A conceptual sediment transport simulator based on the particle size distribution

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A conceptual sediment transport simulator based on the particle size distribution

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ABSTRACT: Sediment transport is a very important process affecting the transport and retention of pollutants that easily adsorb to suspended sediment. The fine sediment plays a dominant role and needs to be properly represented. However, the conventional conceptual sediment transport models do not account for distributions of particle sizes and the differences in behaviour of the different fractions. Our paper presents an analytic solution for this problem using the log-normal probability density function to represent the particle size distribution of the sediment. Sedimentation and resuspension processes are calculated for the particle size distributions by using Hjulstrom diagram for incipient motion and deposition and integrating with Velikanov’s energy of sediment carrying capacity. An application on the Zenne River (Belgium) shows that our conceptual model provides a better representation of the high concentrations and the range of concentrations, as compared to a conventional conceptual sediment transport modelling.

1 INTRODUCTION

Sediment transport plays an important role in characterizing the water quality behaviour of rivers and sewers. Several pollutants are transported attached to the suspended sediments (Jamieson et al., 2005; Lewis et al., 2013; Miller et al., 1982; Ouattara et al., 2011; Viney et al., 2000). In this regard, sedimentation and resuspension processes control the transport and retention of contaminants that show affinity to sediments (Ani et al., 2011; Corbett, 2010; Crabbill et al., 1999; Dhi, 2007; Ongley, 1996). Therefore, estimating the amount of sediment in the water is essential to model the dynamics of attached pollutants.

The sediment-bound pollutant transport is dominated by the suspended sediments (Ongley, 1996; Sartor, 1972). The latter indeed contain much more of the fine particles (as compared to the bed load) as the process of surface erosion tends to be selective towards smaller particles (Asselman, 2000; Kumar & Rastogi, 1987). Therefore, it is important to distinguish the fine sediments from the total suspended sediment (Church & Krishnappan, 1998).

Detailed, hydrodynamic, sediment transport models are able to represent the sediment particles in to different size classes and perform the computation on each fraction (Dhi, 2007; Shrestha, 2013; Young et al., 1989). This approach increases the computation time and cost because the update of each sediment size class has to be maintained at every computation time step and every reach.

On the other hand, surrogate models are ideal for fast computations but they –usually- do not make a distinction between the proportions of the different particle sizes of sediments. Instead, they are based on the stream carrying capacity to determine the resuspension or deposition processes (Viney et al., 2000; Williams, 1980). Consequently, they lack essential detail for simulating the transport of pollutants attached to the sediments.

In order to make maximum use of the computational efficiency and simulate the sediment-attached contaminants, we developed a surrogate sediment transport model that represents the particle size distribution by probability density functions and account for the deposition and resuspension phenomena. Our model calculates the sediment mass and the particle size distribution —i.e. the mean and standard deviation of the distribution—, using a simplified concept for the transport and resuspension processes. We determined the critical particle size of deposition and resuspension separately based on the Hjulström diagram. The
critical conditions of resuspension and deposition of the new conceptual model are applied along with Velikanov’s energy, to calculate the sediment carrying capacity (Velikanov, 1954).

The performance of the new model was tested on the case study of the Zenne river in Belgium. It was compared with a simple continuous stirred tank reactors (CSTR)-based sediment model, combined with Velikanov’s energy.

2 METHODOLOGY

2.1 The study area

A reach of 41 km of the Zenne River—a lowland river in Belgium—was used as a case study for the evaluation of a new conceptual sediment transport simulator. All the tributary rivers and the inlet from the upper reach were considered as point source boundaries. As the river is subject to tidal backwater, the station just upstream of the tidal influence (at Eppegem) was taken as the most downstream point of the model.

Based on the cumulative frequency curves of the sediment particles (Shrestha, 2013), we identified four categories of sediment sources with specific characteristics. Studies show that grain size distributions vary seasonally (Buscombe et al., 2014). No data was available regarding the temporal variability of grain size distributions for the Zenne stations. Therefore, we assumed the sediment characteristics do not change with time.

Sediment concentrations are measured measured by VMM (Flemish environment agency) on an average time interval of one month.

