Optimizing the Power and Energy Consumption of Powered Prosthetic Ankles with Series and Parallel Elasticity

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Abstract

Several works have shown that series and parallel elasticity can reduce peak power and energy consumption in prosthetic ankles. Setting the right stiffness of the elastic elements is essential to unlock this potential. In this work, we perform a thorough optimization of series and parallel elastic elements for a prosthetic ankle driven by a geared DC motor. Through simulation, we study the effect of drivetrain limitations and compare different mechanical and electrical optimization objectives. The results highlight the importance of selecting a motor and gearbox in an early stage of the design process. Drivetrain inertia causes peaks in electrical power in the swing phase, which would go unnoticed in an optimization based solely on mechanical power. Furthermore, limitations of the drivetrain and controller reduce the range of applicable springs. This has a direct influence on the optimized spring stiffness values, which, as a result, are different from other works. Overall, the results suggest that, by integrating motor selection into the early stages of the design process, designs can be made lighter, more compact and more efficient.

Keywords: Compliant actuators, Series Elasticity, Parallel Elasticity, Energy efficiency, Design optimization

1. Introduction

Conventional prosthetic ankles are simple devices, enabling amputees to perform basic tasks. In recent years, they have evolved to advanced powered prostheses, approximating as closely as possible the biomechanical behavior of a healthy ankle. The aim is to reduce the problems associated with lower limb loss and amputees using the rest of their body to compensate for this loss. Amputees tend to walk slower and require a larger amount of energy to walk

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than able-bodied persons [24]. They also show asymmetric gait and joint pains which can be attributed to compensatory behavior at the sound limb [14]. Joint motion, torque and power generation increase at the healthy ankle of amputees when walking [14], effects which lead to a higher susceptibility to develop osteoarthritis in the contralateral limb [17]. By adding active elements in ankle prostheses and being able to provide the same amount of energy of a healthy limb, the metabolic energy of walking can be reduced, as has been shown by Au et al. [2] for the powered ankle-foot prosthesis. Consequently, many active prostheses have been developed in recent years [4].

When looking at the compensatory strategies however, studies have shown the active prosthesis may not lead to a reduction, on the contrary. Ferris et al. suggest that this might be linked to the increased ankle power [7]. Adding active elements to prostheses also increases the weight, which is known to increase the energy cost and asymmetry of the gait cycle [13]. It is therefore necessary to reduce the weight of the actuation unit, which is possible by using series and parallel elasticity for power amplification [15] and energy storage. If the prosthesis were actuated by a geared motor, it would need to deliver a peak output power of about 250 W for a 75 kg person [25]. This requirement can be reduced by adding series elasticity, as applied in the SPARKy prosthesis [11] and the CYBERLEGs prosthesis [8]. With a combination of series and parallel elasticity, the peak torque can be reduced in addition to the peak power, like in the Powered Ankle Prosthesis [2]. Simulations show that series elastic actuation has the potential to reduce the ankle peak power by almost 80%, while parallel elastic actuation can reduce peak power by 66% and RMS power by 50% [23].

But what is the optimal combination of parallel and series elasticity in an actuated prosthetic ankle? This is a difficult question, because the optimal stiffness of the series and parallel elastic element are strongly linked, not only to each other, but also to the gear ratio [1]. An extensive analysis was performed by Grimmer et al., who optimized the springs for energy consumption and peak power [10] for various walking and running speeds. They found that parallel springs can be combined with series springs to reduce peak powers, but series springs alone are better for energy reduction. Eslamy et al. presented a similar analysis which also included unidirectional parallel springs [5]. They concluded that a configuration with series spring and unidirectional parallel spring can further decrease the energy demand. However, the optimization in these two works was based on mechanical energy consumption and mechanical peak power, measured on the motor shaft. It therefore disregards motor limitations, drivetrain dynamics and electrical losses. As demonstrated in [19], neglecting these effects can lead to suboptimal results in terms of electrical energy consumption. Motor inertia, for example, can have a significant impact on SEAs for prosthetic limbs [11] and, more specifically, on their optimized stiffness [3]. Farah et al. also observed the importance of the drivetrain in their simulations of the open and closed loop response of an elastically actuated prosthesis [6]. An evaluation of the drivetrain characteristics is therefore gradually becoming an integral part of the design process of powered prosthetic feet [22][9].

Following these observations, in this work, we will simultaneously address
the design of motor, gearbox and springs for an actuated prosthesis with series and parallel elasticity. More specifically, we will discuss how the operating range of the selected motor, the gear ratio, the drivetrain inertia and the springs influence one another. We will do this by optimizing the gear ratio and parallel/series elasticity of an actuated prosthesis, taking into account the constraints of the selected motor. Our analysis is not confined to the mechanical domain; we will look at both mechanical and electrical power, and discuss the differences between both. We also contemplate about the potential implications for prosthetic design, suggesting that the performance of the actuator can be improved by a concurrent optimization of springs, motor and gearbox.

2. Methods

In this work, we study the optimal design of Series Elastic Actuator (SEA) equipped with a unidirectional parallel spring. This actuator concept, sketched in Figure 1, is similar MIT’s Powered Ankle-Foot Prosthesis [1] and the one studied by Eslamy et al. [5]. The difference is that, in these works, linear compression springs are used. Consequently, the spring stiffness also depends on the choice of the lever arm. We used torsion springs in order remove this dependency. For the analysis presented in this work, we will work with a 150W RE40 Maxon motor. Our calculations showed that this is the smallest suitable motor from Maxon’s brushed DC motor range. The same motor was used in MIT’s Powered Ankle-Foot prosthesis [2] and ASU’s SPARKy 1 [11].

