torques required from the input motors. For most VSA designs, this is not the case. For an extensive overview of VSA designs and their advantages, we refer to the review papers by Vanderborght et al. [219] and Tagliamonte et al. [213].
4.2.4 Variable transmissions

Another way to add a redundant degree of freedom is to use a variable transmission. The actuator can then be described with following equations:

\[ \theta_{\text{out}} = \frac{\theta_1}{n(\theta_2)} \]  
\[ T_{\text{out}} = n(\theta_2)T_1 \]

In this type of actuator, the variable gear ratio – which we assume to be controlled by a secondary motor – creates the redundant degree of freedom. The variable gear ratio affects both the torque and speed, resulting in coupled kinematic and static redundancy.

Over the years, variable transmissions have proven their potential to reduce the energy consumption of cars [38] and wind turbines [132]. These results have sparked interest from researchers in robotics. Variable transmissions show great potential in the field of legged locomotion, prosthetics and exoskeletons. They can be used to shape a motor’s speed-torque curve more favorably for its use in ankle prostheses, enabling a more compact actuator design [211]. Combined with energy buffers such as springs [210] or flywheels [53], they can also form very energy efficient actuation units. Simulations have shown that a knee actuator consisting of an IVT with a flywheel could reduce the actuator’s energy consumption by 85% in walking [6]. Similar reductions have been reported for an IVT combined with a spring [148]. In addition to the improved energy efficiency, variable transmissions can also help to reduce the weight of knee prostheses [124].

The most common CVT types, belt and chain CVTs and toroidal CVTs, tend to have issues with rapid changes in gear ratio [207], while also being heavy and bulky [63]. Small-size CVTs exist, but often have the disadvantage of having a limited range of motion. Furthermore, they are often friction-based, limiting their torque transmission capability [63]. Consequently, the search for the ideal CVT for robotics is still an ongoing research topic [23, 63, 124, 112, 55].

4.3 Equations

4.3.1 Concept

A regular single-stage planetary gearbox consists of a sun gear, a ring gear, planetary gears and a carrier. In order to achieve the highest gear ratio, the sun is typically used as the input and the carrier as output, while the ring is grounded. By using the ring as an additional input, however, the planetary gearbox can be turned into a planetary differential. This component is the basis of the dual-motor actuator presented in this work. The basic schematic of the dual-motor actuator (DMA) is shown in Fig. 4.1. Two input drivetrains are coupled to the sun and ring gear of a planetary differential, while, just like in a regular planetary gear, the output is coupled to the carrier. This topology offers the highest reductions between the inputs and the output (see Table 4.1). As a result,
Figure 4.1: Dual-motor actuator setup. The actuator consists of two drivetrains, coupled to the ring and sun gear of a planetary differential. The load is coupled to the carrier gear, just like in an ordinary planetary gear. Holding brakes, placed in between the motors and their gearboxes, can be controlled to lock the torque, upon which the motor can be switched off.
Table 4.1: Kinematic relationships for different configurations of a planetary differential. The output speed is denoted by $\omega_o$, carrier speed by $\omega_C$, sun speed by $\omega_S$, and ring speed by $\omega_R$. The planetary differential ratio $\rho$ is defined in Eq. (4.8). For a single-stage planetary differential, $1 < \rho \leq 9$. In this case, a configuration where the sun and ring gear are used as inputs yields the highest overall reduction.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
<th>Kinematic relationship</th>
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<tbody>
<tr>
<td>Sun, ring</td>
<td>Carrier</td>
<td>$\omega_o = \frac{1}{1+\rho} \omega_S + \frac{\rho}{1+\rho} \omega_R$</td>
</tr>
<tr>
<td>Sun, carrier</td>
<td>Ring</td>
<td>$\omega_o = \frac{1+\rho}{\rho} \omega_C - \frac{1}{\rho} \omega_S$</td>
</tr>
<tr>
<td>Ring, carrier</td>
<td>Sun</td>
<td>$\omega_o = (1+\rho) \omega_C - \rho \omega_R$</td>
</tr>
</tbody>
</table>

The reductions offered by the planetary differential are exploited in the best possible way, leading to the most efficient design in terms of space.

The input drivetrains each consist of a motor and gearbox with a holding brake. When it is engaged, the motor can be switched off, so that no electrical power is consumed. An additional advantage is that the brakes can hold higher torques than the motor, which is limited by the maximum current it can provide. Without brakes or any other locking mechanism, the maximum torque that can be provided by the actuator would be determined by the weakest motor, as we will explain in section 4.3.3. Having a locking mechanism is therefore essential in order to make the actuator’s operating range as large as possible.

Previously presented concepts have used a cam-based clutch [115] or worm gears [122][14] as locking devices. Worm gears can be designed such that they lock when backdriven, without any need for actuation. With worm gears on both branches, however, the entire actuator becomes non-backdrivable. This makes the actuator less suitable for robotics where, in many cases, backdrivability is desired. For this reason, we used controllable brakes instead of worm gears, like in Girard et al. [81]. The brakes are positioned on the motor shafts, rather than directly on the ring and sun gears. The main advantage of this approach is that the required braking torque is scaled down through the gearbox, so that a smaller brake can be used.

### 4.3.2 Design equations

A schematic of the DMA, along with definitions of the torques $T$ and speeds $\omega$, is given in Fig. 4.2. The signs of the torques and speeds follow the passive sign convention [85], i.e. input powers $T_S \omega_S$ and $T_R \omega_R$ are positive, while output power $T_o \omega_o$ is negative.

The actuator has one static and two kinematic degrees of freedom (DoF). Imposing a certain motion to a load uses up one static and one kinematic DoF, leaving us with one redundant kinematic DoF. We will thus be able to choose the way how both motors contribute to the output speed, while – in steady state – their torques are completely determined by the output torque.
Figure 4.2: Planetary differential with definitions of torque and speed. Definitions respect the passive sign convention.

4.3.2.1 Speeds

The ring and sun drivetrains consist of a geared DC motor. We can thus write

\[ \dot{\theta}_\lambda = n_\lambda \omega_\lambda \]  

(4.5)

where \( n_\lambda (\lambda = R, S) \) are the gear ratios of the planetary gearboxes in the ring and sun drivetrain, respectively. Because of the redundant kinematic DoF, the ring motor angular velocity \( \dot{\theta}_R \) and the sun motor angular velocity \( \dot{\theta}_S \) can be chosen independently. We define the speed ratio \( i \) as

\[ i = \frac{\omega_R}{\omega_S} = \frac{\dot{\theta}_R/n_R}{\dot{\theta}_S/n_S} \]  

(4.6)

The speed ratio \( i \) is the degree of freedom which we would like to control, in such a way that the most energy-efficient operation is achieved. Note that the definition given by Eq. (4.6) becomes undefined for \( \omega_S = 0 \). In static operation, this is not much of an issue, as the sun brake will be engaged in this case. In dynamic operation, however, zero sun speed also occurs when the brake is disengaged. For this reason, we define an alternative speed ratio \( \gamma \)

\[ \gamma = \frac{1}{n_R} \frac{\rho}{1 + \rho} \frac{\dot{\theta}_R}{\dot{\theta}_o} \]  

(4.7)

which will be used instead of \( i \) in our discussions on dynamics and control. Here, we defined the planetary differential ratio \( \rho \) as

\[ \rho = \frac{r_R}{r_S} \]  

(4.8)

This ratio is related to the gear ratio with locked ring \( n_{PG} \) (i.e. the differential is used as an ordinary planetary gearbox) by

\[ \rho = n_{PG} - 1 \]  

(4.9)
Geometry dictates that \( r_R > r_S \), so \( \rho > 1 \). Because of size limitations on the sun gear, practical values of \( \rho \) are between 2 and 9 [99].

The velocity \( \omega_o = \dot{\theta}_o \) of the carrier, which serves as the output, is a linear combination of the sun and ring motor velocities:

\[
\omega_o = J \begin{bmatrix} \dot{\theta}_S \\ \dot{\theta}_R \end{bmatrix}
\]  \( (4.10) \)

with

\[
J = \begin{bmatrix} \frac{1}{n_S} & \frac{1}{1+\rho} \\ \frac{1}{n_R} & \frac{\rho}{1+\rho} \end{bmatrix}
\]  \( (4.11) \)

Combining Eqs. (4.6) and (4.10), we can write the input speeds of the planetary differential as a function of the output speed:

\[
\begin{bmatrix} \dot{\theta}_S \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{n_S(1+\rho)}{1+\rho} \\ \frac{n_R(1+\rho)}{\rho} \end{bmatrix} \omega_o
\]  \( (4.12) \)

or, as a function of \( \gamma \),

\[
\begin{bmatrix} \dot{\theta}_S \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} n_S(1+\rho)(1-\gamma) \\ n_R(1+\rho)\gamma \end{bmatrix} \dot{\theta}_o = Q \dot{\theta}_o
\]  \( (4.13) \)

By differentiation of (4.13) we find the motor accelerations

\[
\begin{bmatrix} \ddot{\theta}_S \\ \ddot{\theta}_R \end{bmatrix} = Q \ddot{\theta}_o + \begin{bmatrix} -n_S(1+\rho) \\ n_R(1+\rho)\gamma \end{bmatrix} \dot{\theta}_o \dot{\gamma}
\]  \( (4.14) \)

as a function of \( \gamma \), \( \dot{\theta}_o \) and \( \ddot{\theta}_o \). Note that a second term appears in Eq. (4.14) because \( \gamma \) is allowed to vary in time.

### 4.3.2.2 Efficiency of planetary differential

Consider the planetary geartrain displayed in Fig. 4.2. We assume that all friction is caused by the meshing of the gear teeth. If both sun and ring are delivering power to the planets, i.e.

\[
T_{oS} (\omega_S - \omega_C) > 0
\]  \( (4.15) \)

\[
T_{oR} (\omega_R - \omega_C) > 0
\]  \( (4.16) \)

the meshing efficiency for the planets with the sun (\( \eta_{SP} \)) and the ring (\( \eta_{RP} \)) can be found by dividing the output power by the input power:

\[
\eta_{SP} = \frac{-T_p^{(S)} (\omega_P - \omega_C)}{T_{oS} (\omega_S - \omega_C)}
\]  \( (4.17) \)
\[ \eta_{RP} = - \frac{T_P^{(R)} (\omega_P - \omega_C)}{T_{oR} (\omega_R - \omega_C)} \]  

(4.18)

\[ T_P^{(S)} \] and \[ T_P^{(R)} \] are the torques on the planets due to meshing with the sun and ring gear, respectively. In the above equations, the speeds with which we multiply the torques are relative to the carrier, as if it is fixed. Indeed: if the sun or ring gear is not moving with respect to the carrier, the gear teeth will not move with respect to each other, and there will be no power transfer. Also note that, if one of the conditions (4.15) or (4.16) is false, the nominator and denominator need to be reversed in the corresponding equation (4.17) or (4.18).

The torque balance is given by

\[ T_{oS} + T_{oR} = T_C \]  

(4.19)

and the power balance, in a reference frame attached to the rotating carrier, by

\[ \eta_{SP} T_{oS} (\omega_S - \omega_C) + \eta_{RP} T_{oR} (\omega_R - \omega_C) = 0 \]  

(4.20)

Note that we could also have found this equation by stating the power equilibrium in a single planetary gear, i.e.

\[ T_P^{(S)} (\omega_P - \omega_C) = T_P^{(R)} (\omega_P - \omega_C) \]  

(4.21)

Indeed, with a fixed carrier and zero planetary inertia, the only powers through the planetary gears are the powers accepted from the ring and sun.

Inserting the speed equation (4.10) into Eq. (4.20), we find (after some small calculations)

\[ \eta_{SP} T_{oS} \eta_{RP} T_{oR} - \eta_{RP} T_{oS} \eta_{SP} T_{oR} = 0 \]  

(4.22)

Combining this with Eq. (4.19), we find the relationship between the torque on the carrier (output) and the sun and ring torques:

\[ T_C = \left( \frac{\eta_{SP} T_{oS}}{\eta_{RP} T_{oR}} + 1 \right) T_{oS} \]  

(4.23)

\[ T_C = \left( \frac{\eta_{RP} T_{oR}}{\eta_{SP} T_{oS}} + 1 \right) T_{oR} \]  

(4.24)

The speed equation (4.10) shows that the output speed is a linear combination of the ring and sun speed with positive coefficients. In other words, the output speed is a weighted average of the sun and ring speed, and will lie somewhere in between these speeds. Furthermore, \( T_{oS}, T_{oR} \) and \( T_C \) are defined in such a way that they always have the same sign. Combining these two facts, we can easily see that

\[ \text{sign} \left[ T_{oS} (\omega_S - \omega_C) \right] = - \text{sign} \left[ T_{oR} (\omega_R - \omega_C) \right] \]  

(4.25)
It is therefore sufficient to check only one of the two conditions (4.15) and (4.16); if one is false, the other one is true. As a result, $\eta_{RP}$ and $\eta_{SP}$ will appear as a product in all equations, regardless of which power flow case we are in. We can thus define $\eta_{PG} = \eta_{RP}\eta_{SP}$ and write

$$T_C = \left(\frac{r_R}{r_S} \eta_{PG}^{\text{sign}[T_C(\omega_S - \omega_C)]} + 1\right) T_{oS}$$  \hspace{1cm} (4.26)

$$T_C = \left(\frac{r_S}{r_R} \eta_{PG}^{\text{sign}[T_C(\omega_R - \omega_C)]} + 1\right) T_{oR}$$  \hspace{1cm} (4.27)

### 4.3.2.3 Torques - static

The meshing efficiency of the planetary gear differential $\eta_{PG}$ can be used in the efficiency function $C_{PG}$ of the planetary differential:

$$C_{PG} = \eta_{PG}^{\text{sign}[T_{o}(\omega_S - \omega_o)]}$$  \hspace{1cm} (4.28)

To make this function more convenient, it can be rewritten as a function of $i$:

$$C_{PG} = \eta_{PG}^{\text{sign}\left[\frac{1}{\rho C_{PG}}T_o \omega_o\right]}$$  \hspace{1cm} (4.29)

With this function, we can rewrite Eqs. (4.26) and (4.27) as

$$T_C = (\rho C_{PG} + 1) T_{oS}$$  \hspace{1cm} (4.30)

$$T_C = \left(\frac{1}{\rho C_{PG}} + 1\right) T_{oR}$$  \hspace{1cm} (4.31)

Eqs. (4.30) and (4.31) demonstrate that the torques on the ring and sun gear are fully determined by the required output. By combining these equations, we can find

$$T_{oR} = \rho C_{PG} T_{oS}$$  \hspace{1cm} (4.32)

indicating that, by design, the torque in the ring branch will be higher than that in the sun branch.

The relationships between the motor torques $T_{m\lambda}$ and the torques on the sun or ring gear of the planetary differential $T_{o\lambda}$ ($\lambda = R, S$) are given by

$$T_{m\lambda} = B_{\lambda} (\omega_o, T_o) \cdot \left[\frac{C_{\lambda}}{n_{\lambda}} T_{o\lambda} + T_{C\lambda}' \text{sign} \left(\frac{\dot{\theta}_{\lambda}}{\lambda}\right) + v_{\lambda} \dot{\theta}_{\lambda}\right]$$  \hspace{1cm} (4.33)

where we introduced a friction torque composed of Coulomb and viscous friction, with friction coefficients $T_{C\lambda}$ (Coulomb friction) and $v_{\lambda}$ (viscous friction). $C_R$ and $C_S$ represent the efficiency function of the gearbox, defined as

$$C_{\lambda} = \eta_{\lambda}^{-\text{sign}[T_{o\lambda} \omega_{\lambda}]}$$  \hspace{1cm} (4.34)
where \( \eta_\lambda \) represents the catalog efficiency of the respective gearbox (\( \lambda = R,S \)). The torque on the motor shaft is multiplied by the braking function \( B_\lambda (\omega_o, T_o) \), defined as

\[
B_\lambda (\omega_o, T_o) = \begin{cases} 
0 & \text{brake engaged} \\
1 & \text{brake disengaged} 
\end{cases} \quad (4.35)
\]

By combining motor equation \( (4.33) \) with torque equations \( (4.30) \) and \( (4.31) \), we obtain

\[
T_{mS} = B_S (\omega_o, T_o) \cdot \left[ \frac{C_S}{n_S} \frac{1}{1 + \rho C_{PG}} T_C + T_{CSS} \text{sign} (\dot{\theta}_S) + \nu_S \dot{\theta}_S \right] \quad (4.36)
\]

\[
T_{mR} = B_R (\omega_o, T_o) \cdot \left[ \frac{C_R}{n_R} \frac{\rho C_{PG}}{1 + \rho C_{PG}} T_C + T_{CR} \text{sign} (\dot{\theta}_R) + \nu_R \dot{\theta}_R \right] \quad (4.37)
\]

Furthermore, the losses between the carrier and the output, which are mostly due to bearings, can be modeled by a combination of Coulomb friction and viscous friction, yielding the following equation for the output torque \( T_o \):

\[
T_o = T_C - T_{CCC} \text{sign} (\omega_o) - \nu_C \omega_o \quad (4.38)
\]

### 4.3.2.4 Dynamics

The static torque equations can easily be extended to dynamic equations. Defining the input vector in dual-motor operation as \( T_{2m} = (T_{mS}, T_{mR})^T \), with \( T_{mS} \) and \( T_{mR} \) the motor torques of the sun and ring motor, respectively, we can write

\[
T_{2m} = [A \ddot{x} + B x + C \text{sign}(x) + D (T_o + T_{CCC} \text{sign}(Jx))] \quad (4.39)
\]

where \( T_o \) is the output torque, \( x \) the state vector

\[
x = \begin{bmatrix} \dot{\theta}_S \\ \dot{\theta}_R \end{bmatrix} \quad (4.40)
\]

and

\[
A = \begin{bmatrix} J_S & 0 \\ 0 & J_R \end{bmatrix} + J_C D \begin{bmatrix} \frac{1}{n_S} & \frac{1}{n_R} \\ \frac{1}{n_S} & \frac{1}{n_R} \end{bmatrix}
\]

\[
B = \begin{bmatrix} \nu_S & 0 \\ 0 & \nu_R \end{bmatrix} + \nu_C D \begin{bmatrix} \frac{1}{n_S} & \frac{1}{n_R} \\ \frac{1}{n_S} & \frac{1}{n_R} \end{bmatrix}
\]

\[
C = \begin{bmatrix} T_{CS} & 0 \\ 0 & T_{CR} \end{bmatrix}
\]

\[
D = \begin{bmatrix} \frac{C_S}{n_S} (\rho C_{PG} + 1)^{-1} \\ \frac{C_R}{n_R} (\frac{1}{\rho C_{PG}} + 1)^{-1} \end{bmatrix}
\]
Here, \( J_S, J_R \) and represent the inertias of the sun and ring, respectively. This includes not only the inertia of the gears themselves, but also that of the motor, gearbox, shaft and couplings on that specific branch. The inertia of the carrier is denoted by \( J_C \). The inertia of the planetary differential’s planets is assumed to be negligible compared to that of the sun, ring and carrier, the two former of which have an entire drivetrain attached to them. Friction coefficients were obtained experimentally, as explained in section 4.6.