2.2 Theories of sediment transport

Bertrand-Krajewski (2006) concluded in his review paper that water quality models often ignore the cohesive nature of sediments for simplicity. Guo et al. (2012), however, showed that Stokes’ law is applicable only to the sedimentation rate of sand particles and not to cohesive sediment. Bertrand-Krajewski (2006) stated that fine sediments are cohesive especially in the presence of bacteria released from effluents of waste water treatment plants and CSOs that glue sediment particles. The cohesive properties of sediments become dominant when the clay fraction is larger than 10% (van Rijn, 1993). The measurement data of the particle size distribution of the sediments in the river Zenne (Shrestha, 2013), however, show that the clay fraction is generally below 10%. Moreover, analysis of historical hydraulic simulation results by Shrestha (2013), show that turbulent flow characteristics prevail in the Zenne river, thus it hampers both floc formation and growth in suspension mode (Church & Krishnapan, 1998). Therefore, we adopted a non-cohesive theory of sediment resuspension. The well-known Shields curve (Shields, 1936) used in sediment transport relates the dimensionless shear stress to the particle Reynolds number (Equation 2.1).

\[
\vartheta = \frac{\tau}{(\rho_s - \rho)gd} = \frac{u_\ast^2 \rho}{(\rho_s - \rho)gd}, \quad Re = \frac{u_\ast}{\nu} = \sqrt{\frac{gRS}{\nu}} \tag{2.1}
\]

where \( \vartheta \) is the dimensionless shear stress; \( Re \) is the particle Reynolds number; \( \tau \) is the bed shear stress [N/m²]; \( \rho_s \) and \( \rho \) are particle density and water density [kg/m³], respectively; \( u_\ast \) is the shear velocity [m/s]; \( \nu \) is the kinematic viscosity of water [m²/s]; \( g \) is acceleration due to gravity [m²/s²]; \( R \) is the hydraulic radius [m]; \( S \) is the reach slope [m/m].

The implicit nature of Shields criteria makes Shields diagram difficult to interpret (Paphitis, 2001) and hence complicates its implementation in conceptual models. A more recent algebraic (Equation 2.2) developed by Soulsby & Whitehouse (1997) is a good representation of Shields criteria (Miedema, 2010; Shrestha, 2013). However, it is only applicable to non-cohesive sediments (Miedema, 2010). Besides, it requires estimating the shear velocity, which is not a trivial task in conceptual models and hence, needs iteration to determine the critical particle size given. Many other empirical approaches have been used to estimate the incipient condition of sediment motion. The reader is referred to Beheshti & Atae-Ashtiani (2008) regarding the empirical s in use. They share the same limitation as the of Soulsby for application in simple conceptual methods.

Hjulström (1939) developed the famous (Southard, 2006) Hjulstrom diagram in the same period as the work of Shields. Hjulstrom diagram relates the critical flow velocity for incipient motion to the particle size. Due to the fact that it is dimensional and can be easily implemented in simple models, the incipient condition in our conceptual model depends on the Hjulstrom diagram. Miedema (2010) has fitted an empirical to the Hjulstrom diagram. The new conceptual model needs to determine the critical diameter of incipient motion corresponding to a given flow velocity thus the Miedema’s empirical should be solved in the reverse direction and this requires iteration. To avoid this complication we fitted a separate empirical (Equation 2.2) to the resuspension curve of the Hjulstrom diagram. For similar reason as the resuspension, we fitted another empirical (Equation 2.3) to the deposition curve of Hjulstrom diagram. This enabled easy programming in our surrogate model and a straightforward evaluation of critical diameters, only based on the mean flow velocity, without having to directly quantify the critical shear stress. The mean flow velocity used in this research is simulated using the Muskingum
routing method of SWAT (Soil and Water Assessment Tool) (Arnold et al., 1995).

