Chaos in solitary VCSELs: asymmetry and noise

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ABSTRACT

We discuss the impact of asymmetries and noise on the nonlinear dynamics of vertical-cavity surface-emitting lasers (VCSEL). We focus in particular on the effects of these features on the chaotic dynamics that can be generated by a free-running VCSEL due to the intrinsic competition between polarization modes taking place in these devices. Experimentally, we observe significant asymmetries especially in the statistics of the chaotic dynamics. We show that these behaviour can be explained theoretically by a combined effect of the system asymmetries and the noise. This work therefore brings new light on the interplay between deterministic and stochastic processes taking place in VCSELs.

Keywords: VCSEL, chaos, nonlinear dynamics, semiconductor lasers

1. INTRODUCTION

Vertical-Cavity Surface-Emitting Lasers (VCSELs) exhibit crucial advantages over the typical edge-emitter structure such as a lower threshold, a circular output beam, and on-chip testing capability. However, their typically circular geometry has no preferred polarization direction à priori, and only the small anisotropies of the laser cavity select the polarization orientation. This selection is however weak and polarization instabilities have quickly been identified as the main drawback of VCSEL devices.\textsuperscript{1, 2} The most representative feature of such instabilities is the so-called Polarization Switching event, where the polarization of the output beam typically switches from the linear polarization at threshold to the orthogonal polarization direction following a change of the injection current or the device temperature.\textsuperscript{1–9} In addition, various dynamical behaviour have been reported accompanying polarization switching events including elliptical polarization, oscillations, quasi-periodic dynamics and noise-induced hopping between these two orthogonal linear polarizations. A couple of years ago, we showed that these instabilities - which can also be seen as a competition between two orthogonal polarization modes - could even lead to a chaotic behavior;\textsuperscript{10, 11} so-called polarization chaos. This dynamics appears as follows when increasing injection current: first the linear polarization otherwise stable at threshold is destabilized and two elliptically polarized states are created. These two states are then destabilized toward periodic solutions followed by a cascade of bifurcations leading to the emergence of two single-scroll chaotic attractors oscillating around the now unstable elliptically polarized states. These two attractors grow for increasing current, and finally merge to form a double-scroll chaotic attractor. Experimentally, once the fast oscillations on the wings of the chaotic attractor has been filtered out, the dynamics can typically be seen as a random-like hopping between two elliptically polarized states.\textsuperscript{10, 11}

From a theoretical point of view, the observation of the polarization chaos in a free-running VCSEL is a significant step forward as it confirms the validity of the spin-flip model (SFM) for VCSEL devices.\textsuperscript{12, 13} But

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it quickly appeared that this framework needs to be extended in order to explain more complex features observed experimentally such as the limit cycle bistability described in. Indeed, because of its intrinsic symmetry, the original SFM model cannot explain asymmetrical behaviours. However, adding a small symmetry breaking mechanism, such as the introduction of a misalignment between the phase and amplitude anisotropies of the laser cavity - as proposed by Travagnin in Ref. - leads to an extended theoretical framework which reproduces accurately the asymmetrical features observed experimentally.

In this contribution, we continue this investigation of asymmetrical behaviours in VCSELs and provide new insight on the underlying mechanism responsible for these features. We already know that the existence of a small misalignment between phase and amplitude anisotropies can have a significant macroscopic impact on the laser dynamics such as switching between two elliptical polarization orientations as described in detail in. But, since chaos is an extremely sensitive dynamics, we expect a major impact of this symmetry breaking on polarization chaos dynamics. Here we focus on the statistics of the mode hopping and report two experimental cases: one showing only a small asymmetry while the other exhibits a strongly asymmetric behaviour.

2. THEORETICAL MODEL - SFM

The model considered here is based on the spin-flip model (SFM) that considers the right and left circular polarizations as two distinct emission processes. However within this framework, as already discussed in some previous work and in the introduction above, the orthogonal axes of the two preferred linearly polarized states are axes of symmetry, in particular for elliptically polarized states. Experimentally however, we observed that this is not an entirely accurate representation of the system as clear signs of asymmetries can be spotted. Interestingly, it appears that introducing a slight misalignment of the cavity anisotropies - the phase and amplitude anisotropies - in the modelling, as first discussed by Travagnin in, leads to an excellent qualitative agreement with experimental observations. This modified SFM model therefore reads as follows:

\[
\frac{dE_\pm}{dt} = \kappa (1 + i\alpha)(N \pm n - 1)E_\pm - (i\gamma_p + \cos(2\theta) \mp i\sin(2\theta))\gamma_s E_\mp
\]

\[
\frac{dN}{dt} = -\gamma (N - \mu + (N + n)|E_+|^2 + (N - n)|E_-|^2)
\]

\[
\frac{dn}{dt} = -\gamma_s n - \gamma ((N + n)|E_+|^2 - (N - n)|E_-|^2)
\]

with \(E_\pm\) the left (-) and right (+) circular polarizations, \(N\) the total carrier population and \(n\) the carrier population difference between the two carrier reservoirs considered for the two separated emission processes. The different time-scales are modeled by the decay rate of the electric field \(\kappa\), the carrier decay rate \(\gamma\) and the spin-flip relaxation rate \(\gamma_s\). The anisotropies inside the laser cavity are defined by the phase anisotropy (or birefringence) \(\gamma_p\), the amplitude anisotropy \(\gamma_a\) and the misalignment angle \(\theta\) between them. Finally, \(\alpha\) is the linewidth enhancement factor and the injection current is \(\mu\).

Unless specified otherwise, we use the following parameter values which are similar to those used in previous works and which allows the emergence of polarization chaos: \(\alpha = 3, \kappa = 600\,ns^{-1}, \gamma = 1\,ns^{-1}, \gamma_s = 100\,ns^{-1}, \gamma_a = -0.7\,ns^{-1}\).