For convenience, the brake function was omitted in the formulas above. It can be included by pre-multiplying Eq. 4.39 with a matrix containing the brake functions:

\[
T_{2m}' = \begin{bmatrix}
B_S(\omega_o, T_o) & 0 \\
0 & B_R(\omega_o, T_o)
\end{bmatrix} T_{2m}
\]  

(4.41)

### 4.3.2.5 Electrical

The following model can be applied to find the motor currents \( I_\lambda \):

\[
I_\lambda = \frac{1}{k_{T\lambda}} T_{m\lambda}
\]  

(4.42)

In this equation, \( k_{T\lambda} \) represents the torque constant of the respective motor. Furthermore, motor voltages \( U_\lambda \) can be calculated with

\[
U_\lambda = k_{T\lambda} \omega_\lambda + R_\lambda I_\lambda
\]  

(4.43)

with \( R_\lambda \) the winding resistance of the motor. The electrical powers \( P_{\text{elec},\lambda} \) are given by

\[
P_{\text{elec},\lambda} = U_\lambda \cdot I_\lambda
\]  

(4.44)

Finally, the total electrical power \( P_{\text{elec}} \) consumed by the actuator is simply the sum of both motor powers:

\[
P_{\text{elec}} = P_{\text{elec},S} + P_{\text{elec},R}
\]  

(4.45)

Here, we neglect the power consumed by the brakes. Whether this assumption is justified, depends entirely on the type of brake that is being used. Bi-stable brakes, for example, do not consume any energy while they are engaged or disengaged; energy input is only required to change their state \[140\]. If the application requires the brake to switch only sporadically between its engaged and disengaged state, such a bi-stable brake will indeed consume a negligible amount of power. In other cases, novel concepts such as statically balanced brakes \[172\] and electroadhesive clutches \[51\] can be used to hold the drivetrain’s position at a low energy cost.

### 4.3.3 Constraints and limitations

Any drivetrain is subject to electrical and mechanical limitations. In this section, we discuss how these affect the design of a DMA.
4.3.3.1 Maximum speed

Typically, a maximum speed is specified on the datasheets of the motor and gearbox. Denoting the maximum motor speeds by $\dot{\theta}_{S,\text{max}}$ (sun) and $\dot{\theta}_{R,\text{max}}$ (ring), we find that the maximum achievable output speed $\omega_{o,\text{max}}$ is given by (dual-motor operation)

$$\omega_{o,\text{max}} = \frac{1}{1 + \rho \left( \frac{\dot{\theta}_{S,\text{max}}}{n_S} + \rho \frac{\dot{\theta}_{R,\text{max}}}{n_R} \right)}$$  (4.46)

Inherently, the ring branch will thus contribute more to the output speed.

4.3.3.2 Maximum continuous torque

High torques lead to high currents in the motor windings. To prevent the windings from burning up, motor manufacturers specify a maximum continuous torque $T_{\lambda,\text{max},\text{cont}}$ ($\lambda = R, S$) for the motor. This leads to following constraint:

$$\text{RMS}(T_\lambda) < T_{\lambda,\text{max},\text{cont}}$$  (4.47)

4.3.3.3 Maximum peak torque

In addition to the rms value of the torque, the peak motor torque is also limited. The maximum motor and gearbox torque, as specified by the manufacturer, must be respected in order to avoid damage or reduced lifetime of the components. Furthermore, the maximum output current of the controller’s power stage may also limit the achievable torque. We will denote the maximum torque that can be provided by the motors, regardless of its cause, with $T_{S,\text{max}}$ and $T_{R,\text{max}}$ for the sun and ring branch, respectively. We can then write the constraint as

$$|T_\lambda| < T_{\lambda,\text{max}}$$  (4.48)

If the actuator is to provide a constant torque, the maximum continuous torques $T_{\lambda,\text{max},\text{cont}}$ ($\lambda = R, S$) will set the limit for the maximum torque that can be provided.

4.3.3.4 Maximum voltage

The motor voltage $U_\lambda$ cannot exceed the voltage $U_{\lambda,\text{max}}$ ($\lambda = R, S$) available from the power source:

$$|U_\lambda| < U_{\lambda,\text{max}}$$  (4.49)

According to Eq. (4.43), the motor voltage links the motor torque to the motor speed. Hence, Eq. (4.49) limits the output speed that can be achieved at certain output torque.
4.3.4 Optimal control

In steady-state conditions, the optimal speed distribution between both motors can be found with a simple parameter sweep. For dynamic problems, this approach is too simplistic. They require solving an optimal control problem (OCP), which can be formulated as:

\[
\min_u \left( \Pi = \int_{t_0}^{t_f} g(x,u,t) dt \right)
\]

\[
\dot{x} = f(x,u,t)
\]

\[
h(x,u,t) = 0
\]

\[
c_{min} \leq c(x,u,t) \leq c_{max}
\]

The goal is to find the control set \( u = (I_{mS}, I_{mR})^T \) which minimizes the cost \( \Pi \), defined as the integral of the cost function \( g(x,u,t) \) with the state \( x \) defined by Eq. (4.40), spanning over time window of \([t_0, t_f]\). In this work, we wish to optimize the efficiency of the DMA with respect to a reference drivetrain. The cost function is thus the total electrical power, which can be written as:

\[
g(x,u,t) = x^T Ku + u^T \begin{bmatrix} R_S & 0 \\ 0 & R_R \end{bmatrix} u
\]

\[
K = \begin{bmatrix} k_{TS} & 0 \\ 0 & k_{TR} \end{bmatrix}
\]

The system has two constraints. First, there is the dynamics of the system, which can be expressed as a set of differential equations \( \dot{x} = f(x,u,t) \). The dynamics of the system were given by Eq. (4.39) and can be rewritten as

\[
\dot{x} = A^{-1} (Ku - Bx - C\text{sign}(x) - D(T_o(t) + T_CC\text{sign}(Jx)))
\]

Second, a path constraint is also present to respect the kinematics of the DMA \( h(x,u,t) = 0 \). The path constraint is derived from Eq. (4.10):

\[
h(x,u,t) = \dot{\theta}_o(t) - \begin{bmatrix} \frac{1}{n_S(1+\rho)} \\ \frac{\rho}{n_R(1+\rho)} \end{bmatrix} x = 0
\]

Finally, we would like to ensure that the control set does not lead to a saturation of the actuator. We will thus add inequality constraints of the form

\[
c_{min} \leq c(x,u,t) \leq c_{max}
\]

where

\[
c(x,u,t) = \begin{bmatrix} x \\ Ku \\ Kx + Ru \end{bmatrix}
\]
contains, respectively, the maximum motor speeds, maximum motor torques, and maximum output voltages (see 4.3.3). The vectors $c_{\text{min}}$ and $c_{\text{max}}$ are given by

$$c_{\text{max}} = -c_{\text{min}} = \begin{bmatrix} \dot{\theta}_{S,\text{max}} \\ \dot{\theta}_{R,\text{max}} \\ \sqrt{2} \cdot T_{S,\text{max,cont}} \\ \sqrt{2} \cdot T_{R,\text{max,cont}} \\ U_{S,\text{max}} \\ U_{R,\text{max}} \end{bmatrix}$$

(4.56)

To apply constraint (4.47) it was assumed that the motor torque follows a sinusoidal trajectory. In that case, the peak torque will be $\sqrt{2}$ times higher than the RMS torque. Therefore, by limiting the motor torques to $\sqrt{2} \cdot T_{\lambda,\text{max,cont}}$, we effectively limit the RMS torque to $T_{\lambda,\text{max,cont}}$. As we will show in section 4.6, where optimal control is applied, the assumption of a sinusoidal motor torque (current) is justified.

The OCP was solved using GPOPS-II [166]. This software transcribes the OCP into a large scale nonlinear programming problem (NLP) and the problem is then solved by using the NLP solver IPOPT [25].

### 4.4 Steady-state analysis

#### 4.4.1 Maximum output torque

According to Eq. (4.32), the torques of the sun and ring motor are linked to each other. This has implications for the maximum torque that can be delivered by the actuator.

The maximum torque depends on the operation mode of the actuator. We will discuss single-motor operation and dual-motor operation separately.

**4.4.1.1 Single-motor operation**

When one of the branches is locked, the maximum output torque is fully determined by the active branch. If the active branch is the sun, we obtain a maximum output torque $T_{o,\text{max},S}$ given by

$$T_{o,\text{max},S} = (\rho C_{PG} + 1) \frac{n_S}{C_S} T_{S,\text{max}}$$

(4.57)

For the ring, the maximum output torque $T_{o,\text{max},R}$ is

$$T_{o,\text{max},R} = \left( \frac{1}{\rho C_{PG} + 1} \right) \frac{n_R}{C_R} T_{R,\text{max}}$$

(4.58)

**4.4.1.2 Dual-motor operation**

In dual-motor operation, both branches are active at the same time. While their speeds can be controlled independently, their torques cannot. In static operation, they are fully
determined by the required output torque, as shown by Eqs. (4.30) and (4.31). As a consequence, the weakest branch effectively limits the output torque. The maximum output torque in dual-motor operation, $T_{o,max,D}$, can thus be written as

$$T_{o,max,D} = \min\{T_{o,max,S}; T_{o,max,R}\} \quad (4.59)$$

In order to maximize the dual-motor operating region, where redundancy can be exploited, the gear ratios must therefore be chosen such that $T_{o,max,S} = T_{o,max,R}$. This yields

$$\frac{n_S}{n_R} = \frac{1}{\rho} \frac{C_S T_{R,max}}{C_R T_{S,max}} \quad (4.60)$$

If similar or identical motors are used, and neglecting the efficiency of the planetary differential, we find

$$\frac{n_S}{n_R} = \frac{1}{\rho} \quad (4.61)$$

Consequently, in order to get the most out of the dual-motor operation, the ring branch will need to deliver higher torques at lower speeds compared to the sun branch. Nevertheless, it might be interesting to make one branch stronger than the other. Such a design would result in an actuator which can deliver high torques at low speeds, as well as high speeds at low torques. Certain applications, such as actuated ankle prosthetics, could benefit from such an actuator design [211].

### 4.4.2 Maximum efficiency

Robotic tasks typically require an actuator to deliver varying torques at varying speeds. The high-power operating points can be expected to have the strongest influence on the overall energy consumption. While the efficiencies of actuators vary over their operating range, motors and gearboxes tend to operate close to their maximum efficiency when they are delivering high powers. Maximizing the maximum efficiency of the actuator is therefore a first step towards maximizing the overall efficiency of the actuator.

To simplify the problem, the maximum efficiency of the sun and ring branch will be defined as $\eta_{Sb}$ and $\eta_{Rb}$. This efficiency includes the motor and gearbox losses, as well as friction losses. Defining the efficiency function of each branch as

$$C_{\lambda,b} = \eta_{\lambda,b}^{-\text{sign}(T_{o,\lambda,\omega_{\lambda}})} \quad (4.62)$$

the electrical power can be written as

$$P_{elec} = C_{Rb} T_{mR} \dot{\theta}_R + C_{Sb} T_{mS} \dot{\theta}_S \quad (4.63)$$

which, after some small calculations, and neglecting losses in the planetary differential and on the output shaft, can be rewritten as a function of the output torque and speed:

$$P_{elec} = \left( C_{Rb} \frac{\rho i}{1+\rho i} + C_{Sb} \frac{1}{1+\rho i} \right) T_o \omega_o \quad (4.64)$$
When both motors are in forward drive, we can write this equation as

$$\eta_{tot} = \frac{T_o \omega_o}{P_{elec}} = \frac{1 + \rho i}{\left( \frac{\rho i}{\eta_{Rb}} + \frac{1}{\eta_{Sb}} \right)}$$  \hspace{1cm} (4.65)

As expected, the efficiency of the actuator depends on the speed ratio $i$. The optimal speed distribution puts the most power demand on the branch with the most efficient drivetrain. A high speed ratio, for example, corresponds to a high ring speed and, consequently, high power in the ring branch. Eq. (4.65) shows that, in this case, the ring branch efficiency dominates the overall efficiency. Note that the planetary gear radius ratio $\rho$ and the speed ratio $i$ appear in couples $\rho i$. This suggests that the parameter $\rho$ – values between 1 and 9 for a one-stage planetary geartrain [99] – is not that important, because it can to some extent be corrected by the speed ratio $i$. However, $\rho$ also has a significant influence on the torques and speeds that need to be delivered by the motors, and consequently on the gearbox choice. A high $\rho$ corresponds to a high planetary differential reduction, leading to a decreased need for high gearbox reductions on the sun and ring motor. Considering that smaller gearbox reductions are, in general, more efficient [49], a higher $\rho$ may allow for a more efficient design of the separate branches. As we will see in section 4.4.4.1, this improves the overall efficiency of the actuator as well.

Eq. (4.65) also shows one very important property of the DMA design: the efficiency in dual-motor operation can never be higher than that of the most efficient branch. This means that, if the actuator is intended to replace a conventional motor with gear reducer, this will only be an improvement in terms of energy consumption if at least one of both branches can be made more energy-efficient than the initial motor-gearbox combination.

Finally, if both drives are identical, i.e. $\eta_{Rb} = \eta_{Sb} = \eta$, the maximum efficiency becomes

$$\eta_{tot} = \eta$$  \hspace{1cm} (4.66)

In other words, with identical drives, the design of neither the planetary gear nor the speed ratio influence the maximum achievable efficiency of the actuator.

### 4.4.3 Power flows

In a conventional geared motor, the imposed motion and the friction losses determine the power required from the motor. For overactuated systems such as the DMA, the user can choose how to distribute the powers over both motors. To do this in an energy-efficient way is not a straightforward task, because the internal power flows have a strong influence on the gear losses inside the actuator.

The power flows that may occur can be divided into eight types, as shown in Fig. 4.3. Which type of power flow occurs, depends on two things: the direction in which power is transferred to the load, as well as the speed ratio. This has important implications for the design, because the power flows influence the efficiency function of the planetary differential and the planetary gearboxes in both branches. The values of the efficiency functions $C_S$, $C_R$ and $C_{PG}$ are listed in the figure for each of the power flow types.
## Figure 4.3: Possible power flows in a DMA configuration based on a planetary differential. The power flows are represented by green arrows, where wider arrows represent higher powers. There are eight types of power flow, all of which can be characterized by the output power $T_o \omega_o$ and the speed ratio $i$ (or $\gamma$). The values of the efficiency functions $C_S$, $C_R$ and $C_{PG}$ depend on the power flow type; their values are listed for each of the eight cases.
The transitions between the different power flow types shown in Fig. 4.3 deserve special interest, because each corresponds to a specific way of using the actuator.

- The extreme case of \( i = \infty \) corresponds to zero speed of the sun branch. At zero speed, friction losses are eliminated, and the only remaining losses are the Joule losses in the motor. These can also be eliminated by engaging a locking device, if is available on the branch.

- At \( i = 1 \), the sun, ring and carrier move at exactly the same angular velocities \( (\omega_S = \omega_R = \omega_o) \). This type of motion is known as “block motion” [168]. The efficiency function \( C_{PG} \), given by Eq. (4.29), becomes

\[
C_{PG} = \eta_{PG}^{\text{sign}(0)} = 1
\]  

Meshing losses are completely eliminated, because the gear teeth of the planets no longer move with respect to those of the sun and the ring.

- At \( i = 0 \), the ring branch is standing still. By engaging a locking device, the losses in this branch can be eliminated completely, just like in the \( i = \infty \) case.

- For a speed ratio \( i = -1/\rho \), both input branches may turn at a certain speed, but will nonetheless combine to an output speed of zero. Obviously, this type of operation is not very efficient for stationary applications. The better option is to lock both branches, so that they can deliver a reaction torque which combines to the required output torque.

### 4.4.4 Case study: replacing a 250W drivetrain

In this section, a reference drivetrain (a 250W Maxon RE65 DC motor with 51:1 planetary gearbox) is replaced by a more energy efficient dual-motor actuator design. In order to make both designs comparable, we will attempt to shape the DMA’s operating range in such a way that it resembles that of the 250W motor. We chose to work with a Maxon 150W RE40 DC motor, one of the most efficient in the Maxon catalog, and combine it with a 90W Maxon RE35 DC motor. In terms of size, this is the smallest combination of motors that is able to span the same operating range as the 250W motor. The parameters of these motors, as well as the reference motor, are listed in Table 4.2. The motors of the DMA are equipped with planetary gearboxes from the Maxon GP42 range, which fits both motors. The gear ratios are chosen in such a way that the DMA covers most of the reference drivetrain’s operating range. If similar solutions are available, the most energy efficient solution is used. Finally, since the friction coefficients are unknown, we will assume \( T_{CC} = 0, \nu_C = 0, T_{CR} = 0 \) and \( T_{CS} = 0 \). All friction is assumed to present itself as viscous friction in the sun and ring branch. The corresponding friction coefficients are derived from the motors’ no-load speed and no-load current:

\[
\nu = \frac{k_T I_{nl\lambda}}{\omega_{nl\lambda}}
\]  

With this formula, we obtain friction coefficients \( \nu_S = 5.2E-6 \text{ Nm/(rad/s)} \) and \( \nu_R = 6.1E-6 \text{ Nm/(rad/s)} \).
### Table 4.2: Parameters of the Maxon motors used in this section.

<table>
<thead>
<tr>
<th>Nominal power</th>
<th>Maxon RE65</th>
<th>Maxon RE40</th>
<th>Maxon RE35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal speed</td>
<td>3710 rpm</td>
<td>6940 rpm</td>
<td>7000 rpm</td>
</tr>
<tr>
<td>Nominal torque $T_{\text{max}}$</td>
<td>485 mNm</td>
<td>177 mNm</td>
<td>101 mNm</td>
</tr>
<tr>
<td>Max. speed $\omega_{\text{max}}$</td>
<td>3000 rpm</td>
<td>12000 rpm</td>
<td>12000 rpm</td>
</tr>
<tr>
<td>No-load speed $\omega_{nl}$</td>
<td>3960 rpm</td>
<td>7580 rpm</td>
<td>7740 rpm</td>
</tr>
<tr>
<td>Max. efficiency $\eta_{\text{max}}$</td>
<td>83%</td>
<td>91%</td>
<td>86%</td>
</tr>
<tr>
<td>Terminal resistance $R$</td>
<td>0.0821 Ohm</td>
<td>0.299 Ohm</td>
<td>0.583 Ohm</td>
</tr>
<tr>
<td>Torque constant $k_t$</td>
<td>55.4 mNm/A</td>
<td>30.2 mNm/A</td>
<td>19.4 mNm/A</td>
</tr>
<tr>
<td>Speed constant $k_b$</td>
<td>172 rpm/V</td>
<td>317 rpm/V</td>
<td>328 rpm/V</td>
</tr>
</tbody>
</table>

### Table 4.3: Overview of DMA designs and their performance. Speed- and torque-dependent efficiencies are calculated with the equations presented in Section 4.3. They include the losses in the motor, gearbox and differential, but not in the additional bearings needed to construct the DMA. The mean efficiency is taken over the entire range of torques and speeds that can be achieved with the actuator; the maximum efficiency is the highest efficiency over the entire operating range. The reference drivetrain, a Maxon RE65 250W with 51:1 reduction, has a maximum efficiency of 64% and a mean efficiency of 51%. Note that the gearbox used in the sun branch of design 3 and 4 has one stage less than the one used in design 1 and 2, making it more efficient.