\[
\begin{align*}
D_{s,f} &= 0.12206U^{-2.02429} \\
D_{s,c} &= 231.9883U^{1.607976}
\end{align*}
\] (2.2)

\[
\begin{align*}
D_{s,\text{deposit1}} &= 10^\left(0.0033\log(10(U)) - 2.5245\right) \\
D_{s,\text{deposit2}} &= 1750 \left(1 - 0.7377 \sqrt{1.9415 - U}\right)
\end{align*}
\] (2.3)

where \( U \) is the mean flow velocity \([\text{m/s}]\), \( D_{s,f} \) and \( D_{s,c} \) are the critical dimensionless grain diameters for the resuspension of the fine and coarse part of the sediment, respectively and \( D_{s,\text{deposit1}} \) and \( D_{s,\text{deposit2}} \) are the critical dimensionless grain diameters for the deposition corresponding to mean flow velocities less than 0.4 m/s and between 0.4 m/s and 1.94 m/s, respectively. The two empirical s are applicable only for dimensionless grain sizes less than 1560. The particles sizes in this study lie in this range.

The critical grain size of resuspension or deposition is determined as a function of the critical dimensionless grain diameter [Equation (2.4)]

\[
d_{ct} = D_\alpha \left(\frac{\nu^2}{g(s-1)}\right)^{1/3}
\] (2.4)

Where \( d_{ct} \) is the critical diameter of resuspension or deposition; \( s \) is the specific gravity of the sediment, \( \nu \) is the kinematic viscosity of water \([\text{m}^2/\text{s}]\); \( g \) is acceleration due to gravity \([\text{m/s}^2]\) and \( D^* \) is the dimensionless grain size.

### 2.3 The particle size distribution

Sediment samples collected during a storm events as well as under dry flow conditions in rivers usually exhibit a lognormal distribution (Abuodha, 2003; Agrawal et al., 2012; Bouchez et al., 2011).

For each of the four categories of the sediment particle size distributions used as a boundary, we fitted a lognormal probability density function by calibrating the mean and standard deviation of the distribution. The log-normal distributions were transformed to normal distributions because the latter has more formulas available for computing the distribution parameters during the mixing of different samples and truncated distributions. The log-normal distribution of the particle sizes was represented using the probability density function shown in Equation (2.5).

\[
p(d) = f_d(d, \mu, \sigma) = \frac{1}{d\sigma \sqrt{2\pi}} \exp\left(-\frac{(\ln(d) - \mu)^2}{2\sigma^2}\right)
\] (2.5)

where \( d \) is the particle diameter and it is always greater than zero*\([10^{-5}\text{m}]\); \( \mu \) is the mean of the log-transformed particle diameters; and \( \sigma \) is the standard deviation of the log-transformed particle diameters.

The mean diameter and standard deviation of the sediment particles is updated during each time step to account for mixing and deposition-resuspension processes. The parameters of two mixed normally distributed samples of suspended sediment samples were evaluated using Equation (2.6) & Equation (2.7). A mixture distribution is treated as a bimodal distribution only if the difference between the mean of the two normal distributions is greater than the sum of the two standard deviations (Schilling et al., 2002). We tested if the combinations of the particle size distributions from different boundaries satisfy the condition for unimodality. The masses of the two sediment samples in a mixture represent their respective weights when applying the s used for computing the mean size and standard deviation of the mixture distribution.

\[
\mu_{\text{mix}} = p_a \mu_a + p_b \mu_b
\] (2.6)

\[
\delta_{\text{mix}}^2 = p_a \delta_a^2 + p_b \delta_b^2 + p_a p_b (\mu_a - \mu_b)^2
\] (2.7)

where \( \mu_{\text{mix}} \) is mean of the mixed samples; \( \delta_{\text{mix}}^2 \) is the variance of the mixed samples; \( \mu_a \) & \( \mu_b \) are variances of the two log-transformed sediment particle size distributions and \( p_a \) and \( p_b \) are the weights of the two samples. \( p_a + p_b = 1 \).

For the sake of simplicity, the population mean and standard deviation were assumed to change according to the sample mean and standard deviation.