Using inverse dynamics, of which the equations are presented in section 2.1, we optimize the stiffnesses of the springs, as well as the equilibrium angle of the parallel spring. The optimization is performed with a parameter sweep, such that a global minimum is guaranteed. The series spring stiffness, parallel spring stiffness and equilibrium angle are varied within a range of 50 to 2000 Nm/rad, 0 to 1000 Nm/rad and -25 to 0 degrees, respectively, with steps of 10 Nm/rad, 10 Nm/rad and 0.5 degrees, respectively. Power and energy consumption are calculated from the inverse dynamics for each combination of series spring stiffness, parallel spring stiffnesses and equilibrium angle. We also impose several constraints based on mechanical and electrical limitations on the drivetrain. Any results which do not satisfy these constraints, specified in section 2.2, are discarded. The optimal results with respect to peak power and energy, both mechanical and electrical, are retained.

We assume that the actuator is able to perfectly track the natural trajectory and torque of a sound human ankle. Ankle data was taken from Winter [25], for an able-bodied person of 75 kg walking at a natural cadence (105 steps per minute). The discrete dataset was carefully filtered in order to get smooth first and second order derivatives of the ankle torque and angle.

2.1. Equations

The torque on the gearbox $T_l$ can be calculated as the sum of the required output torque $T$ and the parallel spring torque $T_s$.

$$T_l = T + T_s$$  \hspace{1cm} (1)
2.1 Equations

Figure 1: Schematic of the actuator with angle definitions. The actuator consists of a Series Elastic Actuator (red) and a unidirectional parallel spring (orange), which is engaged when \( \theta \geq \theta_{eq} \). The ankle angle \( \theta \) is zero when the foot is perpendicular to the leg (denoted by a dashed line). A plantarflexed ankle, as depicted in the figure, corresponds to a negative value of \( \theta \). Note that, for clarity of the presentation, the elastic elements are drawn as compression springs in the figure, although they are modeled as torsional springs in the work.

The parallel spring (stiffness \( k_p \)) is unidirectional, i.e. it is only engaged when the ankle reaches a certain equilibrium angle \( \theta_{eq} \). The torque it provides, \( T_s \), can be written as

\[
T_s = \begin{cases} 
  k_p (\theta - \theta_{eq}) & (\theta \geq \theta_{eq}) \\
  0 & (\theta < \theta_{eq})
\end{cases}
\]  

(2)

The ankle angle \( \theta \) and the equilibrium angle \( \theta_{eq} \) are defined in Fig. 1. As mentioned earlier, we assume that the required ankle torque \( T \) and ankle position \( \theta \) perfectly match the biological ankle data from Winter for an able-bodied person of 75 kg [25].

Equation (2), however, creates a problem at \( \theta = \theta_{eq} \), because it is not differentiable at this angle. Instead of this simple and intuitive equation, we therefore use the more numerically favourable

\[
T_s = k_p \left( \theta - \theta_{eq} + \frac{1}{2} s \sqrt{s^2 + (\theta - \theta_{eq})^2} \right)
\]  

(3)

This function is differentiable for any \( \theta \). The factor \( s \) determines how smooth the engagement of the parallel spring will be. The lower this value, the closer Eq. (3) resembles Eq. (2). We chose \( s = 0.01 \text{ rad}^{-1} \).

The torque on the motor shaft \( T_m \) is given by

\[
T_m = \frac{C}{n} T_l
\]  

(4)
2.2 Constraints

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>Torque constant $k_t$</td>
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</tr>
<tr>
<td>Speed constant $k_b$</td>
<td>3.155 mV/rpm</td>
</tr>
<tr>
<td>Friction coefficient $\nu_m$</td>
<td>5.21e-6 N m s/rad</td>
</tr>
<tr>
<td>Terminal resistance $R$</td>
<td>0.299 Ω</td>
</tr>
<tr>
<td>Terminal inductance $L$</td>
<td>0.0823 mH</td>
</tr>
<tr>
<td>Motor inertia $J_m$</td>
<td>142 g cm$^2$</td>
</tr>
<tr>
<td>Gearbox inertia $J_{tr}$</td>
<td>9.4 g cm$^2$</td>
</tr>
<tr>
<td>Gearbox efficiency $\eta_{tr}$</td>
<td>68%</td>
</tr>
</tbody>
</table>

Table 1: Motor and gearbox parameters for 150W RE40 Maxon motor and GP52 gearbox. Gear ratios below $n = 165$ did not produce any feasible results in our optimizations. Gearbox efficiency varies with the gear ratio, but above $n = 156$, its value remains constant in the datasheet. This justifies the constant value for gearbox efficiency.