<table>
<thead>
<tr>
<th>DMA Ring</th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
<th>Design 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{nom}}$</td>
<td>150 W</td>
<td>90 W</td>
<td>150 W</td>
<td>90 W</td>
</tr>
<tr>
<td>$n_R$</td>
<td>113</td>
<td>156</td>
<td>126</td>
<td>216</td>
</tr>
<tr>
<td>$\eta_R$</td>
<td>72%</td>
<td>72%</td>
<td>72%</td>
<td>72%</td>
</tr>
<tr>
<td>DMA Sun</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{\text{nom}}$</td>
<td>90 W</td>
<td>150 W</td>
<td>90 W</td>
<td>150 W</td>
</tr>
<tr>
<td>$n_S$</td>
<td>113</td>
<td>66</td>
<td>26</td>
<td>15</td>
</tr>
<tr>
<td>$\eta_S$</td>
<td>72%</td>
<td>72%</td>
<td>81%</td>
<td>81%</td>
</tr>
<tr>
<td>Differential</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.5</td>
<td>1.5</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$\eta_{PG}$</td>
<td>94%</td>
<td>94%</td>
<td>94%</td>
<td>94%</td>
</tr>
<tr>
<td>Efficiency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>59%</td>
<td>48%</td>
<td>61%</td>
<td>63%</td>
</tr>
<tr>
<td>max.</td>
<td>67%</td>
<td>64%</td>
<td>68%</td>
<td>71%</td>
</tr>
</tbody>
</table>

### 4.4.4.1 Energy-efficient design

Table 4.3 shows several DMA solutions which cover a similar operating range as the reference drivetrain. In this table, two critical design parameters are varied: the planetary differential ratio $\rho$ and the placement of the two motors. Except for design 2, all designs lead to a higher efficiency than the reference drivetrain. The highest gains in efficiency occur for $\rho=9$. A high $\rho$ will allow decreasing the gear ratio of the sun drivetrain and increase the ring drivetrain gear ratio, as predicted by (4.60). Decreasing the gear ratio of a planetary gearbox is particularly interesting if it leads to a reduction of the number of
4.4 Steady-state analysis

<table>
<thead>
<tr>
<th>Reference drive</th>
<th>Weight</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual-motor actuator</td>
<td>5.8 kg</td>
<td>1.13 dm³</td>
</tr>
<tr>
<td>Ring drive</td>
<td>0.62 kg</td>
<td>0.145 dm³</td>
</tr>
<tr>
<td>Sun drive</td>
<td>0.94 kg</td>
<td>0.186 dm³</td>
</tr>
<tr>
<td>Differential</td>
<td>1.0 kg</td>
<td>0.180 dm³</td>
</tr>
</tbody>
</table>

Table 4.4: Estimated weight and volume of the reference drive and the dual-motor actuator, based on the datasheets of off-the-shelf components. The structure to hold the dual-motor actuator is not included in this calculation, so the estimated weight and volume can be considered as a lower bound. The additional weight can, however, be compensated to some extent by eliminating parts of the separate off-the-shelf components through an integrated design.

gearbox stages, since this will greatly improve gearbox efficiency. Indeed, the gearboxes in the sun branch in designs 3 and 4 ($\rho=9$) have one stage less compared to designs 1 and 2 ($\rho=1.5$), resulting in much higher gearbox efficiencies $\eta_S$. As a result, these high-$\rho$ designs also result in a better overall efficiency. A high value of $\rho$ can thus be considered as a must for an energy-efficient dual-motor design.

The placement of the ring and sun motor is a more difficult subject, because it is closely related to the way the operating range is shaped – something which can be influenced through the choice of the gear ratios of both drivetrains and the way the holding brakes are used. In this regard, Eq. (4.32) can again serve as an important guideline. Knowing that the sun branch can easily be made efficient because it benefits more from the reduction offered by the planetary differential, it makes sense to exploit this branch as much as possible in single-motor operation. This means that the operating range of the sun motor must be maximized, which can be accomplished by choosing a motor with high nominal power. In Table 4.3, design 4, where the 150W motor is used in the sun branch, is indeed the most efficient.

4.4.4.2 Weight, volume and operating range

Reducing energy consumption was the main goal of our optimization. Nevertheless, volume and weight can also be reduced by a dual-motor architecture. The estimated weights and volumes for the reference drivetrain and the DMA are shown in Table 4.4. The weight and volume of the reference drivetrain are almost twice as high as those of the DMA. Even though the estimated weight does not account for the weight of the structure, there appears to be a lot of margin for adding components. An integrated design, which combines the reductions of the individual motors and the planetary differential in a single housing, can also lead to a considerable reduction in weight and volume [81]. The width of the DMA will, however, tend to be larger than that of the single-motor equivalent. The sum of the diameters of the DMA’s composing drivetrains is 84 mm, while the reference drivetrain has a diameter of 81 mm.

Furthermore, we designed the DMA to match the operating range of a conventional
CHAPTER 4. REDUNDANT ACTUATION

4.4.4.3 Efficiency maps

In chapter 2, we introduced efficiency maps as a tool to visualize an actuator’s torque- and speed-dependent efficiency in its entire operating region. Figure 4.4 shows the efficiency map of the most energy-efficient design, design 4, and the reference drivetrain. The map is obtained by means of a parameter sweep on the speed ratio $i$. Figure 4.4 shows that the DMA not only has a higher maximum efficiency, but is also able to operate close to its maximum efficiency in a larger part of its operating range. Furthermore, the DMA is much more efficient than the reference motor at low powers. This is where the losses in the drivetrain have the highest impact, because they are high relative to the total output power for low powers.
power. In fact, when an actuator is backdriven by the load, it may still need to deliver energy if the actuator’s energy losses are higher than the energy flowing in from the load (see section 2.3). In Fig. 4.4, this type of operation is characterized by an efficiency of zero, which corresponds to the dark blue zones at high-torque low-speed and high-speed low-torque. A large part of the reference motor’s operating range consists of such zero-efficiency regions. The DMA, however, is much more efficient in these areas. To understand why, we have to look at the operating modes of the actuator.

### 4.4.4.4 Operating modes and speed ratio \( i \)

Energy-efficient operation of the DMA depends on two crucial elements: a good choice of the operating mode (dual-motor or single-motor operation) and, in dual-motor operation, a well-selected speed ratio. The optimal speed ratio \( i \) and the brake functions \( B_\lambda (\omega, T) \) complement each other\(^2\), and can be mapped together as a function of torque and speed. Such maps are shown in Fig. 4.5, for designs 3 and 4 from Table 4.3.

For both designs, single-motor operation covers a large part of the operating range. In this case, all power will be provided by the unlocked drivetrain. If this drivetrain can be made highly efficient, very high efficiencies can be obtained in single-motor operation.

\(^2\)As we explained in section 4.4.3, the usage of the brakes can be related to the speed ratio. Locking the ring branch corresponds to a speed ratio of \( i = 0 \), while braking the sun branch corresponds to \( i = \infty \).

![Figure 4.5: Optimal operating modes and speed ratios \( i \) for designs 3-4, as specified in Table 4.3. Dark blue denotes single-motor operation with locked ring \( (i=0) \); yellow denotes single-motor operation with locked sun \( (i = \infty) \). In dual-motor operation, the speed ratio is shown. Dual-motor operation is preferred when the required output torque is low compared to the required speed. At high torques and low speeds, single-motor operation is a more efficient choice. For these specific designs, a locked ring is preferable over a locked sun. This applies especially to design 4.](image-url)
This explains the success of design 4, which, as illustrated by the large blue regions in Fig. 4.5, strongly exploits single-motor operation by locking the ring branch.

It appears that dual-motor operation becomes interesting when the demanded torque is low w.r.t. the demanded speed. In dual-motor operation, the speeds can be distributed over both motor branches, significantly lowering the viscous friction losses. At low torques and high speeds, the Joule losses (proportional to torque squared) and the gearbox losses (proportional to the power flow) are less crucial than the viscous friction losses, which are proportional to speed squared. This explains why dual-motor operation is preferred in this region. At lower speeds, however, the decrease in viscous friction losses does not outweigh the additional Joule losses in the second motor branch. It then becomes more interesting to only use one motor and to have the second torque delivered statically by a brake.

Another interesting observation is that the speed ratio never drops below zero. Referring to Fig. 4.3, $i < 0$ corresponds to a situation where one motor acts as a motor while the other works as a generator is not energy-efficient. Indeed: the internal power flow that arises only lead to additional energy losses. Hence, as far as steady state operation is concerned, this type of operation should be avoided.

Finally, it can be noted that, in design 4, the sun brake is only used in a very limited region. In order to save weight, it can be left out of the design, with only a minimal loss of efficiency.

### 4.5 Dynamic analysis

In the previous section, we showed that the DMA could statically deliver high torques (20 Nm) at high speeds (100 rpm) while being lighter and more compact than a traditional motor with gear reducer. But how does the actuator perform in dynamic applications? In order to answer this question, we again compare a specific DMA design to reference drivetrain composed of a motor with gear reducer. This time, the DMA has a 150 W DC motor with 15:1 reduction (sun) and a 90 W DC motor with 129:1 reduction (ring) as inputs, in accordance with the design that was used for the experiments (section 4.6). The reference drivetrain, a 250 W DC motor with 35:1 gear reduction, is designed to cover approximately the same steady-state operating range while respecting the constraints listed in Section 4.3.3. In all these calculations, the supply voltage $U_{\text{max}}$ is assumed to be 30 V for all motors. Parameters of the drivetrains are listed in Table 4.5. The parameters of the planetary differential used in the DMA is listed in Table 4.6.

We start this section by comparing the operating ranges of the two motors. Next, we provide a theoretical discussion of the reflected inertia of a DMA and its impact on the maximum output acceleration that can be achieved with the actuator. We conclude the section by comparing the DMA's efficiency to that of the reference actuator, in a task where sinusoidal oscillations are applied to an inertial load.
Table 4.5: Parameters for the drivetrains of the dual-motor actuator and for the reference drivetrain specified in section 4.5. Friction parameters were obtained from steady-state measurements, as described in section 4.6. The inertias \( J_S, J_R \) and \( J_{ref} \) were obtained from datasheet information and CAD drawings.

<table>
<thead>
<tr>
<th></th>
<th>Ref. drivetrain ((\lambda = ref))</th>
<th>Sun drivetrain ((\lambda = S))</th>
<th>Ring drivetrain ((\lambda = R))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_\lambda )</td>
<td>1.38e-4 kg m(^2)</td>
<td>1.6e-5 kg m(^2)</td>
<td>9.5e-6 kg m(^2)</td>
</tr>
<tr>
<td>( \nu_\lambda )</td>
<td>1.0e-3 Nm/(rad/s)</td>
<td>1.5e-5 Nm/(rad/s)</td>
<td>1.5e-12 Nm/(rad/s)</td>
</tr>
<tr>
<td>( T_{C\lambda} )</td>
<td>0.040 Nm</td>
<td>0.0080 Nm</td>
<td>0.0061 Nm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor</th>
<th>Maxon RE65</th>
<th>Maxon RE40</th>
<th>Maxon RE35</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{nom} )</td>
<td>250 W</td>
<td>150 W</td>
<td>90 W</td>
</tr>
<tr>
<td>( T_{\lambda,\text{max,cont}} )</td>
<td>485 mNm</td>
<td>177 mNm</td>
<td>101 mNm</td>
</tr>
<tr>
<td>( \omega_{\lambda,\text{max}} )</td>
<td>3000 rpm</td>
<td>12000 rpm</td>
<td>12000 rpm</td>
</tr>
<tr>
<td>( R_\lambda )</td>
<td>0.0821 Ohm</td>
<td>0.299 Ohm</td>
<td>0.583 Ohm</td>
</tr>
<tr>
<td>( k_{T\lambda} )</td>
<td>55.4 mNm/A</td>
<td>30.2 mNm/A</td>
<td>19.4 mNm/A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gearbox</th>
<th>Maxon GP82</th>
<th>Maxon GP42</th>
<th>Maxon GP42</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_\lambda )</td>
<td>35</td>
<td>15</td>
<td>129</td>
</tr>
<tr>
<td>( \eta_\lambda )</td>
<td>70 %</td>
<td>81 %</td>
<td>68 %</td>
</tr>
</tbody>
</table>

\[ \begin{array}{|c|c|}
\hline
\rho & 9 \\
\hline
\text{Efficiency of planetary differential } & 88 \%
\eta_{PG} \\
\hline
\text{Carrier inertia } J_C & 12e-6 kg m\(^2\) \\
\hline
\text{Carrier viscous friction } \nu_C & 1e-3 Nm/(rad/s) \\
\hline
\text{Coulomb friction of carrier } T_{CC} & 0.1033 Nm \\
\hline
\end{array} \]

Table 4.6: Parameters for the dual-motor actuator. Friction parameters were obtained from steady-state measurements, as described in section 4.6. The inertia \( J_C \) was retrieved from datasheet information and CAD drawings.
4.5.1 Operating range

The operating range of a drivetrain consisting of a motor and gearbox is limited by the constraints listed in section 4.3.3. All of these constraints are a function of speed and torque, which is why the operating range is typically represented on a speed-torque plane. Figure 4.6 shows the operating range of the dual-motor actuator and the reference drivetrain it is being compared to. As we will explain in section 4.6, it is not energetically favorable to use the brakes when the DMA is performing dynamic motions. For this reason, Fig. 4.6 is constructed with the brakes disengaged at all times, unlike in section 4.4.4.4.

While previous works [122, 81] have reported on the working range in quadrant I, we also offer a view on the negative power quadrants II and IV, which are also of importance in the field of robotics. The main difference between those quadrants is the role of friction and gearbox losses. In the positive power quadrants, these losses increase the torque demanded from the motor. When power is flowing from the load to the motor, however, friction and gearbox losses relieve the motor’s torque requirement by absorbing some of the incoming negative power. For example, due to the gearbox alone, a motor equipped with a gearbox with efficiency $\eta_{tr}$ can deliver $1/\eta_{tr}^2$ times more torque in the negative power quadrants II and IV than in the positive quadrants I and III. This is also visible in Figure 4.6b where, in the negative power quadrants, the reference motor can reach torques that are approximately $1/\eta_{ref}^2 = 2$ times higher than in the positive power quadrants.

Because the voltage is sufficiently high, the torque-speed line does not limit the operating range of the reference actuator. As a result, the operating range is square in each of the four quadrants, restricted by only the maximum continuous torque $T_{ref,\text{max,cont}}$ and the maximum speed $\dot{\theta}_{ref,\text{max}}$. The operating range of the DMA is quite similar in size and shape, except for two additional areas in quadrants I and III, marked in dark grey in Figure 4.6a. The appearance of these areas can be understood by looking at the optimal speed ratio (Figure 4.7). This figure shows that, in the grey areas, the speed ratio $\gamma$ is just below zero. According to Eq. (4.7), this implies that the ring speed has the opposite sign of the sun and output speeds. Because, according to the definitions in Figure 4.2, all torques on the planetary differential have the same sign, the power flow in the ring will be negative. As explained before, this has the advantage that the ring drivetrain – which would otherwise limit the DMA’s output torque – can deliver higher torques to the output. We thus conclude that the operating range can be increased by creating an internal power flow between the sun motor and the ring motor. Although the operating range of kinematically redundant actuators has been analyzed before, this interesting observation has never been reported, because back-driving of motors was prevented by design [122] or simply not considered [81]. Nevertheless, we believe that this is an important feature that can potentially be exploited as an advantage.
Figure 4.6: Operating range of the (a) dual-motor actuator, compared to (b) the reference motor. Specifications of the actuators are listed in Tables 4.5 and 4.6. The constraints associated with the boundaries are indicated on the figure. The reference drivetrain was designed to cover roughly the same operating range. The dual-motor actuator, however, is capable of delivering high torques at low speeds (dark zones), where the reference drivetrain cannot be used. This is made possible by exploiting internal power flows in the dual-motor actuator.
Figure 4.7: Optimal speed ratio in steady-state conditions. The map is obtained by means of a parameter sweep on the ring speed (step size 1 rpm) through its entire range of permissible speeds. All speed ratios are between 0 and 1, except for the dark blue zone, where the speed ratio is just below zero. The optimal speed ratio is close to one in a large part of the operating range, indicating that the ring drivetrain is more efficient than the sun drivetrain. For high speeds, the optimal speed ratio drops, as the ring motor alone cannot provide the required output speed.
Table 4.7: Inertias of the drivetrains and their individual components, reflected to the motor. Because the differential was constructed from an off-the-shelf gearbox, only the total inertia of the differential is known; the separate inertias of the ring and sun were estimated based on datasheet information and design considerations.

4.5.2 Reflected inertia

The inertia of the drivetrain has an important influence on the energy consumption of an actuator performing a dynamic task. An overview of the inertias of the drivetrain components in the DMA design and the reference drivetrain is listed in Table 4.7. This table shows that the contribution of the planetary differential to the total inertia in the dual-motor drivetrains is negligible since, in both sun and ring branch, it is attenuated by the gear ratios of the gearbox in that branch. Furthermore, the inertias of the sun and ring branch are an order of magnitude lower than that of the reference drivetrain.

The inertias of the individual drivetrains, of course, do not reveal much about the overall performance of the DMA. This is best represented by the reflected inertia w.r.t. the input (motor side) and the output (load side), which are discussed below.

4.5.2.1 Inertia reflected to motors

The inertia matrix $A$, defined in section 4.3, contains the inertia reflected to the motors:

$$
A = \begin{bmatrix}
J_S + \frac{C_S}{n_S} \frac{1}{1+\rho C_{PG}} \frac{1}{1+\rho} J_C & \frac{C_S}{n_S n_R} \frac{1}{1+\rho C_{PG}} \frac{1}{1+\rho} J_C \\
\frac{C_R}{n_R n_S} \frac{\rho C_{PG}}{1+\rho C_{PG}} \frac{1}{1+\rho} J_C & J_R + \frac{C_R}{n_R} \frac{\rho C_{PG}}{1+\rho C_{PG}} \frac{1}{1+\rho} J_C
\end{bmatrix}
$$

The inertia matrix contains coupling terms. Neglecting gearbox losses, the coupling terms are identical and given by

$$
\frac{1}{n_S n_R (1+\rho)^2} J_C
$$

There are several ways to make the coupling terms disappear: $n_S n_R \to \infty$, $J_C \to 0$, $\rho \to 0$ or $\rho \to \infty$. Considering that, for a single-stage planetary differential, $1 < \rho$, $\rho \to 0$ is not realistic. Furthermore, the inertia of the load, if any, will need to be included in $J_C$. The carrier inertia $J_C$ can therefore not be neglected either. Consequently, the only practical way to reduce the coupling effect is to use high gear reductions $n_S$, $n_R$ and $\rho$, although $\rho$ cannot exceed a value of 9 for a single-stage planetary differential [99]. High gear reductions, however, comes at the cost of a lower efficiency for the respective planetary gearbox, especially if stages need to be added.
Figure 4.8: Inertia reflected to load for the DMA ($J_{DMA, refl}$, blue) and the 250W reference motor ($J_{ref, refl}$, red). The contributions of the sun ($J_{S, refl}$) and ring ($J_{R, refl}$) are also indicated with dotted lines. For the DMA, the inertia reflected to the load is a quadratic function of the speed ratio $\gamma$.