3. IMPACT OF A SMALL ASYMMETRY

Experimentally, we record the chaotic polarization fluctuations with the setup described in. Using a polarizer oriented at 45° from the linear polarization at threshold allows to clearly observe the characteristic random-like hopping dynamics of polarization chaos and then to perform a statistical analysis of the dynamics. Here we focus on the average dwell-time, i.e. the average time between two successive jumps, or to put it differently the average time the system remains on one side of the double-scroll chaotic attractor. Typically, we expect an exponential decrease of the dwell-time as reported in Refs. In Fig. 1(a), we indeed observe such exponential decrease, but also with a slight discrepancy between the two polarization orientations. Indeed,
Figure 1. Experimental (a) and theoretical (b) evolution of the average dwell-time of the polarization chaos dynamics for an increasing injection current for a small asymmetry of the laser cavity. The dwell-time for the two wings of the chaotic attractor are shown as empty triangle and full squares respectively. The dashed horizontal line in (a) shows the limit of the photodetector bandwidth. In (b), the birefringence and the misalignment angles are set at $\gamma_p = 4.4 \text{ ns}^{-1}$ and $\theta = 0.02 \text{ rad}$ respectively. No noise is considered in the simulation.

The discrepancy between the average dwell-times of the two states.

Theoretically, we successfully reproduce a similar behaviour when introducing a small misalignment angle between the phase and amplitude anisotropy. Thus, with an angle of $\theta = 0.02 \text{ rad}$, we obtain the evolution displayed in Fig. 1(b). We observe that the system exhibit a preference to stay in one polarization mode (one side of the chaotic attractor) than in the other polarization mode; for a given current, the misalignment angle induces a small preference for one side over the other. Nevertheless, we also observe that the preferred side and the discrepancy between the average dwell-times vary significantly when the injection current is increased unlike in the experimental data. Although this point might require further investigations, such study is out of the scope of this paper.

4. RESTABILIZATION OF PERIODIC SOLUTIONS

Since we tested several chaotic devices on the same wafer, we were able to observe a variety of statistical evolution. Although most of the time the mode-hopping statistics evolves as described in the previous section, we also recorded a case with a much stronger asymmetry. This result is displayed in Fig. 2(a). Instead of a smooth exponential decrease similar to the one obtained before, we report here a dramatic change: the average dwell-time is not steadily decreasing, the statistics for the two sides of the chaotic attractor seems to be relatively independent and two peaks around 2.6 and 2.8 mA can be observed. Interestingly, we also observe an inversion of the preferred polarization orientations. In this case, the discrepancy between the average dwell-times reaches much larger levels: up to two orders of magnitude.
With the same theoretical framework, we were not able to reproduce such strongly asymmetric behaviour when changing the system parameters, in particular the birefringence $\gamma_p$ and the misalignment angle $\theta$. In fact, no set of parameters seemed to lead to a re-increase of the average dwell-time. However, by exploring the parameter space, we observed that, in some cases, the system could experience a re-stabilization of the periodic solution oscillating around the two unstable elliptically polarized states. This re-stabilization appears well inside the region of chaotic dynamics and destroys the latter completely. Moreover, the existence of a misalignment angle - though not a requirement to observe this restabilization - makes that the two periodic solutions are stable for two distinct ranges of injection current. So, with increasing injection currents, the system settles first on the periodic solution on one side, and then jumps on the other side to settle on the other solution. Through numerous simulations, we also remarked that these two solutions are only marginally stable: just a small perturbation appears to be sufficient to induce a relatively long transient behaviour before the system settles back on the periodic solution. By introducing a small level of spontaneous emission noise, it is therefore possible to permanently kick the system out of the basin of attraction and thus find back the chaotic behaviour. Yet, by computing the average dwell-time evolution when including spontaneous emission noise, we observe a clear impact of this re-stabilization on the laser dynamics as displayed in Fig. 2(b). We observe that the simulations results show an excellent qualitative agreement with the experimental observations. The two peaks are reproduced accurately, and appears for the range of injection current where the periodic solutions are stable.

To obtain this result, we used a spontaneous emission noise level of $\beta_{sp} = 4.10^{-9}$ which is extremely small compared to the expected level of noise in VCSEL devices (typically around $\beta_{sp} \approx 10^{-4}$, as indicated in Ref.19). However, if larger levels of noise are considered, the size of the peaks decreases until they completely vanish.
5. CONCLUSION

To conclude, we highlight in this contribution the significant impact of asymmetries in Vertical-Cavity Surface Emitting Lasers on their nonlinear dynamics. We pursue the work that has been started in, and show that strongly asymmetrical behaviour can be observed in free-running VCSELs. In particular, we report on critical changes of the statistics of polarization chaos dynamics.

On a theoretical point of view, we demonstrate one more time that the framework of the spin-flip model (SFM) allows to accurately reproduce the dynamics observed experimentally. Thus, we were able to link the strongly asymmetric behaviour reported in the experiment to the re-stabilization of two periodic solutions oscillating around elliptically polarized states. These two periodic solutions being marginally stable, the system is kicked out of their basins of attraction due to the spontaneous emission noise. Yet, we see that the existence of these marginally stable solutions impacts significantly the statistics of the polarization chaos dynamics.

In the end, this work brings new insight on the nonlinear dynamics of VCSELs, and motivates further experimental and theoretical investigations to fully understand the complexity of the devices. Moreover this work provides essential information from an application point of view: in particular for the random bit generation based on polarization chaos dynamics where the statistical evolution of the dynamics might play a crucial role, but also for the development of other applications involving polarization chaos dynamics such as chaos-communication schemes.

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