Here, $n$ is the gear reduction ratio, and $C$ is the gearbox efficiency function

$$
C = \begin{cases} 
1/\eta_{tr} & (P_{mech} > 0) \\
\eta_{tr} & (P_{mech} < 0) 
\end{cases} 
$$

(5)

The use of such an efficiency function is essential for dynamic calculations [20]. $\eta_{tr}$ is the gearbox efficiency as specified in the manufacturer’s datasheet. Motor speed $\dot{\theta}_m$ is influenced by the stiffness of the series spring $k_s$, and is given by

$$
\dot{\theta}_m = n \left( \frac{\dot{T}_i}{k_s} + \dot{\theta} \right)
$$

(6)

Finally, the electrical power consumption $P_{elec}$ is the product of motor current ($I$) and voltage ($U$). These are obtained by applying the motor model

$$
\begin{align*}
I &= \frac{1}{k_t} \left( (J_m + J_{tr}) \ddot{\theta}_m + T_m + \nu_m \dot{\theta}_m \right) \\
U &= LI + RI + k_b \dot{\theta}_m
\end{align*}
$$

(7)

which requires knowledge of several motor and gearbox parameters. These are defined in Table 1, along with their values for the motor and gearbox used in our optimization. Notice the additional contributions to the torque due to motor and gearbox inertia $(J_m + J_{tr}) \ddot{\theta}_m$ and viscous friction $\nu_m \dot{\theta}_m$.

2.2. Constraints

Most optimizations only consider torque and speed at the gearbox shaft. In reality, however, the output torque and speed are limited by mechanical and electrical constraints. A good controller design should therefore ensure that the motor’s speed and torque saturate whenever it reaches its limits, preventing failure of one or more components. The trade-off is a decrease in dynamic performance, since the actuator will not be able to provide the required torque or speed to follow the desired output trajectory. Therefore, setting appropriate constraints in the optimization is equivalent to setting a performance threshold for the actuator.
2.2 Constraints

In this work, we will only consider constraints related to the motor, although other components in the drivetrain may impose additional constraints. The Maxon GP52 gearbox, for example, can only handle 30 Nm continuously and 45 Nm intermittently (i.e. for a duration of one second), according to the manufacturer’s datasheet. In order to not reduce the range of potential solutions too much, these constraints were omitted from the optimization. From our practical experience, we know that the proposed maximum values are very conservative, and may be exceeded at the potential cost of reduced lifetime.

The constraints applied in our optimization are listed below.

2.2.1. Max. motor speed (mechanical)

While motor speed can be pushed up by increasing the supply voltage, the lifetime of the motor and gearbox bearings puts a practical upper limit $\dot{\theta}_m$ on the motor speed:

$$|\dot{\theta}_m| < \dot{\theta}_{max} \tag{8}$$

The speed limit $\dot{\theta}_{max}$ can be obtained from the motor and gearbox catalogs. This restriction can be considered a soft restriction, because motor speed can easily be limited by the controller. Even if the maximum speed is occasionally exceeded by a small amount, this should not directly lead to failure, but at worst to a slightly reduced lifetime. The main issue is the actuator’s reduced capability of following the desired output speed. It is up to the designer to decide whether this could have an adverse effect on the prosthesis’ performance. In this work, we set $\dot{\theta}_{max}$ to the maximum motor speed specified in the data sheet, 12 000 rpm.

2.2.2. Max. motor torque (thermal - mechanical)

Overheating of the motor can occur due to high RMS currents (long-term heating) or high peak currents (short term). Motor manufacturers typically specify a maximum continuous torque $T_{m,max,cont}$, below which overheating should not occur. This leads to the following constraint for long-term heating

$$[k_1 I]_{RMS} < T_{m,max,cont} \tag{9}$$

We consider the long-term heating constraint to be a hard restriction: if Eq. (9) is not satisfied, it indicates that the actuator is not properly designed for the required loads. The maximum continuous torque can be found in the motor datasheet ($T_{m,max,cont} = 177 \text{ mNm}$).

Short-term heating is more difficult to handle. A detailed calculation would rely on parameters such as the motor’s thermal resistance and thermal time constant, as well as the ambient temperature. Very often, however, the motor’s peak torque and current will be determined by the strength of the gearbox or the maximum current output of the controller, which can be characterized by a single torque value. Instead of the thermal calculation, we therefore define
2.3 Optimization objectives

a maximum torque $T_{m,\text{max, int}}$ which satisfies these limitations, and write the following constraint for peak torque:

$$|k_t I| < T_{m,\text{max, int}}$$ (10)

Whether the peak torque constraint should be considered a hard or soft restriction, depends on the case. Exceeding the controller’s maximum current output, for example, can be tolerated in inverse dynamic optimizations, since the current will in practice be limited by the controller, only leading to a reduced performance of the actuator. The torque should, however, not cause mechanical failure of any components. In this work, we used the controller’s maximum output current of 30 A as the most restrictive case for peak torque, so $T_{m,\text{max, int}} = 30A \cdot k_t$.

2.2.3. Max. motor torque (due to available voltage)

The higher the voltage available from the power supply, the higher the torque that can be delivered at a certain speed. If $U_{\text{max}}$ is the maximum available motor voltage, the relationship between torque, speed and voltage becomes

$$\left| T_m + (J_m + J_{tr}) \ddot{\theta}_m + \left( \nu + \frac{k_b k_t}{R} \right) \dot{\theta}_m \right| < U_{\text{max}} \frac{k_T}{R}$$ (11)

This is, again, a soft constraint: if the power source cannot supply the required voltage, this will simply result in a decreased output torque. In our optimizations, we assumed $U_{\text{max}} = 48$ V.

2.3. Optimization objectives

We consider four relevant optimization objectives, all very common in literature:

2.3.1. MPP: minimal mechanical peak power

Mechanical peak power is the highest value of $|P_{\text{mech}}|$ during a gait cycle. The mechanical power $P_{\text{mech}}$ is calculated from the torque and speed at the output of the gearbox,

$$P_{\text{mech}} = \frac{1}{n} T \dot{\theta}_m$$ (12)

$P_{\text{mech}}$ is thus affected by the springs, but not by gearbox losses and drivetrain inertia.