4.5.2.2 Inertia reflected to output

If power losses are neglected, the total inertia reflected to the output, $J_{DMA, refl}$, can be found by stating that the sum of the system’s kinetic energies should be equal to the kinetic energy of the reflected inertia rotating at the output speed:

$$\frac{1}{2}J_{DMA, refl} \dot{\theta}_o^2 = \frac{1}{2}J_C \dot{\theta}_o^2 + \frac{1}{2}J_S \dot{\theta}_S^2 + \frac{1}{2}J_R \dot{\theta}_R^2$$  \hspace{1cm} (4.71)

Finding $J_{DMA, refl}$ would require an inversion of Eq. (4.10). Due to the kinematic redundancy, this is impossible to achieve. As a workaround, we introduced the speed ratio $\gamma$, yielding the inverted relationships (4.13) and (4.14). This results in the following expression for the reflected inertia:

$$J_{DMA, refl} = J_C + J_{S, refl} + J_{R, refl}$$  \hspace{1cm} (4.72)

with

$$J_{S, refl} = n_S^2 (1 + \rho)^2 (1 - \gamma)^2 J_S$$  \hspace{1cm} (4.73)

$$J_{R, refl} = n_R^2 \frac{(1 + \rho)^2 \gamma^2 J_R}{\rho^2}$$  \hspace{1cm} (4.74)

The inertia of a dual-motor actuator is a quadratic function of the speed ratio $\gamma$. The dependence is visualized in Figure 4.8 for the dual-motor actuator studied in this work. The minimal reflected inertia is
\[ J_{Tot,\text{min}} = \frac{(1 + \rho)^2}{J_{S\text{n}}^2} + J_C \]  

(4.75)

and occurs at

\[ \gamma = \frac{1}{1 + \frac{J_{R\text{n}}^2}{\rho^2 J_{S\text{n}}^2}} \]  

(4.76)

Interestingly, the minimal reflected inertia is lower than the reflected inertia at \( \gamma = 0 \), where the speed is completely delivered by the ring motor, and at \( \gamma = 1 \), where the speed is completely delivered by the sun drivetrain. This shows that, in terms of reflected inertia, a dual-motor actuator architecture is not bounded by the inertia of its separate branches. Because the kinetic energy of a drivetrain is proportional to the square of the speed, and the output speed of the DMA is divided linearly over both actuators (Eq. (4.10)), one would expect the dual-motor architecture to be advantageous in applications where high accelerations are required. This will be studied in section 4.5.3.

Nevertheless, even the lowest reflected inertia of the DMA is still slightly higher than that of the reference motor. A disadvantage of the DMA is that it consists of more rotating components than a conventional actuator. In particular, the ring gear and the couplings required to attach the motors to the differential all add inertia to the actuator. This is, however, not the reason for the higher reflected inertia, since the contribution of these components to the total inertia is negligible compared to the inertias of the sun and ring drivetrains. The real reason is the additional reduction through the planetary differential. Although the inertias of the branches themselves are smaller than that of the reference drivetrain (Table 4.7), the reduction from the differential causes their inertia, reflected to the output, to be approximately 4 times (ring) and 2 times (ring) higher than that of the reference drivetrain.

Still, it must be noted that this analysis did not consider the efficiency of the actuator, which also has an impact on the reflected inertia. In this regard, it is important to note that the dual-motor actuator was designed to have a high average efficiency throughout its operating range (see section 4.4.4). The higher efficiency can compensate somewhat for the increased reflected inertia. Furthermore, the reflected inertia was not considered in the design phase. By doing so, it might be possible to conceive a design with lower reflected inertia, without compromising on efficiency.

### 4.5.3 Maximum acceleration

So far, we have established that the DMA has a slightly larger operating range, but a higher reflected inertia. We will now discuss how this translates to the maximum achievable output acceleration.

A common metric to assess the acceleration capability is the ratio between the maximum torque \( T_{\text{max}} \) and the rotor inertia \( J_m \). An analysis similar to the one presented in [91] shows that this metric scales with the motor’s length \( l_m \) and its radius \( r_m \) as

\[ \frac{T_{\text{max}}}{J_m} \sim l_m^{3/2} r_m^{-3/2} \]  

(4.77)
The relationship is in line with catalog data, which exhibits a scaling of $T_{\text{max}}/J_m \sim r_m^{-1.6}$ \cite{242}. A motor's acceleration capability is thus independent of its length, and decreases with its radius according to $r_m^{-3/2}$. In other words, smaller motors can deliver relatively higher accelerations. This appears to be a favorable situation for the DMA where, in essence, we replaced a large motor with two smaller ones. However, the DMA also links the accelerations and torques of the output to those of the motors, which complicates the discussion. Consequently, a more detailed analysis is required to assess whether a DMA really can really reach higher accelerations.

In accordance with Eq. (4.10), the maximum output acceleration is simply the sum of the maximum accelerations of the sun and ring branch:
\[
\ddot{\theta}_{\text{max}} = J \begin{bmatrix} \ddot{\theta}_{\text{S max}} \\ \ddot{\theta}_{\text{R max}} \end{bmatrix}
\] (4.78)

In order to relate the maximum acceleration to the design, we simplify the motor torque (4.39) by neglecting friction:
\[
T_{2m} = Ax + DT_o
\] (4.79)

There are two contributions to the motor torque that must be taken into account. The first part is related to the accelerations of the motors which is strongly influenced by $\gamma$. The second part is related to the desired output torque $T_o$. This torque is divided over both motors according to Eq. (4.39), and consumes current from both motors. The distribution over the motors is determined by the design (parameter $\rho$) and, in contrast to the other term, cannot be influenced by the speed ratio $\gamma$ – except indirectly, through manipulation of the speed-related friction terms.

The maximum acceleration for a specific static output torque is achieved when both motor torques saturate. Defining the vector of saturated motor torques,
\[
T_{2\text{max}} = \begin{bmatrix} T_{\text{S max}} \\ T_{\text{R max}} \end{bmatrix} = \begin{bmatrix} k_{TS}I_{\text{S max, peak}} \\ k_{TR}I_{\text{R max, peak}} \end{bmatrix}
\] (4.80)

Eq. (4.79), at saturation, can be rewritten as
\[
A^{-1} (T_{2\text{max}} - DT_o) = \begin{bmatrix} \ddot{\theta}_{\text{S max}} \\ \ddot{\theta}_{\text{R max}} \end{bmatrix}
\] (4.81)

Left multiplication with $Q$ gives
\[
JA^{-1} (T_{2\text{max}} - DT_o) = \ddot{\theta}_{\text{max}}
\] (4.82)

This equation defines the maximum acceleration $\ddot{\theta}_{\text{max}}$ for a specific output torque $T_o$. If we neglect the efficiencies $C_{PG}, C_S$ and $C_R$ and assume that both motors are identical ($T_{\text{S max}} = T_{\text{R max}} \triangleq T_{\text{max}}$ and $J_S = J_R \triangleq J_m$), we find the maximum acceleration
\[
\ddot{\theta}_{\text{max}} (T_o) = \frac{\frac{1}{1+\rho} \left( \frac{1}{n_S} + \frac{\rho}{n_R} \right) T_{\text{max}} - \frac{1}{(1+\rho)^2} \left( \frac{1}{n_S} + \frac{\rho^2}{n_R} \right) T_o}{J_m + \left( \frac{1}{n_S^2 (1+\rho)^2} + \frac{\rho^2}{n_R^2 (1+\rho)^2} \right) J_o}
\] (4.83)
where we replaced the carrier inertia $J_C$ with $J_o$, the combined inertia of the carrier and load:

$$J_o = J_C + J_{load}$$

(4.84)

Compare to a regular DC motor with gear reducer:

$$\dot{\theta}_{\text{max}}(T_o) = \frac{\frac{1}{n}T_{\text{max}} - \frac{1}{n^2}T_o}{J_m + \frac{J_o}{n^2}}$$

In Eq. (4.83), it is not the sum of actuator inertias $J_S + J_R \approx 2J_m$ but the inertia of a single motor ($J_m$) that appears. This implies that the maximum acceleration scales favorably with the inertias of the DMA drivetrains. On the other hand, if

$$n = \frac{1}{1+\rho} \left( \frac{1}{n_S} + \frac{\rho}{n_R} \right)$$

then

$$n^2 > \frac{1}{(1+\rho)^2} \left( \frac{1}{n_S^2} + \frac{\rho^2}{n_R^2} \right)$$

The static output torque and load inertia are thus reduced more strongly by the conventional solution of a motor with gear reducer. This means that, if the DMA and the conventional actuator are designed to deliver the same static torque output, the conventional actuator will perform better dynamically at high output torques or with high inertial loads. However, as demonstrated in Table 4.7, the inertia $J_m$ of the DMA’s motors can be made smaller than that of a single drivetrain. As a consequence, the DMA starts off with a higher acceleration capability, which nonetheless declines more rapidly with increasing $J_o$ and $T_o$ than that of the reference drivetrain. This is visualized in Fig. 4.9.

### 4.5.4 Energy consumption

In order to evaluate the energy efficiency of the actuator, we calculate the electrical energy consumption in a task where the actuator applies a sinusoidal speed to a variable load. The trajectory is given by

$$\dot{\theta}_{\text{imp}} = \theta_a \Omega \sin(\Omega t)$$

(4.85)

with fixed amplitude $\theta_a = 60^\circ$ and variable frequency $\Omega$. The variable load consists of a variable static torque $T_o$ and a constant inertial load $J_{load} = 11.7 \text{ gm}^2$:

$$T_{load} = T_0 + J_{load} \theta_a \Omega^2 \cos(\Omega t)$$

(4.86)

Based on the inertia matching principle (refer to section 2.3.3.2 or [165]), the reference drivetrain design would be ideal for an inertial load of $J_{load} = 0.17 \text{ kgm}^2$. The actual inertial load is more than ten times lower to reflect the problem of performing dynamic motions with drivetrains designed for high torques, explained in the introduction of this chapter.
Figure 4.9: Influence on the maximum acceleration of (a) load inertia $J_o$ and (b) static torque $T_o$. In Fig. (a), $T_o = 0$ Nm, and in Fig. (b), $J_o = 0$ kgm$^2$. There is an inverse quadratic relationship between $\ddot{\theta}_{\text{max}}$ and $J_o$, and an inverse linear relationship between $\ddot{\theta}_{\text{max}}$ and $T_o$. The DMA can achieve much higher accelerations than the reference actuator at low loads. However, the output torque and, especially, the load inertia have a much stronger effect on the DMA. For high values of $J_o$, the maximum achievable acceleration of the DMA approaches that of the reference actuator, and will eventually drop below it.
By varying the static torque $T_o$ and the frequency $\Omega$, we can evaluate the actuator’s energy consumption for combined static and dynamic loads. The results are shown in Fig. 4.10. In the absence of a static load, the reference drivetrain is capable of reaching slightly higher frequencies than the DMA, but the difference is rather small. Conversely, the DMA is much more capable of dealing with static torques. In accordance with the results from section 4.5.1, the reference drivetrain can only deliver torques of up to 10 Nm at low frequencies, whereas the DMA can reach more than 20 Nm. What also sets both actuators apart, is their energy consumption, presented in Fig. 4.10 as the average electrical power over a cycle. At low frequencies, the electrical energy consumption is quite similar, but the difference increases fast with increasing frequency. At frequencies approaching 2 Hz, the average power of the reference drivetrain easily exceeds 100 W, while the average power of the DMA is only around 30 W. In conclusions, these calculations demonstrate that the better quasi-static efficiency of the DMA outweighs the slight increase in reflected inertia.

4.6 Experimental validation

4.6.1 Test setup

A prototype of the DMA, shown in Fig. 4.11, was constructed using mostly off-the shelf components. The planetary differential consists of a commercial Neugart PLFE 064 planetary gearbox ($\eta_{PG}=94\%$) with a reduction ratio of 10:1 (i.e., $p=9$). Instead of fixing the casing of the gearbox to the structure, a ball bearing is used to constrain the gearbox radially, while allowing it to rotate around its axis. Additionally, a spur gear is mounted on
Figure 4.11: (a) Close-up view of the DMA. (b) Exploded view of the DMA with denotation of the components.
Figure 4.12: Test setup used to validate the case study presented in section 4.4.4. The reference drivetrain (left) and the dual-motor actuator (right), designed with the intention of covering the same operating range as the reference motor, are coupled back-to-back with a torque sensor in between.

The casing (i.e., the ring), which is driven by the ring drivetrain through a 3:1 reduction. In this way, a planetary differential as depicted in Fig. 4.1 is created. The ring drivetrain is a 90W Maxon RE35 motor with a 43:1 Maxon GP42C gearbox ($\eta_R=72\%$); the sun drivetrain is a 150W Maxon RE40 motor with 15:1 Maxon GP42C gearbox ($\eta_R=81\%$). Both motors are equipped with Maxon AB24 holding brakes (24V, 0.4Nm). This design is the same as the one studied in section 4.5, and is equivalent to design 4 in Table 4.3, except for the ring gear ratio $n_R$. The overall ring reduction is thus 129:1, approximately half of the 216:1 reduction specified in Table 4.3. This difference only has a marginal effect on the DMA’s ability to cover the reference motor’s operating range.

Fig. 4.12 shows the test setup that is used to study the actuator concept experimentally. A 250W Maxon RE65 motor with 51:1 Maxon GP81A gearbox, which served as the reference drivetrain in section 4.4.4, is coupled to the DMA with a ETH Messtechnik DRBK-50 torque sensor in between, so that the torque provided by the drivetrains can be measured. The positions and speeds are obtained from Maxon HEDL 5540 encoders (500 CPT) mounted on each of the motors. A Maxon EPOS3 70/10 controller imposes a constant torque on the 250W motor, while a constant speed is imposed on the DMA’s ring motor (by means of a Maxon MAXPOS 50/5) and sun motor (by means of a Maxon EPOS3 70/10). The motor currents are retrieved from the EPOS3 and MAXPOS controllers, while the voltage at the motor terminals is obtained from a custom-made differential amplifier circuit board. The available sensors thus allow for a direct measurement of the mechanical and electrical power consumption of the actuators.
Figure 4.13: Power consumption of the dual-motor actuator and the reference motor studied in section 4.4, for a range of constant speeds and torques. The dual-motor actuator consumes less energy than the reference motor, especially at high speeds.
4.6 Experimental validation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency of planetary differential $\eta_{PG}$</td>
<td>88 %</td>
</tr>
<tr>
<td>Efficiency of sun gearbox $\eta_{S}$</td>
<td>81 %</td>
</tr>
<tr>
<td>Efficiency of ring gearbox $\eta_{R}$</td>
<td>68 %</td>
</tr>
<tr>
<td>Sun drivetrain viscous friction coefficient $\nu_{S}$</td>
<td>1.5e-5 Nm/(rad/s)</td>
</tr>
<tr>
<td>Ring drivetrain viscous friction coefficient $\nu_{R}$</td>
<td>1.5e-12 Nm/(rad/s)</td>
</tr>
<tr>
<td>Carrier viscous friction $\nu_{C}$</td>
<td>1e-3 Nm/(rad/s)</td>
</tr>
<tr>
<td>Coulomb friction of sun drivetrain $T_{CS}$</td>
<td>0.0080 Nm</td>
</tr>
<tr>
<td>Coulomb friction of ring drivetrain $T_{CR}$</td>
<td>0.0061 Nm</td>
</tr>
<tr>
<td>Coulomb friction of carrier $T_{CC}$</td>
<td>0.1033 Nm</td>
</tr>
</tbody>
</table>

Table 4.8: Empirical friction coefficients for the DMA design.

4.6.2 Steady-state experiments

4.6.2.1 Power consumption

First, we would like to verify whether the proposed actuator actually succeeds in decreasing the energy consumption, compared to the 250W geared DC motor. To do so, we imposed a range of speeds and torques within the operating ranges of the actuators. The speed ratio was then hand-tuned to its optimal value. Fig. 4.13 shows the experimentally obtained electrical power consumptions for speeds of -20, 20, -50 and 50 rpm, for various torques within the actuator’s operating range. Throughout most of the operating range, the DMA is indeed more efficient than the reference motor. The decrease in electrical power consumption is small for low speeds (+/-20 rpm), but increases for higher speeds (+/-50 rpm), where the DMA consumes approximately 20 W less than the reference motor.

4.6.2.2 Validation of the theoretical model

As mentioned at the beginning of this section, the model described in section 4.3 contains many a priori unknown friction coefficients. With the available experimental data, however, we can estimate these unknown coefficients.

We imposed a range of randomly distributed speeds and torques, and calculated the expected currents based on Eq. (4.42) with the imposed output speed and torque as input. These values were then fitted onto the steady-state current measurements, with the friction coefficients $T_{C\gamma}$, $\nu_{\gamma}$, $\eta_{\lambda}$ ($\gamma = R, S, C$) and $\eta_{PG}$ as fitting parameters. The resulting parameters are listed in Table 4.8 and the fit is shown in Fig. 4.14. There is a good match between the measurements and the fitted model, especially when the drivetrains are in forward drive. The model proposed in section 4.3 can therefore be considered adequate for steady-state operation.

4.6.2.3 Optimal speed ratio

Based on the improved model with empirically-defined friction coefficients, we can find the real optimal speed ratios for the actuator. This is done by performing a parameter
Figure 4.14: Currents obtained from experiments and the theoretical model, for various operating points. There is a good correspondence between the model and the measurements.

Figure 4.15: Optimal operating modes and speed ratio, based on the empirical model of the tested actuator. Dark blue denotes single-motor operation with locked ring; yellow denotes single-motor operation with locked sun. In dual-motor operation, the speed ratio is shown. This result demonstrates that, in the presence of Coulomb friction and/or low gearbox efficiencies, single-motor operation should be exploited as much as possible.

sweep on $i$. The resulting speed ratios and operating modes are shown in Fig. 4.15.

The differences with the tentative catalog-based model, of which the results were presented in Fig. 4.5, are considerable. Regarding the ring brake, little has changed. It is, again, used throughout a large part of the region, especially where the torque is relatively high compared to the speed. The sun brake, however, is used much more extensively. In fact, dual-motor operation is limited to those operating points that cannot be reached
with a single motor. This can be explained by the friction from additional drivetrain components.

Recall that, in the theoretical model, the friction in the ring and sun branch was modeled as viscous friction, with friction coefficients of a similar order of magnitude. The actual friction coefficients, listed in Table 4.8, are very different. The additional bearings that are required in a DMA design, as well as the spur gear used to couple the ring drivetrain to the planetary differential, add to the total friction in the system. This friction presents itself mostly as Coulomb friction. Furthermore, according to Table 4.8, Coulomb friction is fairly evenly distributed over the ring and sun branch. This is not the case for viscous friction, which is much more present in the sun branch. Knowing that viscous friction causes power losses proportional to $\omega^2$, while Coulomb friction losses are only proportional to $\omega^1$, it becomes obvious that the sun branch should be avoided at high speeds. This explains why, in Fig. 4.15, the sun drivetrain is locked whenever the output speed is relatively high compared to the output torque. Furthermore, the speed ratios in the dual-motor region are much higher than in Fig. 4.5. This means that, here too, the ring motor is used more extensively in order to reduce viscous friction losses in the ring branch.

### 4.6.3 Dynamic experiments

In order to validate the results from the dynamic analysis, the experimental test setup was modified by replacing the load motor with a flywheel (Figure 4.16). The flywheel has an inertia of $J_{load} = 11.7 \text{ gm}^2$, the same as in Section 4.5.4. The DMA is still the same as in section 4.6.1. In these experiments, the holding brakes were constantly open, leaving the motor shaft to rotate freely. The ETH Messtechnik DRBK-50 torque sensor remains mounted between the actuator and the load, but a US Digital E6 encoder (2000 CPT) is added to the setup to measure the output position.
The imposed speeds are obtained from optimal control, as explained in section 4.3.4. A sinusoidal speed is imposed at the output, according to the specifications in Section 4.5.4. Imposed frequencies are 0.25, 0.5, 1 and 1.5 Hz.