After a deposition, the upper tail of the distribution extending up to the critical diameter of settlement was used to compute the mass of the settled sediments. In order to determine the statistical parameters of the mixture of normal distributions, it was important to quantify the mean and standard deviation of the truncated normal distribution using the statistical formulae published by Barr & Sherrill (1999) and Vernic et al. (2009). Accordingly, the mean and standard deviation of the upper truncated normal distribution were evaluated using Equation (2.8) & Equation (2.9), respectively. The mean and variance of the lower truncated normal distribution corresponding to the sediment settling to the bed were evaluated using Equation (2.10) & Equation (2.11), respectively.

\[
E(d|d < d_{cr}) = \mu - \sigma \phi(\phi^{-1}(\mu) - \mu) - \Phi(\phi^{-1}(\mu) - \mu)
\] (2.8)

\[
\text{Var}(d|d < d_{cr}) = \frac{1}{2} \left[1 + \Phi(\phi^{-1}(\mu) - \mu) - \frac{1}{1 - \Phi(\phi^{-1}(\mu) - \mu)}\right]
\] (2.9)

\[
E(d|d > d_{cr}) = \mu + \phi(\phi^{-1}(\mu))
\] (2.10)

\[
\text{Var}(d|d > d_{cr}) = \frac{1}{\phi(\phi^{-1}(\mu))}
\] (2.11)

Where \( d_{cr} \) is the critical diameter*\([10^{-5}\text{m}]\) of deposition; \( \beta = (\ln(d_{cr}) - \mu)/\delta \); \( \lambda(\beta) = \phi(\beta)/(1 - \Phi(\beta)) \);
\[ \delta(\beta) = \lambda(\beta)/[\lambda(\beta) - \beta]; \ E(d|d < d_{cr}) \] is the mean of log transformed particle sizes of the sediment remaining in suspension; \[ \text{Var}(d|d < d_{cr}) \] is the variance of log transformed particle sizes of the sediment remaining in suspension; \[ E(d|d > d_{cr}) \] is the mean of log transformed particle sizes of the settling sediment; \[ \text{Var}(d|d > d_{cr}) \] is the variance of log transformed particle sizes of the settling sediment.

We calculated the fraction of sediment mass in the channel settling to the bottom using Equation (2.12).

\[
Frac_{sett} = 1 - \text{erfc} \left( \frac{-\ln(d_{cr}) - \mu}{\delta \sqrt{2}} \right)
\]

(2.12)

Where \( Frac_{sett} \) is the settling fraction of sediment mass in the channel; \( \text{erfc} \) is the complementary error function.

For a known settling fraction of sediment mass in the channel, we determined the corresponding critical diameter of the sediment using Equation (2.13).

\[
d_{cr} = \exp \left[ \mu + \delta \sqrt{2} (\text{erfinv}[1 - 2(1 - Frac_{sett})]) \right]
\]

(2.13)

Where \( \text{erfinv} \) is the inverse of error function.

Resuspension was assumed to take place only if the settled sediment is available in the bottom sediment reservoir. Therefore, no new detachment of sediments from the banks or bottom of the river was considered.

The mixing of distributions and updating of the moments of mixed distribution was also performed for the bottom reservoir. Similar to the sediment in the suspension, the particle sizes of the bottom sediment reservoir were also assumed to follow lognormal distribution. We determined the re-suspending fraction of the bottom sediment mass using Equation (2.14).

\[
Frac_{resus} = 0.5 * \text{erfc} \left( \frac{-\ln(d_{cr, resus}) - \mu}{\delta \sqrt{2}} \right)
\]

(2.14)

Where \( Frac_{resus} \) is re-suspending fraction of the sediment mass from the bottom sediment reservoir; \( d_{cr, resus} \) is the critical particle diameter for resuspension.