2.3.2. MEC: minimal mechanical energy consumption

In most works, the mechanical energy consumption is calculated as the integral of the absolute value of the mechanical power $P_{\text{mech}}$:

$$E_{\text{mech, abs}} = \int |P_{\text{mech}}| dt$$ (13)

By definition, however, energy should be calculated without absolute value, and (13) therefore does not represent the actual mechanical energy. The concept
behind this cost function is that power losses are proportional to the power flow through the components. It is assumed that, by reducing the power flowing in and out of the actuator, the energy losses will be minimized. This method does, however, not account for differences in loss mechanisms between the individual components of the actuator, which could potentially shift the actual minimum.

2.3.3. EPP: minimal electrical peak power

Electrical peak power is the highest value of $|P_{elec}|$ during a gait cycle. The electrical power $P_{elec}$ can be calculated by multiplying the motor current $I$ and voltage $U$:

$$P_{elec} = UI$$

(14)

2.3.4. EEC: minimal electrical energy consumption

Electrical energy consumption is calculated as

$$E_{elec} = \int P_{elec}dt$$

(15)

This energy corresponds to the energy available at the motor terminals. To what extent negative energy can effectively be recovered, depends on the design of the controller and battery. For a discussion on the consequences of incomplete energy regeneration capability for energy optima, we refer to [21].

3. Optimization of spring stiffness

In this section, we present a thorough analysis of the optimization and its results. We discuss how different optimization criteria influence the optimal series and parallel spring stiffness, assuming a fixed gear ratio of $n = 250$, the energy-optimal choice. Analyses of power flows and the power loss profile are presented to provide a better understanding of the results.

3.1. Optimization results

The optimization results are summarized in Table 2. For comparison, the powers and energies of the rigid actuator, i.e. the same geared DC motor without springs, are also shown. These values are hypothetical since, unlike for the actuator designs with series and parallel compliance, the rigid actuator would not be able to satisfy the optimization constraints. It can only follow the imposed ankle trajectory if it is overpowered, which is only possible for a limited amount of time. This demonstrates how the capabilities of an actuator can be augmented by adding series and/or parallel springs. Furthermore, the SEA with unidirectional parallel spring yields much lower energy consumption and peak powers than a rigid actuator. Electrical energy, for example, can be reduced by almost 40%, and electrical peak power by no less than 63%.

In general, electrical peak powers are typically 2-3 times higher than the mechanical ones. This indicates that a great deal of power is lost in the drivetrain. Nevertheless, whether the optimization is performed based on the mechanical or
3.2 Power flow analysis

The power balance for the ankle prosthesis can be defined as

\[ P_{\text{bio}} + P_{\text{inertia}} + P_{\text{loss}} = P_p + P_s + P_{\text{elec}} \]  

These power flows, resulting from the electrical peak power (EPP) and electrical energy (EEC) optimizations, are plotted in Fig. 3. From this figure, we can learn several things: the importance of drivetrain inertia in the swing phase, the influence of optimization objectives on the equilibrium angle of the parallel spring, and how well these results match with tests on actual prototypes. However, as mentioned, electrical properties of the system do not make much of a difference. Results mainly depend on whether the objective is peak power or energy reduction. Energy reduction requires the series spring to be as stiff as possible, whereas some series compliance is desired in order to minimize the peak power. In the latter case, the parallel spring should be relatively compliant and tuned to very negative equilibrium angles. This is visualized in a torque-angle plot (Fig. 2). For EPP, the spring is engaged from mid-swing to late pre-swing for EPP. Conversely, when energy consumption is minimized (EEC), the prosthesis should be equipped with a stiff parallel spring with an equilibrium angle close to zero. This way, the spring will only be active during stance and the early pre-swing phase.

How well do these results match with tests on actual prototypes? Little experimental data is reported in literature, but there is one comparable work by Sup et al. [18]. The actuation concept in this work is the same as the one suggested by our MEC and EEC optimizations, i.e. a rigid actuator with unidirectional parallel spring. Sup et al. reported an average electrical power of 45 W, which corresponds to an electrical energy consumption of approximately 31 J. This value is very comparable to our optimal value of 35.2 J. The small difference with our results can easily be explained by the different drivetrain designs.

### Table 2: Optimization of an SEA (series spring stiffness \( k_s \)) with unidirectional parallel spring (stiffness \( k_p \), equilibrium angle \( \theta_{eq} \)) for an active ankle prosthesis. The prosthesis is designed to mimic the ankle of an able-bodied person weighing 75 kg, at natural walking speed. Results are presented for four different optimization criteria: mechanical peak power (MPP), electrical peak power (EPP), mechanical energy consumption (MEC) and electrical energy consumption (EEC). Peak powers and energy consumption of a rigid actuator (i.e. no springs) is shown for reference.