Interestingly, none of the solutions to the optimal control problem made use of the brakes. This suggests that, for dynamic motions, brakes are no asset for the DMA. A likely explanation is that, upon disengagement of the brakes, high accelerations will be needed to bring the motor up to speed again, causing high Joule losses.

4.6.3.1 High-frequency measurement

We first present a high-frequency measurement in order to gain a better understanding of how the DMA works, and to demonstrate the validity of the model. Figure 4.17 shows the speed, currents and electrical power consumption for the measurement at 1.5 Hz.

**Speed distribution** The speed trajectories of the sun and ring are roughly sinusoidal, their frequency in line with that of the imposed trajectory. Between 0.14-0.2 s and 0.48-0.53 s, the ring speed is capped at approximately 10 000 rpm. This is due to saturation of the voltage, which was limited to 30 V (see section 4.3.3). To ensure that the correct output speed is still reached, the sun delivers a slightly higher speed during these time intervals, giving its speed profile a sawtooth-like appearance.

Higher-frequency components are avoided in order to prevent additional accelerations, which are the cause of most of the torque delivered by the motors. This means that the speed ratio $\gamma$ should roughly be constant throughout most of the cycle. Indeed, in Fig. 4.18, which shows the (imposed) speed ratio over a range of cycles with different frequencies, we observe that $\gamma$ tends to stay close to a single value for most of the cycle. For frequencies of 0.5 Hz and below, $\gamma = 1$ at all times, meaning that the sun is held still and all speed is being delivered by the ring branch – the most efficient branch in the design. Because, at this low frequency, the dynamics can be neglected, this result can be found by tracing the required combinations of output torque and speed on the map with the optimal speed ratio (Fig. 4.7). With a peak output torque of 0.12 Nm and a maximum output speed of 30 rpm for the 0.5 Hz measurement, it is easy to see that the speed ratio $\gamma = 1$ indeed corresponds to the optimal speed ratio according to the map. At higher frequencies, the average value of $\gamma$ decreases towards the value that minimizes the reflected inertia, as discussed in section 4.5.2.

Another interesting observation is that, at zero speed, i.e., when the acceleration is maximal, the speed ratio briefly exits the range [0,1]. As remarked in section 4.4.4.4 and illustrated in Figure 4.3, speed ratios outside of this range give rise to internal power flows. This is another example of how, in highly dynamic motions, the decrease of individual motor speeds and accelerations dominates the choice of the optimal speed distribution.

**Current** Many of the conclusions in this work are derived from the dynamic actuator model presented in section 4.3. Its validity is proven by the excellent agreement between
Figure 4.17: Modeled (red) and measured (blue) speeds, voltages, currents and electrical powers in the ring and sun branch, for a sinusoidal output speed with a frequency of 1.5 Hz. The measured values are averages over at least ten cycles. The $3\sigma$ confidence interval is shown as a grey fill.
Figure 4.18: Imposed speed ratio $\gamma$ over a range of sinusoidal cycles with different frequencies. At low frequencies, only the ring branch – the most efficient branch – is used, corresponding to a speed ratio of $\gamma = 1$. When the frequency is increased, $\gamma$ takes on an average value between 0 and 1 in order to reduce the reflected inertia. Upon velocity reversals (0%, 50% and 100% of cycle), $\gamma$ can take on values above one or below zero to smoothen the acceleration profiles of the individual motors.
the measured currents (blue) and the currents estimated from Eqs. (4.39)-(4.42) (red). The currents in both drivetrains are dominated by the inertial torque of the respective drivetrain, and therefore follow their acceleration pattern.

Electrical power Fig. 4.17 shows that the peak electrical power in the ring branch is approximately three times higher than that of the sun branch. But how much of this power makes its way to the load, and how much is lost as heat? To gain a better understanding of the power consumption, we present four important contributions to the electrical power in Fig. 4.19: the power related to the inertia of the load \( J_\text{load} \dot{\theta}_o \ddot{\theta}_o \), the power related to the inertia of the drivetrain \( J_\lambda \dot{\theta}_\lambda \ddot{\theta}_\lambda \), the Joule losses \( R_\lambda I_\lambda^2 \) and the friction losses, which include the power lost through viscous and Coulomb friction, as well as the gearbox losses.

The power flow related to the load is almost negligible compared to the power flows related to the inertia of the branches. In both branches, most of the input power is used to move the inertia of the drivetrain itself. This is a result of using a drivetrain designed for high torques to perform dynamic motion. Such drivetrains are composed of high torque motors and/or high gear reductions, leading to drivetrain inertia which is much higher than the inertia of the robot’s links. When the payload is removed, most power will move back and forth from the motor to the drivetrain inertia.

Inertia acts as an energy buffer, and therefore, the net energy consumption of these power flows, however big they are, is zero. They do, however, have an influence on the required motor torque and, consequently, on the Joule losses. These are much higher in the ring branch, where the acceleration is the highest. The winding resistance in the ring branch \( R_R = 583 \, \text{m}$$\Omega) is also almost twice as high as that of the sun branch \( R_S = 299 \, \text{m}$$\Omega).

The other source of losses is friction. Friction losses are related to the speed of the drivetrains, and therefore are not affected by the drivetrain inertia. The losses are similar in the sun and ring branch, but their contribution relative to the total power flow is clearly higher in the sun branch, which has the highest friction coefficients (see Table 4.8).

4.6.3.2 Energetic comparison

The measurements also allow for a direct comparison with the simulated energy consumption in Section 4.5.4. The calculated average power over a cycle is shown in Figure 4.20 and compared to the measurements on the setup. The measured values are slightly higher than the predicted values, indicating that there are still some unmodeled losses in the system. Nevertheless, the quantitative behavior is similar. Figure 4.20 also provides a good visualization of the difference between the reference drivetrain (red) and the DMA (blue). The energy consumption of the reference drivetrain rises much more quickly than that of the DMA. At a frequency of 1.5 Hz, the DMA only consumes 18% of the energy consumed by the reference drivetrain.
Figure 4.19: Power flows for a sinusoidal oscillation at 1.5 Hz, averaged out over ten cycles. The power flows were estimated from the position and current measurements.

Figure 4.20: Average power consumption in case of a purely inertial load of 12 gm². The estimated power consumption obtained from the optimization is denoted by the dashed lines, while experimentally obtained powers are denoted by crosses. For reference, the estimated power consumption of the reference drivetrain is plotted in red. There is a good match between the experiments and the estimations.
4.7 Discussion

In this chapter, we studied a dual-motor actuation concept with a planetary differential and holding brakes. This actuator is potentially more efficient than a conventional motor-gearbox solution because of two reasons. First, the actuator has a redundant kinematic degree of freedom, meaning that output speed is a weighted sum of the two motor speeds. This redundancy can be exploited by distributing the speeds in such a way that the overall efficiency of the actuator is the highest. A second option is to use the holding brakes to lock one of the two branches. In this case, the locked branch no longer contributes to the output speed, but the losses in this branch are also completely eliminated. What we end up with by using the brakes, is equivalent to an actuator which can be sized to the operating point, and is therefore more efficient.

4.7.1 Influence of friction and usage of brakes

An important finding is that the results depend strongly on the amount of viscous and Coulomb friction and the way they are distributed over the branches. This was not discovered in previous research, where friction was systematically neglected to simplify the analysis. In our setup, an approach where single-motor operation is maximized, turned out to be the most energy-efficient solution for steady-state operation. In this case, the sun, which exhibited high viscous friction, would be locked at high speeds, while the ring branch (lower gearbox and motor efficiency) is locked at high torques. When viscous friction is high in both branches, however, dual-motor operation could be a better strategy at high speeds.

With dynamic loads, this strategy no longer applies. In simulations where optimal control was applied to find the optimal speed distribution for a sinusoidal output trajectory, the brake was left completely unused. The logical explanation is that, any time a brake is disengaged, high startup currents would be needed from the motor. Instead, the preferred strategy is to keep both motors in motion, minimizing their required acceleration. This was clearly visible in our experiments, where optimal control always yielded the smoothest possible speed trajectories for both motors. Brakes would most likely only be used if they can be engaged during the entire motion, something which is only possible if a single motor can perform the motion on its own.

4.7.2 Internal power flows

The usage of the brakes also affects the power flows in the actuator. In stationary operation, internal power flows – characterized by a negative speed ratio $i$ – cause unnecessary losses, and should therefore be avoided. This is a well-known principle which many researchers use as a guideline. With a dynamic load, however, this is no longer necessarily true. Firstly, at output speed reversals, short internal power flows may reduce motor accelerations. But more importantly: internal power flows can extend the steady-state operating range in the absence of brakes. If the power flow in the most loaded branch is
reversed, friction can be used to absorb a part of the output power, lowering the torque requirement from the motor. By exploiting this phenomenon, the DMA is capable of providing high torques at low speeds, where the single-motor alternative is not. In previous analyses, the benefit of internal power flows in terms of operating range was not discovered because internal power flows were a priori prevented by non-backdrivable elements, or because the actuator was only used in steady-state operation.

4.7.3 Dynamic performance

An interesting feature of the DMA is that the speed ratio $\gamma$ can be chosen in such a way that the inertia of the actuator, reflected to the output, is smaller than that of its composing drivetrains. Nevertheless, we also found the reflected inertia of our DMA design to be slightly higher than that of a single-motor alternative. This does not have to be a disadvantage in terms of dynamic performance, though. The maximum acceleration of the DMA was shown to be higher than the single-motor reference drivetrain, although the advantage declines with the inertia of the load. The DMA, thanks to its extended operating range, was also able to provide higher accelerations when a static torque was added to the load. Furthermore, its energy consumption is considerably lower than that of its single-motor equivalent.

If the accelerations are low, most power will go through the branch with the least amount of friction, just like in steady-state operation. For higher accelerations, the optimal speed ratio tends to move towards the speed ratio that minimizes the reflected inertia. In all our trials with sinusoidal output speeds, $\gamma$ remained fairly constant, indicating that rapid changes in $\gamma$ should be avoided because of the additional energetic costs that would result from the high accelerations related to the high-frequency motion.

4.7.4 Reliability

A well-known potential advantage of redundant actuators, which has not been discussed so far, is their damage-tolerant behavior (reliability). Failure of one of the drivetrain components is catastrophic for traditional actuators, which typically consist of a single motor and gearbox. Although failure of an individual component is more likely in redundant actuators, which consist of a higher number of components, the redundant branch(es) can compensate for the loss of one branch. Although the actuator will not be able to operate at its full capabilities, it may be able to maintain its functionality. This is a crucial aspect for actuators in safety critical systems (e.g. airplanes) or remote robotic missions.

The effectiveness of this approach however depends on the type of redundancy and on the way how the branches fail. An overview is given in Table 4.9. If failure of a branch leads to its output being blocked, the output motion of a kinematically determinate actuator will be limited by the elasticity of the coupling between the failed branch and the output. If this coupling is rigid, the actuator will not be able to move at all. By introducing kinematic redundancy, the output will be able to move, albeit with reduced speed. Conversely, if the branch disconnects, e.g., because of a broken coupling or a failed
<table>
<thead>
<tr>
<th></th>
<th>Statically redundant</th>
<th>Kinematically redundant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch blocked</td>
<td>Output blocked</td>
<td>Output speed reduced</td>
</tr>
<tr>
<td></td>
<td>Infinite output torque</td>
<td>Output torque unaffected</td>
</tr>
<tr>
<td>Branch disconnected</td>
<td>Output speed unaffected</td>
<td>Freely moving output</td>
</tr>
<tr>
<td></td>
<td>Output torque reduced</td>
<td>No output torque</td>
</tr>
</tbody>
</table>

**Table 4.9**: Failure modes and their impact on the output of the actuator. The DMA fits in the second column, as it is a kinematically redundant and statically determinate actuator.

power supply, a statically determinate actuator will no longer be capable of providing an output torque. Such a failure would be unacceptable for robots which lift major payloads. For this reason, robotic manipulators are often equipped with normally closed holding brakes. An alternative solution would be to introduce static redundancy: here, the parallel motor units will still be able to provide a part of the required output torque, so that the payload can be put down gently.

From the perspective of fault-tolerance, the choice between both types of redundancy will also be affected by the application. Kinematically redundant actuators such as the DMA could, for example, be the preferred actuation for performance-augmenting exoskeletons, where a restriction of the user’s movements is more of a concern than the loss of actuator torque. On the other hand, humanoid robots would most likely benefit more from statically redundant actuators, since a disconnected motor in a kinematically redundant actuator would cause the robot to collapse under its weight.

### 4.7.5 Challenges and limitations

Kinematically redundant actuators have numerous potential benefits, but these benefits do not come without a cost. Additional components bring extra possibilities, but also extra design parameters and constraints. The result is a much more complex design process which, if not performed correctly, can easily lead to an ineffective actuator design. If individual components are not being used in the most optimal way, the greater number of components will inevitably result in a higher cost, weight and volume, and possibly even a lower energy efficiency. Making components work together in harmony, taking advantage of each component’s strength while minimizing the effect of its weakness, is the key to successful redundant actuator designs.

An important part of the dual-motor actuator is the differential that connects both motors to the output. In section 4.4.1, we explained how this component also links the motor torques and, consequently, the constraints of the two motors. As a result, it is easy to lose a part of a motor’s working range in a redundant actuator design. For constant and well-defined loads, the input drivetrains can be designed to have a matching torque limitation in order to get the most out of the actuator. Dynamic loads, however, are more complicated to deal with, because the differential’s kinematic and static constraints also affect the motor’s capability to accelerate to certain speeds. This makes the optimal design of the actuator very task-dependent, with reduced versatility as an inevitable consequence.
Another important challenge is the control of redundant actuators. For well-defined tasks such as the ones we imposed in this chapter, optimal control can be used to find the optimal power distribution offline. More complex tasks may however require more advanced controllers with state feedback. Applying this type of control to a redundant actuator is not a straightforward task. In chapter 5, we will discuss this issue in further detail.

4.8 Conclusion

The goal of this chapter was to assess how redundancy can be implemented on an actuator-level, with the aim of making the actuator more energy-efficient, more compact and lighter. To answer this question, we focused on a simple concept in which two motors are coupled to a single output through a planetary differential. This dual-motor actuator (DMA) was then subjected to an extensive theoretical and experimental analysis, covering both steady-state and dynamic operation. We succeeded in building a DMA which was more efficient than a reference drivetrain in steady-state operation, thanks to the possibility of directing the working points of the two motors to more energy-efficient regions on the motors’ efficiency maps. Furthermore, the theoretical analysis indicated that a DMA, like a variable transmission, can adapt its reflected inertia to the task. As a result, a DMA can be designed to deliver high torques with high energy-efficiency, while still being able to reach high accelerations. Such requirements are typical of robots which interact with their environment, but are very hard to meet by conventional actuator designs.

A downside of the concept is that the additional motor, the mechanism linking both inputs – a differential, in this specific case – and, possibly, the brakes, also introduce new constraints. Moreover, the design links the constraints of both motors to each other. The result is a complex design process, where poor design choices can have a large impact on the actuator’s operating range and, consequently, its versatility. The analysis presented in this chapter gives some rough guidelines on how to deal with this intricacy, but specific design rules are nearly impossible to devise for this class of actuators. Consequently, the design of a redundant actuator will most likely remain a complicated, iterative process.

In conclusion, our results indicate that, with some effort, the DMA can be used as an energy-efficient alternative to a classic drivetrain consisting of a motor with speed reducer. Its capability of providing higher torques at low speeds, combined with its ability to divide the acceleration over two motors, makes it a very suitable solution for applications which require a wide range of torques, speeds and accelerations.
Chapter 5

Series Elastic Dual-Motor Actuation

5.1 Introduction

In the previous chapters, we have discussed the potential of series elasticity and redundancy for the energy-efficient actuation of robots. The logical continuation of this research is to combine both concepts in a single actuator. Rather than the actual design, the biggest challenge is to find a control architecture for such an actuator. The controller’s task is not only to use the degree of freedom in the most optimal way, but also to deal with the decoupling of the actuator and load dynamics induced by the spring. In this chapter, we will present a control framework for this type of problem and apply it to a hopping robot. State-of-the-art control approaches for hopping and series elastic actuation are combined with an actuator-specific control allocation algorithm, which aims at exploiting the redundancy to reduce the electrical power consumption of the actuator. The applicability of our controller is proven by implementing it on a hopping robot consisting of a two-link segmented leg, similar to the human leg.


5.1.1 Redundant actuation for hopping robots

A remarkable amount of research on hopping robots has been performed over the past half-century [195]. The interest in this topic is explained by the fact that hopping can be seen as a subtask of legged locomotion [202]. The first studies on hopping were, in fact, a part of the Raibert’s well-known work on legged robots [182]. Several insights from hopping can be transferred to walking robots, prosthetics and exoskeletons, applications which have gained in importance in recent years. Despite years of research in these fields, the agility with which humans and animals are able to move, is still unrivaled by current
5.1 Introduction

A key feature of the human muscle is the elasticity of tendons, which act as a spring in series with our motor units. These elastic tissues play a crucial role in jumping motions [28, 7]. By using the elasticity of the tendons, the hopping motion – and locomotion in general – can be performed in a much more energy-efficient way [5]. For this reason, several researchers have implemented springs in hopping robots [194, 129, 93, 16]. Based on the notion that the knee joint needs to stiffen as it is flexed more in order to ensure stable hopping [201, 186], the stiffness of the knee actuator is often nonlinear in compliant designs [220]. The ability to vary stiffness is also an important feature, because the stiffness of the leg determines the hopping frequency [65] and varying stiffness allows us to adapt to the surface stiffness [69]. Motivated by these observations, Variable Stiffness Actuators have been implemented in several robotic hoppers [220, 235].

Another remarkable feature of the human muscles are their motor units. These consist of multiple muscle fibers, which can be activated according to the need [94, 128]. In contrast to series elasticity, redundancy on joint-level has only been researched to a limited extent in the field of robotics. Nevertheless, redundancy – introduced by continuously variable transmissions (CVTs) – has been very successful in the automotive sector [146]. Several authors have demonstrated that biomechanical applications, such as exoskeletons and prosthetics, can also benefit from such concepts, especially when combined with energy buffers such as springs and flywheels. In ankle prosthetics, for example, a variable transmission with spring can be used to shape the motor’s speed-torque curve to the biological ankle profile in walking [211]. Simulations show that the energy requirement of knee prosthetics for walking can be reduced by up to 85% by combining an infinitely variable transmission (IVT) with a flywheel [6]. Similar reductions are obtained when the IVT is used in conjunction with a spring [148]. In spite of the great energetic advantages predicted by these works, practical proof for the energy-efficiency of these systems is still missing. Building redundant actuation units that combine variable transmissions with energy buffers is difficult, especially if they need to be compact and energy-efficient [63]. Furthermore, energy-efficient online control of such complex actuators is a very tough problem, which is still largely unsolved [125].