2.4 The sediment carrying capacity

The sediment transport capacity of a stream flow is the steady flux of sediments that the flow can transport (Prosser & Rustomji, 2000). In our model, the sediment transport capacity of a given flow is imposed by using Velikanov’s energy (Velikanov, 1954), as implemented by Zug et al. (1998) and used by Shrestha (2013) [Equation (2.15)].

\[
\begin{align*}
CT_{\text{min}} &= \eta_1 \frac{s \rho_w U}{s - 1} \omega_s I \\
CT_{\text{max}} &= \eta_2 \frac{s \rho_w U}{s - 1} \omega_s I
\end{align*}
\]

(2.15)

where \( CT_{\text{min}} \) and \( CT_{\text{max}} \) are the minimum and maximum sediment concentrations, respectively that the stream can carry; \( \eta_1 \) and \( \eta_2 \) are the critical sedimentation and erosion efficiency coefficients, respectively; \( s \) is the specific gravity of the sediment; \( \rho_w \) is the density of water, \( U \) is the mean flow velocity, \( \omega_s \) is the settling velocity and \( I \) is the channel slope.

At the beginning of each calculation time step, the total mass of suspended sediment in each river reach is updated, by representing a reach as a continuously stirred reservoir. The latter mass is then compared to the sediment carrying capacity of the stream.

If the sediment in suspension is less than the minimum carrying capacity, resuspension takes place, as governed by the re-entrainment criteria and the sediment mass available in the bottom reservoir. If the sediment in the suspension is less than the maximum carrying capacity but greater than the minimum carrying capacity, the critical condition of deposition is checked based on the critical diameter of settlement. If the sediment mass exceeds the maximum carrying capacity, the coarsest particle sizes are deposited.

2.5 The model comparison

The new method, based on the particle size distribution and further called PSD method, was compared to the widely adopted method of sediment transport that assumes a complete mixing and calculates the suspended sediment mass using only the sediment carrying capacity (e.g. Neitsch et al., 2009; Viney & Sivapalan, 1999; Williams, 1980). The latter method is further called the SCC method. In the SCC model, a linear reservoir concept was used for complete mixing and transport of sediment and the Velikanov’s energy was used for stream power (transport capacity). Unlike the PSD method, the SCC method does not impose the critical diameter condition for deposition and resuspension. For a fair comparison, resuspension in both models was enabled only when there was sufficient deposited sediment mass in the bottom reservoir.

3 RESULTS AND DISCUSSION

Based on experimental data for the Zenne basin during dry weather periods (Shrestha, 2013), log-normal distribution functions have been fitted to the different boundaries of the river system (Table 3.1).

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Location parameter</th>
<th>Shape parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSOs</td>
<td>1.75</td>
<td>1.08</td>
</tr>
<tr>
<td>WWTPs</td>
<td>2.26</td>
<td>0.69</td>
</tr>
<tr>
<td>Tributaries</td>
<td>1.00</td>
<td>1.80</td>
</tr>
<tr>
<td>Canal</td>
<td>0.20</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Table 1. Mean and standard deviations of the log-transformed normal distributions of particle sizes (*10^-5 m).
As long as the difference between the mean values of two normal distributions is not greater than the sum of the respective standard deviations, the mixture distribution would be considered unimodal (Schilling et al., 2002). In this regard, only the difference between the mean of the log-transformed normal distribution of WWTPs (waste water treatment plants) and the canal is slightly greater than the sum of the respective standard deviations (Table 3.1). Therefore, a mixture of sediment between any two of the four boundaries was assumed unimodal.

The two empirical equations we fitted to the Hjulstrom diagram reproduces the deposition and resuspension curves (Fig. 1). A comparison of the simulated sediment concentrations by the new method and the SCC approach, after comparable calibration efforts, revealed that both methods simulate comparable low concentrations (Fig. 2). Both methods seem to slightly overestimate the low concentrations compared to the VMM observations.

The PSD method simulated much higher high sediment concentrations than the SCC method. Before making any judgement of which method performed better, it is important to assess the reliability of the observed data.

The observed peak concentrations from the VMM are significantly lower as compared to observations with more recent data of much higher temporal resolution that were obtained from the Flemish Hydraulics Research institute (FHR). The latter institute takes sediment samples every seven hours from representative depths, while the VMM samples were collected with buckets from the water surface. Studies show that sediment concentration increases with depth (Agrawal et al., 2012; Bouchez et al., 2011; van Rijn, 1993).
and hence concentration near the water surface are normally significantly lower than the depth averaged concentration. Based on this fact, we believe that the data from FHR are more realistic and that the actual peak concentrations are thus probably higher than what the VMM data suggest.