<table>
<thead>
<tr>
<th>( k_s )</th>
<th>MPP</th>
<th>EPP</th>
<th>MEC</th>
<th>EEC</th>
</tr>
</thead>
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<tr>
<td>( k_p )</td>
<td>0</td>
<td>280 Nm/rad</td>
<td>300 Nm/rad</td>
<td>640 Nm/rad</td>
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<tr>
<td>( \theta_{eq} )</td>
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<td>-8.5°</td>
<td>-0.5°</td>
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<tr>
<td></td>
<td>252 W</td>
<td>87.2 W</td>
<td>89.1 W</td>
<td>229 W</td>
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<td></td>
<td>615 W</td>
<td>305 W</td>
<td>235 W</td>
<td>475 W</td>
</tr>
<tr>
<td></td>
<td>35.4 J</td>
<td>26.5 J</td>
<td>26.2 J</td>
<td>20.3 J</td>
</tr>
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<td></td>
<td>64.3 J</td>
<td>41.0 J</td>
<td>40.9 J</td>
<td>35.2 J</td>
</tr>
</tbody>
</table>
3.2 Power flow analysis

Figure 2: Torque-angle plot of the ankle, with optimized unidirectional parallel springs for electrical peak power (red) and electrical energy (blue). At angles below the equilibrium angle $\theta_{eq}$, the parallel spring does not contribute to the output torque. At angles above $\theta_{eq}$, torque increases linearly with increased dorsiflexion. Optimizing for electrical energy consumption (EEC) yields a higher spring stiffness than an optimization for electrical peak power (EPP), indicated by the steeper slope in the EEC case. Higher stiffness is accompanied by a higher $\theta_{eq}$, in order to keep the spring stiffness profile closer to the angle-torque characteristic of the ankle.
3.2 Power flow analysis

Figure 3: Power flows optimized for electrical peak power (a) and the electrical energy (b). The biological power profile $P_{bio}$ (blue) and the electrical power profile $P_{elec}$ (green) are plotted along with three power flows responsible for the most significant power shifts: the parallel spring ($P_p$, red), the series spring ($P_s$, yellow) and the inertia of the gearbox and motor ($P_{inertia}$, purple). Positive powers indicate that power is injected into the environment, i.e., all powers are defined from a motor perspective. The high inertial power peak is caused by the interaction between series and parallel spring, when the latter is engaged. This is reflected in the electrical power as well.

spring, and the emergence of power peaks due to the interaction between the series spring and the unidirectional parallel spring. These topics are discussed below.

3.2.1 The importance of drivetrain inertia during swing

Because of the relatively low inertia of the foot, the biological power during swing phase is close to zero. The motor, however, still consumes a significant amount of power in this phase, especially when optimized for electrical peak power. Most of it is due to its own reflected inertia, which tends to be bigger than the inertia of the foot due to the high gear ratios typical of prosthetics. This phenomenon has also been observed in electrical measurements on the SPARKy 1 prosthesis [11]. Note that the inertial torque is not included in the optimizations based on mechanical power and energy. Consequently, these types of optimization are likely to be suboptimal in case of high accelerations and high drivetrain inertia.

3.2.2 Equilibrium angle of the parallel spring

Looking at the parallel spring, we observe that it provides a power to the output which roughly follows the required biological output power. The main difference between the electrical peak power (EPP) and the electrical energy (EEC) optimization is the equilibrium angle of the parallel spring. As shown previously in Fig. 2, this equilibrium angle is a lot more negative in the EPP
3.3 Power loss analysis

optimization ($\theta_{eq} = -8.5^\circ$) than in the EEC optimization ($\theta_{eq} = -1^\circ$). This manifests itself in two power bumps at mid-stance (73-83% gait cycle) and terminal swing (88-97%) for the EPP objective, but not for EEC, where the parallel spring is disengaged. In an electrical energy optimization, such bumps are to be avoided, since motors are generally inefficient at delivering low powers. Conversely, these bumps are completely irrelevant for the peak power optimization as long as they are not higher than the peak power during the stance phase. For this reason, the optimization engages the parallel spring at a more negative angle in order to further reduce the peak power during stance, even though this is unfavorable during the swing phase. Also notice that, despite the distinct difference in equilibrium angles, the power profile of the parallel spring in stance is still quite similar for both objectives. This is achieved by lowering the spring stiffness for more negative equilibrium angles. In Section 4.1, we will see that this is a general trend.

3.2.3. Power peaks due to interacting springs

Another important issue is the strong power peak at around 55% gait cycle, which is caused to the interaction between the unidirectional spring and the series spring. The engagement of the unidirectional parallel spring is modeled by Eq. (3). The second derivative of this equation, $\ddot{T}_s$, resembles a pulse at $\theta = \theta_{eq}$. Since the acceleration of the motor is given by

$$
\dot{\theta}_m = n \cdot \left( \frac{\ddot{T} + \ddot{T}_s}{k_s} + \dot{\theta} \right)
$$

(17)

maintaining a continuous output speed at the moment of engagement requires a high acceleration from the motor, and thus a powerful burst of torque. In Fig. 3, this presents itself as a strong peak in $P_{inertia}$ and in the electrical motor power $P_{elec}$ at 57% gait cycle (EPP) and 54% (EEC). Furthermore, according to Eq. (17), the power peak should increase as the series spring gets more compliant. The optimization therefore yields high series stiffness values to minimize its influence on the actuator dynamics. Indeed, in Fig. 3, the power attributed to the series spring is nearly zero for both objectives.

3.3. Power loss analysis

Figure 4 gives a clearer view on how the springs affect the power losses. The EPP objective yields power losses which do not exceed 75 W, while the EEC objective, at its peak, causes almost 170 W to be lost between 40-60% gait cycle. Nevertheless, the power losses of the EEC objective are very low over the rest of the gait cycle. In the EPP power profile, multiple peaks of approximately 40 W appear, which add up to a greater overall energy loss than the EEC objective. Two of these peaks, which are not present in the EEC optimization, indeed occur in the swing phase, as indicated in our previous discussion. This confirms that preventing their appearance is one of the keys to reducing energy consumption.
4. Influence of drivetrain

In this section, we discuss the influence of the selected motor and gearbox on the optimal stiffnesses and the resulting peak powers and energy consumption. First, the gear ratio is varied to study its effect on the selection of springs and the resulting power and energy consumption (subsection 4.1). Next, we repeat the analysis with drivetrain inertia reduced by 50%, in order to evaluate its influence on the results.