5.1.2 Control allocation for dynamic systems

Over-actuated or redundant systems are very common in aeronautics and aerospace engineering. In such systems, the redundant degrees of freedom must work together to ensure a correct operation of the aircraft or spaceship, posing a challenging control problem. The field of study that deals with the control of such systems is called “control allocation” [109]. Multiple decades of research on this topic have yielded several methods and algorithms to combine the multiple inputs to a single output, possibly with the aim of reducing a certain performance metric. These algorithms have been studied thoroughly and their advantages and disadvantages are well-known [161]. However, nearly all of these methods are based on the assumption that the actuator dynamics do not have an influence on the control allocation problem. As a consequence, traditional control allocators have
difficulties with rapidly varying control commands [125, 160].

In recent years, several researchers in the field of robotics have taken up an interest in redundant actuation on joint-level. The additional redundant degree(s) of freedom can be used for practical reasons, e.g. overcoming transmission nonlinearities [159], but they can also be exploited to achieve more advanced goals such as increased energy efficiency [122, 134]. The problem with robotics is that, unlike in aeronautics, usually the actuator dynamics cannot be neglected. This is especially the case for highly geared actuators, where the reflected inertia of the drivetrain has a strong impact on the required torque [199]. As a result, traditional control allocation algorithms often lead to unsatisfactory results when applied to problems in robotics [125]. Control allocation for such applications is an active field of research, and custom solutions that suit a particular system are still the most common [109].

Another approach is to pre-compute the inputs offline using optimal control theory. This strategy has been applied to redundant actuators on many occasions [127, 173, 32]. The method is suitable for well-defined applications with cyclic motions, but fails when the actuator interacts with an unknown environment or when robustness to perturbations is required. Hopping robots are a good example of such an application, as the hopper needs to be able to adapt to changes in, for example, the stiffness or the height of the ground [136, 111, 145]. Furthermore, the optimization problem is typically solved with the square of the mechanical power ($P_{\text{mech}}^2$) or the square of the torque ($T_m^2$) as cost functions [82]. These cost functions do, however, do not take the nonlinear dynamics of gearbox and, especially, bearing friction into account. Instead, it is assumed that either gearbox losses ($P_{\text{mech}}^2$ approach) or Joule losses ($T_m^2$ approach) are dominant. For simple actuators, this is often the case, but in the context of complex actuators with many components, the assumption can be questioned. Here, the losses of the components add up and, moreover, affect each other. Consequently, the power taken from our energy source – which is, ultimately what we really want to minimize – becomes a complex function of the speeds and torques in the system (see section 2.5). Motivated by these considerations, we put forward the hypothesis that a cost function based on the electrical power ($P_{\text{elec}}$) can lead to better results, even if small approximations are required for implementation in a high-speed online controller.

### 5.1.3 Outline of the chapter

The chapter is structured as follows. In section 5.2, we describe the Series Elastic Dual-Motor Actuator (SEDMA), which adds a series elastic element to the DMA presented in chapter 4. We also introduce the Marco Hopper II setup, a robotic hopper which will be used to test the proposed controller. In section 5.3, we establish the equations that describe the SEDMA. The actual control framework and its different layers are explained in section 5.4. In section 5.5, we present experiments on the Marco Hopper II, which prove the applicability of the SEDMA and the proposed control architecture. The control allocator, a crucial layer of the control framework, is compared to other state-of-the-art solutions in section 5.6. After discussing possible improvements (section 5.7), the chapter is concluded in section 5.8.
Figure 5.1: The Series-Elastic Dual-Motor Actuator on the MARCO Hopper II setup. (a) Schematic, (b) Actual setup.
5.2 Test setup

5.2.1 Series Elastic Dual-Motor Actuator (SEDMA)

A schematic of the actuator, which we will name Series Elastic Dual-Motor Actuator (SEDMA), is depicted in Figure 5.1a. It consists of a dual-motor actuator (DMA) like the one presented in chapter 4, combined with a series spring. The main purpose of the spring is to protect the DMA’s planetary differential from the impacts at touchdown, which can be several times higher than the robot’s weight [110]. Furthermore, the spring can relieve the motor by absorbing energy during knee flexion and re-injecting it during the extension phase of the jump.

The design of the DMA is identical to the one presented in chapter 4, except for the ring drivetrain, which now consists of a 200W Maxon EC-4pole motor with a 15:1 reduction provided by a planetary gearbox. The additional 3:1 reduction provided by the spur gear pair that couples the drivetrain to the planetary differential’s ring is maintained. The sun drivetrain, consisting of a 150W Maxon RE40 DC motor with a planetary gear reducer of ratio 15:1, is unchanged with respect to the design from chapter 4.

5.2.2 MARCO Hopper II

In order to validate the control approach, the SEDMA was implemented on the MARCO Hopper II test bench (Figure 5.1b). The setup, introduced in [157], represents a human leg with an actuated knee. The robotic leg is approximately half the size of the human leg. Two links of equal length \( l = 250 \text{ mm} \), mass \( m_l = 0.17 \text{ kg} \) correspond to the shank and thigh. A mass \( m_h = 1.8 \text{ kg} \) is attached to the hip joint in order to mimic the trunk; a mass \( m_f = 0.30 \text{ kg} \) represents the foot. A cable between the hip and the foot has two functions: it prevents overextension of the knee, but also transfers kinetic energy from the hip to the foot to enable hopping. Furthermore, the hip and foot are mounted on a linear bearing to ensure a purely vertical motion.

The leg is actuated through a Bowden cable, connected to a pulley (radius \( r_{pulley} = 4.3 \text{ cm} \)) at the knee joint (mass \( m_k = 0.23 \text{ kg} \)). With this configuration, only an extension torque can be applied to the knee – flexion torques are not required for hopping to match to the Spring-Loaded Inverted Pendulum (SLIP) model [27]. The other end of the Bowden cable is connected to the series spring of the SEDMA, described previously. An overview of the setup’s parameters is given in Table 5.1.

The hopper is equipped with Bürster potentiometers (S/N 8709-5250) measuring the distance between the hip and foot, as well as the position of the foot relative to the ground. The extension of the spring is measured with another Bürster potentiometer (S/N 8709-5150). A SCAIME ZF100 sensor is used to sense the force in the Bowden cable. Finally, the position and speed of the spindle is inferred from the two incremental encoders (500 CPT) on the motors of the SEDMA.
5.3 SEDMA equations

The proposed control strategy separates the DMA from the spindle and the spring. For this reason, we will derive the equations of these two subsystems separately in this section.

5.3.1 Spring and spindle

The relationship between the output of the DMA $\theta_C$ and $x_o$, the output position of the SEDMA, is determined by the series spring (stiffness $k_s = 19.87 \text{ N/mm}$) and the spindle (gear ratio $n_{sp} = 314 \text{ rad/m}$), leading to the following equation:

$$x_o = \frac{\theta_C}{n_{sp}} - \frac{F}{k_s} \quad (5.1)$$

The output force of the SEDMA, $F$, appears in this equation as a result of Hooke’s law. The spindle relates this force to the carrier torque $T_C$:

$$F = C_{sp} n_{sp} T_C \quad (5.2)$$

where we defined the efficiency function of the spindle

$$C_{sp} = \eta_{sp}^{\text{sign}(F \cdot x_o)} \quad (5.3)$$

with $\eta_{sp} = 85\%$ the efficiency of the spindle, obtained from its datasheet.

5.3.2 Dual-Motor Actuator (DMA)

In this section, we will shortly reprise the equations of motion of the DMA from chapter 4. For the schematic of the planetary differential with definitions of torques and speeds, we refer to Fig. 4.2.

5.3.2.1 Speeds

The relationship between the carrier speed $\dot{\theta}_C$ and the input speeds $\dot{\theta}_S$ and $\dot{\theta}_R$ of the sun and ring motor, respectively, is given by

$$\dot{\theta}_C = C_{\theta} \begin{bmatrix} \dot{\theta}_S \\ \dot{\theta}_R \end{bmatrix} \quad (5.4)$$

![Table 5.1: Parameters of the Marco II setup.](image)
\[ C_\theta = \begin{bmatrix} \frac{1}{n_S (1+\rho)} & \frac{\rho}{n_R (1+\rho)} \end{bmatrix} \]  

where \( n_S \) and \( n_R \) are the reductions of the planetary gearboxes on the sun and ring branch, respectively, and \( \rho \) defined by Eq. (4.8). Utilizing the speed ratio \( i \), defined by Eq. (4.6), we can invert the speed equation (5.4), yielding

\[
\begin{bmatrix} \dot{\theta}_S \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{n_S (1+\rho)}{1+\rho i} \\ \frac{n_R (1+\rho i)}{1+\rho i} \end{bmatrix} \dot{\theta}_C \]  

(5.6)

By differentiation, we find the accelerations

\[
\begin{bmatrix} \ddot{\theta}_S \\ \ddot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{n_S (1+\rho)}{1+\rho i} \\ \frac{n_R (1+\rho i)}{1+\rho i} \end{bmatrix} \dot{\theta}_C + \begin{bmatrix} \frac{-n_S (1+\rho)\rho}{n_R (1+\rho)(1+\rho)} \\ \frac{-n_S (1+\rho)\rho}{n_R (1+\rho)(1+\rho)} \end{bmatrix} \dot{\theta}_C i 
\]  

(5.7)

### 5.3.2.2 Torques

The relationship between the state vector \( q = (\dot{\theta}_S \quad \dot{\theta}_R)^T \) and the input vector \( T_m = (T_{mS}, T_{mR})^T \) can be written as

\[
T_m = J\dot{q} + Bq + C\text{sign}(q) + D(C\theta q) 
\]  

(5.8)

with

\[
J = \begin{bmatrix} J_S & 0 \\ 0 & J_R \end{bmatrix} + J_C D \begin{bmatrix} \frac{1}{n_S} & \frac{1}{1+\rho} \\ \frac{n_R (1+\rho)}{1+\rho i} & \frac{1}{1+\rho} \end{bmatrix}
\]

\[
B = \begin{bmatrix} v_S & 0 \\ 0 & v_R \end{bmatrix} + V_C D \begin{bmatrix} \frac{1}{n_S} & \frac{1}{1+\rho} \\ \frac{n_R (1+\rho)}{1+\rho i} & \frac{1}{1+\rho} \end{bmatrix}
\]

\[
C = \begin{bmatrix} T_{CS} & 0 \\ 0 & T_{CR} \end{bmatrix}
\]

\[
D = \begin{bmatrix} \frac{C_S}{n_S} (\rho C_{PG} + 1)^{-1} \\ \frac{C_R}{n_R} (\frac{1}{\rho C_{PG}} + 1)^{-1} \end{bmatrix}
\]

The efficiency functions are defined by Eqs. (4.34) and (4.29). The inertias of the sun and ring branch are denoted with \( J_S, J_R \). They represent the inertia of the entire branch, i.e., they include the inertia of the motor and gearbox, but also the inertia of the gears inside the planetary differential and, in the case of the ring, the spur gear transmission between the ring branch and the differential. Similarly, \( J_C \) consists of the inertia of the carrier, but also includes the inertia other parts in the output drivetrain, most notably the spindle. Friction is modeled as a combination of viscous friction (coefficients \( v_S, v_R, v_C \)) and Coulomb friction (coefficients \( T_{CS}, T_{CR}, C_{CC} \)) on the sun and ring branch, as well as on the output of the DMA, i.e. the carrier. These coefficients, along with other relevant parameters, are listed in Tables 5.2 and 5.3. The friction coefficients are the ones that were obtained experimentally in chapter 4, where we also showed that the proposed model of the DMA provides a very good fit to dynamic measurements.
### Table 5.2: Parameters for the planetary differential.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency of planetary differential $\eta_{PG}$</td>
<td>88 %</td>
</tr>
<tr>
<td>Inertia of the output (carrier) $J_C$</td>
<td>12e-2 kg m²</td>
</tr>
<tr>
<td>Carrier viscous friction $\nu_C$</td>
<td>1e-3 Nm/(rad/s)</td>
</tr>
<tr>
<td>Coulomb friction of output (carrier) $T_{CC}$</td>
<td>0.1033 Nm</td>
</tr>
</tbody>
</table>

---

### Table 5.3: Parameters of the drivetrains of which the DMA is composed. The inertias $J_S$, $J_R$ and $J_{ref}$ were obtained from datasheet information and CAD drawings.

<table>
<thead>
<tr>
<th>Drivetrain</th>
<th>Sun drivetrain ($\lambda = S$)</th>
<th>Ring drivetrain ($\lambda = R$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drivetrain inertia $J_{S\lambda}$</td>
<td>1.4e-7 kg m²</td>
<td>8.7e-8 kg m²</td>
</tr>
<tr>
<td>Viscous friction coefficient $\nu_{S\lambda}$</td>
<td>1.5e-5 Nm/(rad/s)</td>
<td>1.5e-12 Nm/(rad/s)</td>
</tr>
<tr>
<td>Coulomb friction coefficient $T_{CC\lambda}$</td>
<td>0.0080 Nm</td>
<td>0.0061 Nm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor</th>
<th>Sun drivetrain ($\lambda = S$)</th>
<th>Ring drivetrain ($\lambda = R$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal power $P_{nom}$</td>
<td>150 W</td>
<td>200 W</td>
</tr>
<tr>
<td>Nominal torque $T_{\lambda nom} = T_{\lambda max, cont}$</td>
<td>177 mNm</td>
<td>135 mNm</td>
</tr>
<tr>
<td>Max. speed $\omega_{\lambda max}$</td>
<td>12 000 rpm</td>
<td>25 000 rpm</td>
</tr>
<tr>
<td>Terminal resistance $R_{\lambda}$</td>
<td>0.299 Ohm</td>
<td>0.102 Ohm</td>
</tr>
<tr>
<td>Torque constant $k_{T_{\lambda}}$</td>
<td>30.2 mNm/A</td>
<td>13.6 mNm/A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gearbox</th>
<th>Sun drivetrain ($\lambda = S$)</th>
<th>Ring drivetrain ($\lambda = R$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear ratio $n_{\lambda}$</td>
<td>91/6</td>
<td>91/6</td>
</tr>
<tr>
<td>Gearbox efficiency $\eta_{\lambda}$</td>
<td>81 %</td>
<td>81 %</td>
</tr>
</tbody>
</table>
5.3.2.3 Electrical domain

The total electrical power consumed by the DMA is the sum of the electrical powers of both motors:

\[ P_{elec} = U_S \cdot I_S + U_R \cdot I_R \]  \hspace{1cm} (5.9)

with voltages \( U_\lambda \) and currents \( I_\lambda \) given by

\[ I_\lambda = \frac{1}{k_{T_\lambda}} T_{m_\lambda} \]  \hspace{1cm} (5.10)

\[ U_\lambda = k_{T_\lambda} \dot{\theta}_\lambda + R_\lambda I_\lambda \]  \hspace{1cm} (5.11)

In these equations, \( k_{T_\lambda} \) is the torque constant of the motor and \( R_\lambda \) is the motor’s winding resistance.

5.3.2.4 Constraints

The DMA has several limitations, which must be respected by the controller.

**Maximum speed**  The motor speeds \( \dot{\theta}_S \) and \( \dot{\theta}_R \) should not exceed the maximum speeds specified in the motor datasheet:

\[ |\dot{\theta}_S| < 12 \, 000 \text{ rpm} \]

\[ |\dot{\theta}_R| < 25 \, 000 \text{ rpm} \]

**Maximum motor torque**  Another constraint is the maximum motor torque, which is limited for several reasons. First, motor manufacturers often specify a maximum (peak) torque to avoid mechanical damage to the motor. Second, the power stage of the controller has a current limit, which may impose an even stricter constraint on the torque.

And finally, high motor torques (above the maximum continuous torque) may lead to failure through excessive heat generation in the windings if they are sustained for a long period of time, even if they are below the motor’s maximum peak torque. While the commercial Maxon EPOS3 drive in the last stage of our controller has a built-in protection against overheating, this issue should already be considered in the design of the control allocator in order to obtain the most optimal results. Based on these considerations, the motor torques \( T_{mS} \) and \( T_{mR} \) were limited to following values:

\[ |T_{mS}| < 350 \text{ mNm} \]

\[ |T_{mR}| < 250 \text{ mNm} \]

which roughly corresponds to twice the maximum continuous torque \( T_{\lambda,max,cont} \) of the respective motor.
5.4 Control framework

Figure 5.2: Control schematic.

Maximum voltage  The voltage output of the power source puts a limit on the motor voltage $U_S$ and $U_R$:

$$|U_\lambda| < 48 \text{ V}(\lambda = R, S)$$

Following Eq. (5.11), this constraint limits the speed that can be reached at certain torque.

5.4 Control framework

The control framework, represented schematically in Fig. 5.2, consists of two main parts. The first is the high-level virtual model control, which calculates the desired force to be generated by the actuator, with the aim of mimicking human hopping. The second is a force controller, which ensures that the SEDMA generates the force required for hopping. This controller calculates the required currents for both motors, based on the desired output force. In this section, we explain how the control framework was implemented on the Marco Hopper II test bed.

5.4.1 Virtual Model Control

The MARCO Hopper II is controlled by means of Virtual Model Control (VMC) [177]. In this control strategy, the dynamics of the hopper are replaced with that of a single-mass harmonic oscillator. It is commonly accepted that this model, known as the spring-loaded inverted pendulum (SLIP) model, describes the dynamics of running and hopping well [27]. In previous work, the VMC-based control strategy was successfully applied to the MARCO Hopper II with a traditional motor-gearbox as actuator [157]. In this work, we use the same VMC controller as high-level controller to calculate the force required from the SEDMA. This is the top level of our control architecture.

The controller is implemented as a state machine. It distinguishes between the flight phase and the stance phase, which is split into two parts: an extension phase and a com-
Figure 5.3: State machine used for the Virtual Model Control. Transition from stance to flight occurs when \( l_{\text{leg}} \) (the distance between hip and foot, i.e. the length of the virtual harmonic oscillator) approaches its maximum value, which is limited by the knee stop. The stance phase is initiated when the position of the foot \( x_f \) approaches the ground (threshold \( x_f = 5 \text{ mm} \)). The transition between the compression and extension phase of stance is made when \( l_{\text{leg}} < x_0 \), a threshold which can be chosen arbitrarily, but with physical limitations of the setup in mind.

In the flight phase, the hopper is allowed to move freely, and the commanded force is zero. As soon as it reaches the stance phase, the VMC is activated. The commanded spring force, derived in [157], is

\[
F_d = \frac{2l \cos \left( \frac{\phi}{2} \right)}{r_{\text{pulley}}C_{\text{cable}}} \left[ \frac{m_b + \frac{1}{2}m}{m_v}k_v \left( l_0 - l_{\text{leg}} \right) + \frac{1}{2}m_l g \right]
\]

(5.12)

where \( l_0 = 40 \text{ cm} \) is the maximum (extended) length of the virtual spring, \( l_{\text{leg}} \) is the distance between the hip and foot, \( \phi \) is the knee angle as defined in Fig. 5.1, and \( C_{\text{cable}} \) an efficiency function for the Bowden cable, defined as

\[
C_{\text{cable}} = \begin{cases} 
\eta_{\text{cable}} & \text{(stance-extension)} \\
1/\eta_{\text{cable}} & \text{(stance-compression)} 
\end{cases}
\]

with \( \eta_{\text{cable}} = 85\% \) (a value derived from experimental data). Other parameters were defined in section 5.2. The mass of the virtual harmonic oscillator \( m_v \) is set to 1 kg, leaving

\[1\text{ During the compression phase, the hip and the foot move towards each other, causing the virtual spring to store energy (compress). During the extension phase, the virtual spring releases its energy by extending, causing the foot and hip to move away from each other.} \]
the virtual stiffness $k_v$ to determine the hopping pattern. According to [157], the minimum virtual stiffness required for hopping is

$$k_v > \frac{2m_v g}{l_0 - x_0} = 140 \text{ N/m} \quad (5.13)$$

where $x_0 = 26 \text{ cm}$ is the length of the virtual spring at its maximum compression$^2$. This stiffness corresponds to the value of $k_v$ for which the ground reaction force of the equivalent single-mass oscillator becomes zero at its apex point, enabling the oscillator to leave the ground. In our experiments, we met the requirement set by Eq. (5.13) by choosing $k_v = 175 \text{ N/m}$.