Unfortunately, the FHR data do not overlap with the simulation period used in this research. Thus we could not make quantitative evaluation of the model performance against the new data set; instead we used the analogy from comparison of the overlapping dataset of both data sources for the recent period (2011–2014) (Fig. 3). From the comparison, the VMM observed peak sediment concentrations turned out to be significantly underestimated. Given the fact that the period of simulation (2007–2010) and the good quality FHR data (2011–2014) are close to each other and there is no significant difference in the rainfall and discharge of the two periods, it is normal to expect similar distributions of sediment concentrations for both periods. From the analogy, the VMM data used in this research for the earlier period (2007–2010) were also underestimated (Fig. 3). Based on this fact, we considered the overestimation of the simulated peak sediment concentrations by the new simulator compared to the VMM data as an affirmative trend. The quantile comparison of the box plot at Eppegem station shows that there is a good agreement for high concentrations and range of distribution between the PSD simulated concentrations and the FHR data (Fig. 3).

The SCC method on the other hand simulated seriously underestimated high concentrations compared to the FHR data (Figs. 2 & 3). Both models show slight overestimation of the low concentrations compared to the VMM data (Fig. 2). Given the fact that the low concentrations of the VMM data are slightly underestimated compared to the FHR data (Fig. 3), the simulations of the low concentrations by both models is acceptable. The SCC method simulates very narrow range of concentration compared to the concentration distribution of the good quality data of FHR (Fig. 3).

The advantage of the PSD model for simulating realistic high concentrations and similar concentration range as the good quality data of FHR makes it advantageous over the SCC method. The advantage of the PSD method over the SCC method is attributed to the fact that the PSD method updates the median size of the sediments at every time step and thus calculates a realistic settling velocity and hence the sediment carrying capacity of the river. Besides imposing the carrying capacity as a limit, it imposes the critical condition for the incipient motion, and this contributed for a better performance.

4 CONCLUSIONS

Quantifying the fine proportion of the suspended sediment load is essential for investigating the transport of adsorbed pollutants. The usual approaches in hydrodynamic models are computationally expensive and the approaches in conceptual sediment transport models do not account for the distributions of sediment particle sizes and their dynamics (Viney and Sivapalan, 1999; Willems, 2010). We presented an analytic solution using log-normal probability density functions to represent the particle size distributions of sediment. The empirical frequency distributions of the sediment particle sizes of the Zenne River was acceptably represented by log-normal distribution functions. The proposed conceptual model applies the critical condition for the initiation of motion and deposition by means of a simple algebraic representing the Hjulstrom diagram. The eroded or deposited sediment mass is determined based on an analytical evaluation of the area under the probability density function extending from the tail to the critical sediment size of deposition/resuspension.

A comparison of the simulation result from the new method and the conventional CSTR – sediment carrying capacity (SCC) based approach at two measuring stations along the Zenne River shows that the new method performs better than the SCC method when qualitatively compared to the distribution of high sediment concentrations and the range of the observation of the good quality data from FHR. The SCC method indeed systematically underestimated the high concentrations and simulated very narrow concentration
range while performing equally good with the PSD method for the low concentrations.

The PSD method simulated the extreme high concentrations better than the conventional SCC method because it accounts for a dynamic median size of the sediments and thus adapts realistic settling velocity and hence the sediment carrying capacity of the river. It also imposes the critical condition for the initiation of motion, apart from the carrying capacity and this contributed for better performance.

Inspite of the promising performance of the new simulator, the particle size distributions used in this study are limited to samples collected during low and average flow conditions and thus do not represent the storm conditions due to data limitation. Accounting for the seasonal variability of the particle size distributions could be a subject of future researches. In cases where the particle size does not follow a unimodal distribution, the new PSD method might not give realistic simulation results because it is based on the assumption of normal distribution.

We believe that this work enhances the simulation of sediment-adsorbed pollutant transport in simple conceptual water quality models.

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REFERENCES


Figure. The theoretical probability distribution fitted to the empirical cumulative distribution of the particle sizes