4.1. Influence of gear ratio

An important consideration in any actuator design is the choice of the gearbox. High gear ratios can be used to match high-speed motors to the required output characteristics, but in general, they come at poor efficiencies. Furthermore, they increase the reflected inertia of the motor, which can be a problem for a high-speed application such as prosthetic ankles. Well-chosen series and parallel springs may provide a solution, because they allow to reduce the speed and/or torque required from the motor, such that the gear ratio can be decreased. In this section, we will discuss the influence of the gearbox on the feasible stiffness combinations as well as the resulting power and energy consumption.

4.1.1. Influence on motor constraints

Figure 5 shows which constraints are violated for various combinations of series and parallel stiffness. The gear ratio was fixed to $n = 180$ (Fig. 5a)
4.1 Influence of gear ratio

Figure 5: Feasibility of the motor and for gear ratios of (a) $n = 180$ (b) $n = 360$. The equilibrium angle $\theta_{eq}$ is tuned to the optimal value for every combination of series and parallel stiffness. The optimal stiffness combinations found by minimizing mechanical peak power (red) or mechanical energy consumption (green), not taking into account the motor constraints, are indicated as dots. For both gear ratios, these stiffness combinations fail to respect the maximum continuous torque that can be delivered by the motor.

and $n = 360$ (Fig. 5b), two values on either side of the range of gear ratios that lead to feasible solutions. The figures give insight into how the gear ratio influences the choice between parallel and series springs. Two main conclusions can be drawn. First, with high gear ratios, more compliant parallel springs can be used, but the constraint on motor speed is more easily exceeded. Conversely, low gear ratios make the design more likely to exceed the peak torque, such that high accelerations cannot be achieved. In both cases, series springs are not allowed to be very compliant. Second, optimizations that do not take motor constraints into account easily lead to designs that violate these constraints.

As explained in section (2.2), the most important motor limitation is the maximum continuous torque. Figure 5 shows that most spring combinations fail to respect this constraint. Parallel springs can be helpful in this regard, because well-tuned parallel springs allow to decrease the torque on the motor. Indeed, within a certain range of parallel stiffnesses, several combinations of series and parallel springs appear which lead to a feasible design. For high gear ratios ($n = 360$, Fig. 5b), the range of parallel spring stiffnesses is quite wide, spanning from 900 Nm/rad down to low values of 200 Nm/rad. Compliant parallel springs can be used because the torque output of the geared DC motor is increased by the gearbox, reducing the need for high torques from the parallel spring. The disadvantage is that the gearbox also reduces the attainable output speed, leading to the appearance of a large area where the motor’s maximum speed is exceeded. Speed is no longer a problem at low gear ratios ($n = 180$, Fig. 5a), but the geared motor’s output torque is decreased, meaning that the
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Motor’s maximum torque (or current) is reached more easily. This is especially a problem at low series stiffnesses, due to the peak in motor acceleration that appears whenever the parallel spring is engaged or disengaged. As explained in section 3.2, motor acceleration increases when more compliant series springs are used, causing higher peak torques. This explains why, in Fig. 5a, the maximum peak torque is exceeded for series stiffnesses between approximately 500-700 Nm/rad.

Also plotted on Fig. 5 are the optimal stiffness combinations found by minimizing mechanical peak power (red) or mechanical energy consumption (green), not taking into account the motor constraints. The results are consistent with those from Grimmer et al. [10] for bidirectional springs: MEC optimization yields an SEA without parallel spring, whereas MPP optimization yields a combination of series and parallel elasticity. Interestingly, neither of both results fit within the motor’s operating range for the two gear ratios presented above. Twice, the violated constraint is the continuous motor torque. This could have detrimental consequences: if these optimized stiffnesses would be used in combination with the RE40 motor, the windings would burn up within a number of cycles. The only solution is to select a more powerful motor, leading to a heavier and bulkier design, or to settle for decreased actuator performance by lowering the demanded output torques or speeds. By already selecting a motor in the optimization phase and setting appropriate constraints, such problems can be avoided.

4.1.2 Influence on optimal spring stiffness

Figure 6 shows how the optimal spring stiffnesses \( k_s \) and \( k_p \) and the optimal equilibrium angle \( \theta_{eq} \) evolve with the chosen gear ratio. This figure confirms the strong influence of optimization constraints on the optimized spring stiffnesses. We notice a strong preference for stiff series springs for all objectives except MPP. The optimal stiffness of the parallel spring depends on whether peak power or energy consumption is minimized. In the former case, compliant parallel springs can be used, while the latter objective requires stiff springs.

In the previous section, we already demonstrated the strong effect of the optimization constraints on the optimal solution for two extreme values of the gear ratio. Figure 6 shows that this can be extended to the entire range of gear ratios. Gear ratio does not affect the MPP or MEC objectives, and therefore one might expect a single value for the optimized springs stiffnesses and equilibrium angle. Yet, the optimized springs show a clear dependence on the chosen gearbox due to the constraints. This confirms limitations of the drivetrain and controller as a dominant factor in the design.