### 5.4.2 SEDMA force control

The force control of the SEA is performed with a cascaded controller, consisting of an inner speed loop and an outer force loop. This is a well-established control approach for SEAs [35], which works well even when friction is high [247].

The force control loop takes the desired spindle force $F_d$ from the VMC, given by Eq. (5.12), as a set value. The measured spindle force $\hat{F}$, which is estimated from the measured motor positions (found by integrating the state measured vector $\dot{\hat{q}} = \begin{bmatrix} \dot{\theta}_S & \dot{\theta}_R \end{bmatrix}^T$) and the measured output position of the SEDMA ($\hat{x}_o$) using equations (5.1) and (5.4), serves as feedback. The error $F_d - \hat{F}$ is then fed into a PI controller, which generates a desired speed for the DMA. This is handled by the inner speed loop, which consists of a control allocator and a low-level controller. The control allocator translates the desired DMA output speed into desired speeds for the individual motors, $\dot{q}_d$. The desired speeds are then used as setpoints for the low-level controller which, using a feedback linearization scheme, generates current commands for the individual motors. The details of the control allocator and the low-level control are explained below.

#### 5.4.2.1 Control allocation

The purpose of the control allocator is to translate the desired output speed $\dot{\theta}_{C,d}$ to motor speeds $\dot{\theta}_S$ and $\dot{\theta}_R$. We calculate the optimal speed distribution offline and implement it in a three-dimensional lookup table.

The electrical power consumption $P_{elec}$, as modeled in Section 5.3, can be written as a function of 5 variables: $\dot{\theta}_C$, $\ddot{\theta}_C$, $i$, $i$ and $T_C$. This means that the optimal speed ratio depends not only on the output, but also on the previous speed ratio. To simplify the problem, we propose to approximate the acceleration of the sun and ring motor, given by Eq. (5.7), as follows:

$$\begin{bmatrix} \ddot{\theta}_S \\ \ddot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{n_S(1+\rho)}{1+\rho i} \\ \frac{n_R(1+\rho)i}{1+\rho i} \end{bmatrix} \dot{\theta}_C \quad (5.14)$$

$^2 l_0$ and $x_0$ correspond to the highest and lowest position of the hip during the ground contact phase.
We thus neglect the rate of change of the speed ratio \( i \), making the speed ratio independent of previous solutions. As a result of this approximation, the electrical power \( P_{elec} \) becomes a function of only the output variables \( \dot{\theta}_C, \dot{\theta}_C \) and \( T_C \), as well as the speed ratio \( i \). This simplifies the problem considerably, but the approximation needs to be validated. We assess the validity of the approximation in section 5.5.3, and discuss the implications on the control allocation in section 5.7.

By minimizing the \( P_{elec} \) with respect to \( i \), we can thus find an optimal speed ratio

\[
i_{opt} = f(\dot{\theta}_C, \dot{\theta}_C, T_C)
\]

which is only a function of the DMA’s output. Combining this function with Eq. (5.6), we have a relationship between the motor speeds and the output speed. Consequently, the control allocation problem is solved.

In practice, the function (5.15) is implemented as a 30x30x30 lookup table (LUT) with evenly spaced speed, torque and acceleration breakpoints. The lookup table is calculated with a parameter sweep on \( \dot{\theta}_R \) (step size 1 rpm), where each solution is checked against the constraints specified in Section 5.3.2.4. We chose \( \dot{\theta}_R \) as a parameter rather than \( i \), because the latter does not have a finite range of possible values. In the control algorithm, linear interpolation and extrapolation is used to calculate intermediate points. The desired speed \( \dot{\theta}_{C,d} \) and the resulting acceleration \( \ddot{\theta}_{C,d} \) are fed to the LUT, along with the estimated output torque \( \hat{T}_C \). This implicitly assumes perfect tracking at the output, but allows the controller to adapt to changes in output torque. The output of the LUT, the ring speed \( \dot{\theta}_R \), is then used to calculate the sun speed \( \dot{\theta}_S \) by means of Eq. (5.4). Consequently, the vector \( q_d \), which serves as the input for the low-level controller, is completely defined.

5.4.2.2 Feedback linearization

The equations for feedback linearization are derived from the SEDMA’s equations of motion (5.8), where we consider \( q \) as the control variable and the vector \( T_m = (T_{mS}, T_{mR})^T \) as the input. Defining the control error as

\[
e = \dot{q}_d - \dot{q}
\]

we can write the suggested control law

\[
T_m = J\dot{q}_d + Pe + I \int e \, dt + Bq_d + C \text{sign}(q_d) + D \left( \hat{T}_C + T_{CC} \text{sign}(C_\theta q_d) \right)
\]

with \( P \) and \( I \) 2x1 matrices which contain the controller gains. This controller combines feedforward acceleration and friction compensation with feedback of the sensed torque \( \hat{T}_o \), which is calculated from the estimated force \( \hat{F} \) by means of Eq. (5.2). Finally, the torques obtained from Eq. (5.16) are used as set points for the Maxon EPOS3 controllers in the sun and ring branch, which are both operated in Cyclic Synchronous Torque mode.
Figure 5.4: Foot and hip position for the MARCO II hopping robot. Hopping is initiated from a squat position (hip height 200 mm), meaning that the virtual spring is loaded at the beginning of the trial. After a short transient, the hopping height converges to a constant value of 24 mm, at a hopping frequency of 1.02 Hz.

5.5 Experiments

In this section, the control approach suggested in section 5.4 is applied to the Marco Hopper II test setup presented in section 5.2. We start by discussing the hopping pattern that was generated in experiments. Next, we examine the measured speeds and accelerations that resulted from the control allocation algorithm. We finish by comparing the allocated speeds from the experiments to those that would be generated by an optimal control algorithm.

5.5.1 Hopping pattern

Figure 5.4 shows the first ten seconds of a hopping trial. A constant hopping height of 24 mm could be achieved after only a short transient. This height is in line with the experiments previously presented by Oehlke et al. [157], although the hopping frequency is lower (1.02 Hz compared to 1.5 Hz). This is most likely due to the way how friction is compensated in Oehlke et al., i.e., by increasing the virtual spring stiffness during the upward motion. The resulting energy injection may be higher than strictly needed for compensating the friction, and lead to an increase in hopping frequency.

The preferred hopping frequency of humans corresponds to approximately half of the bouncing system’s resonance frequency [40]. For humans, the preferred frequency is between 2 Hz and 2.3 Hz [65, 187]. Based on the antiresonance frequency – the energy-optimal frequency of a Series Elastic Actuator [227] – the optimal hopping frequency for the Marco Hopper II with SEDMA would be 1.34 Hz, which is higher than the selected frequency of 1.02 Hz. At frequencies below the optimal frequency, a low turning point is needed to achieve an appreciable hopping height, resulting in a higher flexion of the knee joints [13]. Consequently, the hops start to resemble a series of squat jumps. This is the case for the measurements in Fig. 5.4.
Figure 5.5: (a) Speeds of the sun, ring and carrier; (b) Speed ratio. The speed of the carrier, which serves as the output of the dual-motor actuator, is a weighted sum of the sun and ring speeds (see Eq. (5.4)). In the configuration under study, the latter is dominant.

While the hopping frequency can be increased by raising the virtual stiffness $k_v$ [27], it actually makes sense to hop at a lower frequency from a biomechanical point of view. In low-frequency hopping, the knee has a higher contribution to the total hopping torque [119, 96]. At higher frequencies, relatively more torque is required from the ankle. Taking into consideration that the knee is the only actuated joint on the hopping robot, low-frequency hops can be considered more natural for the MARCO Hopper II.

### 5.5.2 Online speed distribution

Figure 5.5 shows how the output speed is distributed between the sun and ring. We emphasize that this behavior is not pre-programmed, but merely a result of the online control algorithm. The speeds of the ring and carrier converge quickly to a stable cycle, after a small transient at the start. According to the speed equation (5.4), the ring contributes $\rho = 9$ times more to the output speed than the sun. This is evident in Figure 5.5a, where the ring speed almost follows that of the output. The sun mostly serves to make the overall motion more energy-efficient and to provide speeds and accelerations beyond the ring drive’s capabilities.

In terms of speed ratio (Figure 5.5b), we see both negative and positive speed ra-
5.5 Experiments

Figure 5.6: Estimated and actual acceleration during a hopping trial. The acceleration of the ring is very similar to that of the output. As a consequence, the estimated acceleration strongly resembles the actual acceleration. The sun, however, follows a very different acceleration pattern, leading to a poorer approximation.

5.5.3 Accelerations

In section 5.4.2.1, we proposed a simplification to the equation for the acceleration (Eq. 5.7). In this subsection, we will examine the validity of this approximation by comparing the estimated accelerations to the actual accelerations during the hopping experiment. The results are shown in Figure 5.6.

Notable in these plots are the strong deviations that occur when $\dot{\theta}_S \approx 0$. Because of the way how the speed ratio $i$ was defined (Eq. (4.6)), the optimal speed ratio $i$ then makes a step from $+\infty$ to $-\infty$ or the opposite. This can also be seen in Figure 5.5b, where the speed ratio jumps from +10 to -10 approximately every 0.5 s. The strong variations in $i$
lead to a strong contribution of the neglected term in Eq. (5.14), which is proportional to the variation of the speed ratio $\dot{i}$. Nevertheless, the approximation is quite smooth throughout most of the motion.

As mentioned in the previous section, the ring’s contribution dominates the output speed, and this also applies to the acceleration in Figure 5.6. The acceleration of the ring is nearly the same as that of the output - albeit scaled by the planetary differential with a ratio of 9:10. As a consequence, the approximation of the ring acceleration is quite good, too, because the simplified formula for the acceleration essentially assumes that the accelerations of the individual motors are proportional to the output acceleration. Conversely, acceleration profile of the sun is very different from that of the output. This results in an estimated acceleration which is only good at very low accelerations. At higher accelerations, the neglected term, which is proportional to the output speed $\dot{\theta}$, becomes significant, making the approximation invalid.

### 5.5.4 Comparison to optimal control

In the suggested control architecture, the limited accuracy of the approximation has an influence on the speed distribution, but not on the actual output speed. As a consequence, the accuracy of the hopping controller is not affected. But what are the consequences for the optimality of the speed distribution? In an attempt to answer this question, we took the measured output speed and torque of the DMA, and optimized the speed distribution off-line, allowing us to take the exact dynamics into account. Using the commercial software package GPOPS [166], we applied optimal control (OC) with three cost functions:

1. **Sum of mechanical powers** $\left(T_{oR}\omega_{oR}\right)^2 + \left(T_{oS}\omega_{oS}\right)^2$: This cost function aims at reducing the gearbox losses in both branches. Friction and gearbox losses are neglected in the calculation of the torques $T_{oR}$ and $T_{oS}$.

2. **Sum of motor currents** $I_{mR}^2 + I_{mS}^2$: This cost function, which in traditional actuators corresponds to torque squared, aims at reducing the Joule losses in both motors. Friction and gearbox losses are neglected in the calculations.

3. **Electrical power** $P_{elec}$: This cost function includes all friction coefficients and works with the exact value of the acceleration. In the OC formulation, it is given by

   \[
   P_{elec} = k_T S \dot{\theta}_S I_{mS} + R_S I_{mS}^2 + k_T R \dot{\theta}_R I_{mR} + R_R I_{mR}^2 
   \]  

   \[(5.17)\]

If the model is exact, this equation yields the exact electrical energy consumption of the actuator.

The first and second strategy are common in literature and have the advantage of not requiring prior knowledge of friction coefficients. The third strategy is equivalent to the online controller, but uses the exact acceleration instead of the approximation. Considering that this approach uses the complete model of the actuator, which was shown to represent the actuator dynamics in an accurate way, the results from this optimization
Figure 5.7: Optimal speed distributions, calculated with three different offline optimization strategies: optimal control with a $T^2$ cost function, with a $P^2_{\text{mech}}$ cost function and with a $P_{\text{elec}}$ cost function. The output torque and output speed of the DMA, as measured during the experimental trial presented in Figs. (5.4), (5.5) and (5.6), was used as input data for the optimization. The three resulting speed distributions are compared to the measured speed distribution from the online controller (yellow).
should reflect the actual energy-optimal distribution well, and can therefore be used as a baseline.

In Figure 5.7, the optimal speed distributions resulting from the three optimizations are compared to the speed distribution obtained from the online controller. An interesting observation is that, among the different controllers, the ring speeds look more alike than the sun speeds. This is due to the specific design of the DMA, where the ring contributes much more to the output speed. For this reason, the ring speed tends to resemble the output speed, especially when the output speed is close to the maximum that can be reached by the actuator. Conversely, because the sun has less impact on the output speed, there is more room for changes in its speed profile.

The ring speeds obtained from the online controller and from the optimal control based on torque are quite close to the ones obtained from optimal control based on electrical power, which we take as the reference for the energy-optimal speed distribution. In contrast, optimal control based on mechanical power makes more use of the sun branch, of which the speed is brought to saturation for large parts of the cycle. This solution is the furthest from the energy-optimal solution (OC - $P_{elec}$). This comes as no surprise, considering that the hopping motion requires high speeds and accelerations from an actuator that was, in fact, designed for high torques at relatively low speeds (see chapter 4). In such cases, the $T^2$ cost function, which includes the inertia of the drivetrains, performs better than the $P_{mech}^2$ cost function, which does not.

Apparently, the OC - $P_{elec}$ optimization yields a solution where the sun branch is not used. This situation arises when the required speed can be delivered by a single motor, and accelerations are insufficiently high to make a distributing the speeds over the two branches favorable. In this case, only the most efficient branch – the ring branch – is used (see section 4.5). The $T^2$ cost function follows this trend by yielding low sun speeds. When $P_{mech}^2$ is used as cost function, however, the sun speed is consistently brought to saturation, the most energy-inefficient solution. The measured speed profile lies somewhere in between, but displays an additional oscillation during the upward motion. The acceleration that results from this oscillation have an adverse effect on the energy consumption. In fact, Table 5.4 shows that the online controller has a higher energy consumption than any of the considered optimal control strategies. This is partially explained by the approximation of the acceleration, but also by the fact that the online controller does not predict future states. Unlike the optimal control optimizations, where the entire timespan of the task is taken into account, the online controller brings the speed distribution to the optimal value for that specific point in time. Consequently, the resulting speed distribution may drift to high accelerations. This is a major disadvantage of the proposed control allocator.

5.6 Comparison with related work

The idea behind the control allocator proposed in this work is to generate an optimality map offline and implement it into the offline controller. The concept was proposed earlier by Girard and Asada [82], where the authors used dynamic programming (value iteration) to generate a 2D optimality map in state space, which was subsequently approximated
5.6 Comparison with related work

<table>
<thead>
<tr>
<th>Energy per cycle (J)</th>
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<tbody>
<tr>
<td>OC - $P_{elec}$</td>
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<tr>
<td>OC - $T^2$</td>
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<tr>
<td>OC - $P_{mech}$</td>
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<tr>
<td>Measurement</td>
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| Table 5.4: Energy consumption for different cost functions. Optimal control (OC) with electrical power ($P_{elec}$) as a cost function yields the best results, and can be seen as a reference. If the square of the motor torque ($T^2$) is used, a similar energy consumption is obtained. The measured energy consumption, which was obtained with the online controller, is the highest. It is slightly higher than the energy consumption that would result from optimal control with $P_{mech}$ cost function. |

Another solution, proposed in [81], is based on nullspace projection. Despite its elegance, this approach has several downsides. First, nullspace projection can yield solutions that violate the actuator’s limits, causing one of the motors to saturate. Additional action is required to prevent this from happening; for example, the control allocator’s output could be post-processed to respect the constraints. In our approach, the LUT can easily be programmed to not yield solutions that do not respect the limitations, provided that such a solution exists. The proposed solution will also be the most energy-efficient out of all feasible options, something which is difficult to guarantee with a post-processed nullspace projection. Second, nullspace projection demands linear equations. Its performance is therefore affected by the nonlinearities of (gearbox) friction. Conversely, in our approach, the system’s nonlinearities can be taken into account when the LUT is calculated.

One of the main advantages of the proposed control allocator is that it is based on a detailed model of the electrical energy consumption. We hypothesized that the resulting controller can be more energy-optimal compared to those based on a traditional $T^2$ optimization (e.g. [82, 83]) or $P_{mech}^2$ optimization, where electrical losses and gearbox losses are not taken into account. This was verified in simulation, taking our experimental data as an input. Unfortunately, the online controller came out as the worst solution, yielding a higher energy consumption than any of the offline optimal control optimizations. This result might, however, be related to the fact that the entire trajectory is optimized at once, so that accelerations can be anticipated. If the $T^2$ or $P_{mech}^2$ minimization would be implemented online, the results might be different. Furthermore, in applications where the required accelerations are small or where the inertia of the drivetrain components is low, the impact of the error on the approximated acceleration will decrease. The online controller will then outperform the $T^2$ or $P_{mech}^2$ optimal control approaches thanks to the detailed model of friction in the drivetrain. Another interesting advantage of the online...
CHAPTER 5. SERIES ELASTIC DUAL-MOTOR ACTUATION

controller is w.r.t. offline optimal controller is that the optimality is not affected by unknown output forces, because torque feedback is used as an input to the control allocator. We can conclude that the control allocator can definitely be improved, but the overall control framework achieves its goal of enabling stable hopping in unknown environments.

5.7 Challenges and future work

The control architecture presented in this chapter enables stable closed-loop hopping with the SEDMA, while concurrently distributing the control effort over both actuators in an online fashion. One of the main issues with the proposed control allocator is the approximation of the acceleration, which is required to make the optimal solution independent of the past states. This enables to map the optimal speed distributions in a 3D lookup table instead of a 4D lookup table, which would be much more computationally expensive. The approximation loses its validity at high accelerations, resulting in a less optimal speed distribution. Furthermore, the proposed control allocator cannot anticipate future accelerations of the motors, which may lead the controller to drift towards excessively high motor accelerations. A control allocation strategy which deals with these issues in an effective way would be a major improvement to the controller.

Another aspect which was not addressed in this chapter is the selection of the series spring. Finding the (energy-)optimal spring stiffness for a hopping robot is, however, not an easy task. A work by Pelc et al., where a hopping robot was controlled with an artificial neural oscillator, proved that there is a clear relationship between a system’s natural frequencies and the optimal spring stiffness [167]. A potential strategy would therefore be to choose the stiffness of the spring such that the natural frequency of the system – resonance for parallel elastic actuators, antiresonance for series elastic actuators – is matched to the desired hopping frequency. Vu et al. tested this strategy on a hopping robot with a variable parallel elastic element on the knee joint, but found that the optimal hopping frequency was consistently higher than the resonance frequency [235]. The strong linearizations and other simplifications that are needed to calculate the theoretical resonance frequency could be a cause for this discrepancy.