In general, the series spring is required to be relatively stiff in order to prevent power peaks. In fact, all optimization objectives except MPP demand the series spring to be as stiff as possible for most gear ratios. This is in contrast with the parallel spring, especially for the energy optimization objectives MEC and EEC. Here, the parallel spring’s stiffness is fairly constant, at a relatively high value of approximately 600 Nm/rad. For the peak power objectives EPP and MPP, the parallel spring is only required to be this stiff at the lowest gear
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Figure 6: Influence of gear ratio on spring selection for different optimization objectives. The optimal series spring stiffness $k_s$ (top), parallel spring stiffness $k_p$ (middle) and equilibrium angle $\theta_{eq}$ (bottom) are plotted as a function of the different gear ratios $n$. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple).
4.1 Influence of gear ratio

Figure 7: Influence of choice of gear ratio on mechanical and electrical peak power and energy consumption. The values in this graph correspond to the optimal settings of spring stiffness and equilibrium angle. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple). The mechanical energy consumption plot shows the net mechanical energy input required for walking (16.5 J). It is independent of the gear ratio and optimization objective, since springs are only a medium of energy storage; they do not have the ability to dissipate or generate energy over an energy cycle.

ratios. As the gear ratio increases, the geared DC motor is able to deliver more torque to the output, and the need for the parallel spring decreases. This is reflected in the gradually decreasing parallel stiffness values.

Finally, the optimal equilibrium angle $\theta_{eq}$ (bottom of Fig. 6) shows a trend which is nearly identical to that of parallel stiffness. High parallel stiffnesses $k_p$ come with small $\theta_{eq}$ while low $k_p$ comes with higher $\theta_{eq}$. As explained in section 3.2, this is required for the parallel spring to deliver the optimal amount of power during stance without causing power peaks during swing.

4.1.3 Influence on optimization objectives

Figure 7 demonstrates the effect of different optimization objectives on the optimal gear ratio.

Achieving low mechanical peak power demands sufficiently high gear ratios. The gradual decrease in MPP as the gear ratio increases to 280 reflects the movement towards a broader operating region of the motor. Between $n = 280$ and $n = 320$, the selected spring stiffnesses remain roughly the same (see previous section), and the mechanical peak power converges to its optimal value. Conversely, electrical peak power exhibits a minimum at a lower gear ratio, ap-
proximately at $n = 260$. This is explained by the growing torque due to the motor’s reflected inertia, which increases with gear ratio. This inertial torque, which does not affect mechanical peak power, leads to an increase in electrical peak power at high gear ratios.

When it comes to electrical energy consumption, there is no clear difference between the MEC and EEC objective. This is consistent with our findings in previous sections. In section 3, we drew the same conclusion for a fixed gear ratio of $n = 250$, and in section 4.1.2, the optimized spring stiffnesses were similar for both objectives. If the springs are optimized for peak power instead of energy consumption, the electrical energy consumption will be much higher. This also proves that both objectives are not necessarily related, as previously remarked by Grimmer et al. [10].

4.2 Influence of actuator inertia

As mentioned in the introduction, several authors have identified the potential impact of motor and gearbox inertia on the design of active prostheses. In this section, we will evaluate the influence of inertia on the selection of spring stiffness.

In dynamic applications, actuator inertia will affect the torque that has to be provided by the motor. This will also affect the choice of gear reduction. It is well-known that, for purely inertial loads and a lossless drivetrain, the optimal gear ratio is found by matching the load inertia $J_{\text{load}}$ to the reflected drivetrain inertia $n^2 (J_m + J_{\text{tr}})$. The resulting gear ratio,

$$n = \sqrt{\frac{J_{\text{load}}}{J_m + J_{\text{tr}}}}$$

maximizes the actuator’s acceleration capability [16] and energy efficiency [12]. This principle, called “inertia matching”, implies that the optimal drivetrain design is defined as soon as the load is known.

Unfortunately, a simple analytical solution as Eq. (18) does not exist for prostheses. Here, the load is far from purely inertial, since most of the torque on the ankle joint results from interaction with the environment. The only way to study the effect of inertia, is to repeat the optimization with a different drivetrain inertia. In this section, we reprise the analysis presented in sections 4.1.2 and 4.1.3 with $J_m$ and $J_{\text{tr}}$ reduced by 50%.

4.2.1 Influence on optimization objectives

Figure 8 presents the peak powers and energy consumptions as a function of gear ratio, with reduced drivetrain inertia. Comparing these results to the ones obtained with full actuator inertia (Figure 7), the minimal mechanical peak power remains roughly the same (100 W). This is logical: mechanical output power does not depend on drivetrain inertia; it only affects the MPP optimization results through its impact on the constraints. In contrast, electrical peak power is strongly influenced by the decreased inertial torque. At the optimal configurations, it is reduced from 234 W to 113 W for the optimal
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Figure 8: Influence of choice of gear ratio on mechanical and electrical peak power and energy consumption, when actuator inertia is reduced by 50%. The values in this graph correspond to the optimal settings of spring stiffness and equilibrium angle. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple).
4.2 Influence of actuator inertia

![Graph showing the influence of gear ratio on spring selection for different optimization objectives, when actuator inertia is reduced by 50%](image)

Figure 9: Influence of gear ratio on spring selection for different optimization objectives, when actuator inertia is reduced by 50%. The optimal series spring stiffness $k_s$ (top), parallel spring stiffness $k_p$ (middle) and equilibrium angle $\theta_{eq}$ (bottom) are plotted as a function of the different gear ratios $n$. Optimization objectives are mechanical peak power (MPP, blue), electrical peak power (EPP, red), mechanical energy (MEC, yellow) and electrical energy (EEC, purple).

value. The impact of decreased inertia is not as strong in terms of electrical energy consumption (decrease from 35 J to 32 J). Recall that inertia itself is an energy-neutral element, i.e. it does not dissipate energy. The reduced energy losses are a secondary effect of the decrease in inertial torque, which results in less Joule losses.