On the Marco Hopper II, we selected the most compliant spring that would fit in the setup, taking the maximum deflection of the spring into consideration. The selected spring stiffness of 20 N/mm is much higher than the spring stiffness corresponding to an antiresonance frequency of 1.02 Hz, the hopping frequency of our experiments: depending on the way how the problem is simplified, a stiffness of around 8-12 N/mm would be required. It is therefore possible that better results could be obtained with a more compliant spring, although more research (simulations) would be needed to confirm the optimal spring stiffness.

Still, there was another issue with the setup. The linear bearings on the Marco II exhibited a lot of friction. As a consequence, during the downward motion of the hopper, a lot of the potential energy was dissipated. This energy, which could otherwise be stored in the spring, could no longer be used to reduce the motor’s speed requirements. Due to the speed limitations of the motors, the hopper was only able to reach hopping frequen-
cies of around 1.3 Hz. At the experimental hopping frequency of 1 Hz, jumps resembled a squat jump. Literature indicates that, in human squat jumps, the contractile elements (motor units) need to deliver more power, because they need to compensate for the potential energy that could not be stored on the elastic elements [7, 65]. Interestingly, this corresponds exactly to the behavior of the SEDMA on the Marco Hopper II, demonstrating that the hopping behavior generated by the controller actually follows the patterns in biomechanics.

Unfortunately, the inefficient use of the spring that resulted from the setup’s issues did not allow us to draw conclusions about the energy-efficiency of the SEDMA, which was one of the project’s initial goals. With more energy-efficient bearings and, perhaps, an adapted choice of spring stiffness, the spring could effectively be used to reduce the energy consumption, and the true energetic benefits of a combination of springs and redundancy could be evaluated. With the current setup, this was impossible. Nevertheless, the disappointing results once again illustrate the importance of matching the actuator to the load. Moreover, they demonstrate how difficult it can be to achieve this with a complex actuator, whose components not only interact with the load, but also with each other.

5.8 Conclusion

In this chapter, we have demonstrated that the principles of series elastic actuation and redundancy can be combined in a single actuator. We proposed a control framework to tackle the complex control problem of dealing with the separated dynamics of the load and the motor, as well as distributing the power requirements over the redundant degrees of freedom. The control framework was validated on a physical test setup, where a series elastic dual-motor actuator was used to actuate the knee joint of a robotic hopper. The results show that the controller is able to generate a repetitive hopping behavior while distributing the power over the two motors. Considering that hopping is a subtask of legged locomotion, this work paves the way for series elastic redundant actuation of legged robots, prostheses and exoskeletons.
Chapter 6

Discussion and future work

6.1 Discussion of research questions

Below, we give a summary of the main research questions and their answers.

How does the drivetrain affect the energy consumption of actuators in robotics?

Actuators in robotics deliver rapidly changing torques at varying speeds, and their operating points often lie all four motor quadrants. As a result, the modeling of a robot’s energy consumption becomes a very complicated task. Gearbox efficiency and Coulomb friction, for example, lead to nonlinearities in the motor torque and power consumption. Motor efficiency also varies over the motor’s operating range. Most of the time, it is lower than the maximum efficiency specified in the catalog. Moreover, the inertia of the drivetrain often causes a motor’s torque requirement to be higher than the load torque itself. And finally, drive electronics can consume a surprisingly high amount of energy. For all these reasons, which were explained in more detail in chapter 2, the electrical energy consumption of an actuator is typically much higher than its mechanical energy output. In conclusion, the drivetrain often has a very strong impact on an actuator’s energy consumption, and therefore deserves a lot of attention during the design of energy-efficient actuators.

What is the role of negative power in robotics?

Unlike in many other applications, actuators in robotics experience both negative and positive power flows. Unfortunately, strong misconceptions exist about negative power flows. An often-used but false claim is that “the absorption of energy requires an (equal) amount of energy from the motor”. This is fundamentally untrue: electric motors can be used as generators, and examples exist in robotics where energy is effectively regenerated.
This is confirmed by our findings in chapter 2, where we managed to regenerate a small amount of negative energy from the motion of a pendulum.

If the intention is to store this energy in a battery, a 4-quadrant controller is needed; otherwise, this energy will be dissipated in a braking resistor. Furthermore, an energy-efficient actuator design is very important to maximize the amount of energy that can be regenerated. A DC motor, for example, will not be able to regenerate energy at high torques and low speeds, because the Joule losses are higher than the power supplied to the motor by the load. As a consequence of such losses, the amount of energy that can be regenerated by robots is often small and does not justify the cost of additional electronics. This is a strong motivation for the use of (mechanical) energy buffers such as springs, which allow recycling this energy in a more efficient way.

**How complex must the model of the actuated system be in order to provide a reasonable estimate of the power and energy consumption?**

The energy consumption of an actuator can be found through integration of its (electrical) power. Following our discussion on the role of negative power, it is incorrect to calculate the energy consumption with the integral of the absolute value of power – something which is not uncommon in robotics. The difficulty in obtaining a good estimate of energy consumption lies in capturing all the losses of the system. Any unmodeled loss will lead to an underestimation of the actual energy consumption.

The direction of power flow is a very important consideration in robotics, and this should be reflected in the model. Nonlinearities in the power profile can only be predicted with Coulomb friction and, more importantly, a nonlinear gearbox efficiency function. Gearbox and motor inertia also play a very important role. When accelerations are high and torques are low, the inertia of the drivetrain may, in fact, dominate the power profile. It must be noted that the inertia itself does not cause additional losses. It does, however, have an influence on the current flowing through the motor and, consequently, on the Joule losses, which are often the most important source of losses in an actuator.

**Can the efficiency of actuators be improved by applying motor and gearbox models in model-based optimizations?**

One of the main contributions of this work was the assessment of motor and gearbox losses in various actuator types. As explained above, these losses have a strong impact on the electrical energy consumption of an actuator. Based on this observation, we hypothesized that model-based optimizations yield better results when electrical energy consumption is applied as a cost function, instead of cost functions based on mechanical power and motor torque.

Interestingly, a cost function based on the square of mechanical power was shown to result in a similar spring configuration for an active ankle prosthesis with elastic elements (section 3.5). Additionally, in our tests on the Marco Hopper, we found that the optimal speed distribution of the SEDMA, obtained by applying optimal control with a cost
function based on the square of the motor torque, was similar to the one obtained with electrical energy as a cost function (section 5.5). These results indicate that simple cost functions may suffice if the power consumption is dominated by the mechanical output power (ankle prosthesis) or by the Joule losses of the motor (SEDMA).

In other cases, the better option is to take motor and gearbox losses into account. In section 3.3.2, for example, we observed that the optimal stiffness of the PEA in terms of mechanical energy consumption does not coincide with the optimum in terms of electrical energy consumption. A suboptimal spring stiffness can easily lead to a much lower efficiency, and therefore, a model of the electrical power consumption is definitely recommendable for the optimization of parallel elastic actuators.

**How can elastic elements contribute to energy efficiency?**

Elastic elements have the capability of reducing the speed and torque requirements of an actuator by exchanging power directly with the load. Practically, springs can be arranged in series or parallel with the drivetrain. If the resulting elastic actuator delivers a torque that matches the torque-angle characteristic of the spring, a series arrangement decreases the motor speed, while a parallel spring decreases the torque required from the motor. In dynamic motions, the speed reduction enjoyed by a Series Elastic Actuator is combined with a slight decrease in torque, resulting in a very high energy efficiency for this type of actuator. On the other hand, Parallel Elastic Actuators can be used to compensate for static torques. Considering that the torque-related Joule losses are often the most important source of dissipation in an actuator, parallel elastic elements can provide huge energetic benefits for many actuator designs. It is, however, important to tune the design of the elastic element to the imposed load and motion, and to select a matching motor and gearbox. Failure to do so may result in strongly reduced actuator bandwidth, range of motion or torque output.

**Can redundancy be exploited to make actuators more energy-efficient?**

Redundant actuators allow dividing the required output power over different motors so that, on average, the motors are used more efficiently. Furthermore, if its input drivetrains are equipped with locking mechanisms, a motor can be shut down during operation when it is not needed. The results from chapter 4 confirm that such a redundant actuator can be designed to deliver constant torques and speeds more efficiently than a traditional motor with gear reducer. In dynamic applications, kinematically redundant actuators were shown to make better use of the motors’ acceleration capability by dividing the acceleration requirement over the motors. Thanks to this feature, an actuator sized for high torques can still deliver high accelerations. However, the higher number of drivetrain components and the redundant power flow paths also complicate the design and control of redundant actuators. This complexity needs to be dealt with in order to get the most out of the concept in terms of cost, weight, volume, energy efficiency and bandwidth.
Can series elasticity be combined with redundancy in legged robots?

In chapter 5, we presented experiments where a series elastic dual-motor actuator was implemented on a hopping robot. The control framework that was proposed in this chapter was successful in generating repetitive closed-loop hopping. The main challenge in combining elasticity with redundancy is to manage the interaction between the load, spring and the motors. The energy efficiency of the series elastic dual-motor actuator strongly depends on the selected spring and the effectiveness of the control allocation algorithm – but getting these aspects right is not straightforward. In conclusion, our results prove that series elasticity and redundancy can be combined, but they also demonstrate the difficulty in achieving energy efficiency with such an actuator.

6.2 Future work

While the extensive analyses presented in this dissertation have already led to many interesting insights regarding elastic and redundant actuators, several topics still deserve further attention. Below, we discuss several possible extensions to this work.

6.2.1 Complexity of the loss models

The loss models that were introduced in chapter 2 and employed throughout this dissertation are simplified representations of reality. A few examples:

- Friction was represented as a combination of viscous friction and Coulomb friction. This model does not account for the dynamics of friction and neglects the Stribeck effect.

- In reality, gearbox efficiency is both speed- and, especially, load-dependent. Nevertheless, in this work, gearbox losses were assumed to be proportional to the output power of the gearbox. This assumption may lead to an overestimation of the actuator’s efficiency at low powers, as gearbox efficiency tends to drop with decreasing torque [49].

- The motor losses were modeled as a combination of Joule loss and viscous friction, sometimes with an additional Coulomb friction term. While this model provide a good description of the foremost loss mechanisms (Joule loss, eddy current and remagnetization loss), improvements are possible. Heating, for example, can have a strong impact on the winding resistance and, consequently, on the resulting Joule losses. Furthermore, the torque constant of the motor may drop due to demagnetization of the permanent magnets [80]. The efficiency of gearboxes also exhibits temperature-dependence [48].

The use of simplified models is motivated by the desire of getting a good overall view of the loss distribution in an actuator. More elaborate models exist, but require more parameters, which are not specified for most commercial components and therefore need to be
determined empirically for each specific component. This can take a lot of time and, more importantly, will not necessarily lead to a better insight into the loss distribution, as different components typically share similar speed- and torque-dependent loss mechanisms. This justifies the simplified models used in this thesis.

6.2.2 Combination of series and parallel elasticity

Most of our analysis in chapter 3 focused on designs with a single elastic element, either in series or in parallel. Combined series and parallel elasticity was briefly addressed in the discussion on elastic elements for an ankle prosthesis (section 3.5). The conclusion from this theoretical discussion was that, in terms of energy consumption, an actuator with series and parallel elastic elements does not perform better than a simple parallel elastic actuator. There is very little fundamental research on combined parallel and series elasticity – a rare example can be found in [19] – and practical examples of such actuators are also scarce. They have been implemented in active prostheses [12, 78, 108], robotic hands [183] and warehouse robots [135], but these examples cover most of the state of the art. For a good assessment of the advantages and disadvantages of combined series and parallel elasticity, more fundamental research is needed, and more research prototypes need to be built and tested for a wide range of applications.

6.2.3 Other forms of redundancy

In chapter 4, we explored the potential of redundancy by studying a specific actuator: a kinematically redundant actuator with a single-stage planetary differential as linking mechanism. To generalize the results of this chapter, more designs should be studied. Multi-stage or compound differentials, for example, open up new possibilities to decrease the energy consumption even further. Another option would be to exploit static redundancy instead of kinematic redundancy. While, in dynamic applications, kinematically redundant actuators have several benefits (section 4.5) that do not apply to statically redundant actuators, the latter may have specific advantages of their own. The evaluation of statically redundant actuators and other topologies, such as motors combined with a CVT or IVT, can therefore be seen as another interesting research track.

6.2.4 Combining elasticity and redundancy

In chapter 5, we presented the “Series Elastic Dual-Motor Actuator” which combined series elasticity with kinematic redundancy. The main goal of this chapter was to demonstrate that such an actuator can be controlled on an actual robotic system. Still, many research options are left open regarding the combination of elasticity and redundancy. Elastic elements could, for example, be used in parallel to the redundant actuator. They could also be integrated into the redundant actuator design, e.g. by combining an elastic actuator with a rigid actuator or by connecting one of the components of a multi-stage differential to a compliant element. Several examples can already be found in literature:
springs have been implemented in statically redundant actuators [134], kinematically redundant actuators [151], variable stiffness actuators [213, 219] and with CVTs [124, 148]. In short, there are many ways to combine elasticity and redundancy, and a study of different concepts could very well lead to some new interesting actuation concepts.

### 6.2.5 Locking mechanisms

Locking mechanisms and clutches are receiving an increased amount of attention in the robotics community. They are especially known to be a powerful tool for controlling the energy flows between a spring and the load [175], which is why they have appeared in several elastic actuator designs [134, 190, 174], often with the goal of (dis)engaging a parallel spring [51, 92, 130].

In this dissertation, locking mechanisms – in the form of holding brakes – were a part of the dual-motor actuator that was presented in chapter 4. Our results showed that the holding brakes were, indeed, very useful for reducing the energy consumption when the load was static (section 4.4). Interestingly, with dynamic loads, the holding brakes were left unused in our optimizations, indicating that they do not contribute to a higher energy efficiency (section 4.5). The question remains whether this conclusion can be extended to other mechanisms and other loads, especially if springs are implemented in the redundant actuator. To gain an understanding of how locking mechanisms should be used in redundant actuators, and in which cases they can lead to energy reduction in dynamic applications, more fundamental research is needed.

### 6.2.6 Control of redundant actuators

The challenging problem of controlling redundant actuators in highly dynamic systems was explained and illustrated in practice in chapter 5, where we designed a controller for a robotic hopper. While the controller succeeded in achieving stable hopping – the task of the high-level control – the simple control allocator proposed in this chapter did not entirely meet the secondary objective of distributing the power over the motors in the most energy-efficient way. Since the benefit of having multiple motors is determined entirely by the ability to achieve a more efficient power distribution, the development of energy-efficient control allocation schemes can be considered a research priority for the development of redundant actuation concepts.

### 6.2.7 Extension to multiple degrees of freedom

In this work, we assumed that the torques and speeds that should be delivered by the actuator are known. This is, for example, the case for actuated prosthetics and humanoid robots, where the aim is to imitate the motion of a human leg. In many applications, however, a robot’s task is simply to perform a motion from point a to point b in a certain time, but the exact path to follow is undefined. This opens up the possibility of choosing the
motion of the robot and its individual motors in such a way that the overall energy consumption is as low as possible. Energy-efficient path planning or motion planning, as this is called, has been the subject of several works throughout the years [234, 70, 243, 142], and can be considered as the first step towards energy-efficient actuation. Despite its importance, motion planning was not considered within the scope of this work. Nevertheless, considerable benefits can be obtained by coupling motion planning to actuator design, especially when elastic elements are involved. Motion planning can then aim at generating joint motions that excite the actuators’ natural dynamics, which can be tuned by selecting the correct spring stiffnesses. Several authors have successfully applied such an approach to robotic arms [218, 153, 137] and humanoids [216, 196, 246] with elastic actuators.

Furthermore, robots typically consist of multiple motors, which can make a robot redundant with respect to a certain task\(^1\). By carefully planning the trajectories of the joints, a kinematically redundant manipulator, just like a redundant actuator, can use its motors in a more energy-efficient way. Moreover, the redundant manipulator only requires one additional motor to obtain a lower energy reduction for all motors. Conversely, a redundant actuator relies on an additional motor to reduce the energy consumption of only the specific joint it is actuating. This thought raises several important questions. Is redundancy on a joint-level actually more beneficial than redundancy on a robot-level? Could both be combined in a single robot? And if so, what is the optimal configuration, and how does it depend on the robot’s degrees of freedom and the design of the redundant actuator? These questions could not be answered within the scope of this work, but are very relevant for the assessment of redundant actuation concepts.

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\(^1\)In robotics, actuated joints are referred to as a robot’s "degrees of freedom". If the number of degrees of freedom of a robot exceeds the number of degrees of freedom needed for the task, the robot is said to be redundant.
In this work, we have explored different ways of increasing an actuator’s efficiency in the field of robotics. Unlike many other works, the whole drivetrain was taken into account to ensure that the electrical energy consumption is minimized. We focused on two actuation paradigms: the introduction of springs and the creation of redundant degrees of freedom. The two proposed solutions have one obvious downside: they consist of adding components to a drivetrain, each of which bring along additional cost, weight, volume, energy losses and design complexity. Fortunately, the increased complexity can be turned into an advantage, because it opens up new possibilities for using the components in a more effective way.

Generally speaking, compliant and redundant actuators owe their energy efficiency to the manipulation of power flows. Springs enable temporary energy storage in an energy buffer, while redundant actuators rely on the creation of new power flow paths to other active elements. Both concepts open up the possibility of diverting power flows away from components which are less efficient for a specific part of the task. Practically, this will result in a decrease of speed or torque in one or more components of the actuator – possibly at the cost of increasing it in another component. Considering that each component has its own characteristic loss mechanisms, which make it better at either delivering high torques or working at high speeds, the challenge is to make all these components work together in the most optimal way. A well-designed compliant or redundant actuator will thus harmonize the components, playing each on its strengths while avoiding its weaknesses. The result is an actuator in which the components are smaller, lighter and more energy-efficient – but also more numerous.

The obvious question is whether the benefits gained from the introduced components actually outweigh the additional costs. Based on the energetic analyses presented in this thesis, the answer is yes. In our experiments on the pendulum setup, for example, a Series Elastic Actuator (SEA) yielded reductions in energy consumption of up to 78% compared to a conventional motor with gearbox. However, this specific case study presented the SEA with the ideal circumstances to exploit its full potential: a load which can be com-
pensated perfectly by the actuator’s well-chosen spring. If the SEA was forced to deliver a static load, its performance dropped considerably, to the point where a parallel spring arrangement became the more energy-efficient choice. This example highlights the strong connection between elastic actuators and the loads they are facing.

Redundant actuators present a possibly more versatile alternative, because their operating range can be shaped by design, independent of the load. The typical L-shaped operating range of kinematically redundant actuators is particularly interesting. Their capability of delivering very high torques at low speeds while being able to reach very high speeds at low torques corresponds exactly to the requirements of many robotic systems. Usually, these requirements are met by large motors, which are underpowered for most of the task and struggle with their own reflected inertia. Kinematically redundant actuators, being a much better match for such tasks, can be much more efficient than these large motors.

To summarize, compliant and redundant actuators obtain their energy efficiency by closely matching the design to the application. A loss of versatility is an inevitable side-effect, which must be taken into consideration when such actuators are selected for a certain application. Nevertheless, with the right match-up between the task and the actuator, remarkable energetic benefits can be achieved. For this reason, we believe that compliant elements and redundant actuation structures will be an integral part of tomorrow’s energy-efficient actuators.
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