4.2.2 Influence on optimized spring stiffnesses

So what is the impact of drivetrain inertia on the optimized spring stiffnesses? In Figure 9, the optimized spring stiffnesses are plotted as a function of gear ratio with drivetrain inertia reduced by 50%. A major difference with the results in Figure 6 is the increased usage of the series spring, especially for the MPP and EPP objectives. The parallel spring design remains roughly the same for the MEC and EEC objectives. The MPP and EPP objectives, however, yield very different parallel spring designs. With reduced inertia, the equilibrium angle is lowered to the point where the spring is engaged throughout the entire gait cycle. In agreement with our results from the previous paragraphs, the parallel spring stiffness is lowered accordingly.
4.2.3. Influence on optimal gear ratio

Equation (18) predicts an increase in gear ratio when the drivetrain inertia is reduced. Indeed: the range of gear ratios has clearly shifted to higher values in figures 8 and 9. The same applies to the optimal gear ratios. Minimal electrical power consumption, for example, was previously achieved for \( n = 250 \). With half of the drivetrain inertia, the optimal gear ratio has shifted to \( n = 310 \). This can also be interpreted in terms of reflected inertias. While drivetrain inertia was decreased by 50\%, the reflected inertia only decreased by 23\% percent. This demonstrates that, as predicted by the inertia matching principle, a change in motor inertia will be opposed by a change in gear ratio.

Note that the efficiency of gearboxes decreases with their gear ratio. This may cancel out the energetic benefits gained from the decreased inertial torque.

5. Implications for the design of prosthetic ankles

The results presented in this work teach us several important things regarding the design and optimization of prosthetic ankles with series and parallel elements.

A first aspect that was discussed in this work, is the difference between electrical and mechanical power flows. Based on the biological power profile, nearly no power would be required during the swing phase. Nevertheless, accelerations in the swing phase - as well as in late stance - are high. This leads to surprisingly high power peaks in these phases, since the motor needs to accelerate its own inertia as well as that of the gearbox. Failing to provide the required accelerations may, however, compromise ground clearance. This must be carefully assessed, because ground clearance prevents the user from stumbling and falling, the most important task during swing. It is therefore recommendable to take the inertia of the drivetrain into consideration in a design phase.

Another goal of this work was to evaluate the effect of different objectives on the optimization. As one may expect, peak power-based and energy consumption-based optimizations lead to very different results, because the former only applies to a single point in the gait cycle, whereas the latter takes the entire cycle into account. Whether the optimization is performed in the mechanical or electrical domain makes less of a difference. The common assumption that (electrical) energy consumption can be minimized by minimizing the (mechanical) power flow in the drivetrain, as specified by Eq. (13), thus seems to be valid in this specific case. This is a valuable insight, because the latter cost function is much easier to apply.

Regarding the selection of springs, our results demonstrated a general trend of favoring designs with only parallel elasticity. On a superficial level, this result confirms that of Eslamy et al. [5]. Nevertheless, the approach followed in our work is very different from that in Eslamy et al. Our analysis has demonstrated that the optimal design strongly depends on the choice of the motor – something which was not taking into account in the aforementioned work. Resulting from the interaction between the series spring and the unidirectional parallel spring,
designs combining series and parallel elasticity were found to require torques beyond the motor's limits. Such solutions were discarded in our optimization, which is the reason why our optimized series springs tend to be stiff. Not taking motor limits into account in the design phase, like in the approach followed by Eslamy et al., may lead to selecting a larger motor than strictly necessary. A more effective strategy would be to choose the motor first, and then optimize the springs within the operating range of the selected motor. Of course, this may require several iterations in order to find the smallest possible motor.

Finally, the choice of gearbox mainly depends on the required output torques and speed, but the optimal gear ratio is also influenced by the inertia of the drivetrain. Choosing a correct gear ratio is important, because it will determine the range of stiffnesses available for a certain motor. Springs can therefore only be exploited if a matching gear ratio is chosen. Gearboxes, however, have several disadvantages. Firstly, they increase the reflected inertia of the drive. As mentioned earlier, prosthetic ankles require considerable accelerations, which can lead to high inertial torques caused by the inertia of the motor and gearbox. Secondly, gearbox efficiency decreases with increasing gear ratio. This is particularly relevant when the inertia of the drivetrain is low, because, as our results have shown, low-inertia drivetrains will require higher gear ratios. In that case, the most energy efficient design could well be one with suboptimal springs in combination with a smaller, more efficient gearbox. The gear ratio should therefore just as much be considered a part of the optimization as the springs themselves.

6. Conclusion

In this work, we explored the connections between gear ratio, drivetrain inertia, motor constraints and spring selection in actuated ankles with series and parallel elasticity. Our analysis shows that these design variables all have a distinct influence on each other. Motor constraints, in particular, have a strong impact on the design and performance, and should already be part of the design process in an early stage. Gear ratio and drivetrain inertia were also identified as parameters with a distinct influence on spring selection. In conclusion, the results clearly motivate an approach where gearbox, motor and springs are taken into account simultaneously. Potential advantages are decreased energy consumption, better dynamic performance and reduced size and weight, all essential aspects for actuators in prosthetics.

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