Design of focal beam shaping system through irradiance and phase control

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ABSTRACT

Focal beam shaping (FBS), or laser beam shaping at focus, is required in many laser applications. The most common approach is to use a phase element and a Fourier transform lens to generate at the focal plane of the lens the desired irradiance pattern, usually a flat-top. The shaping quality depends strongly on a dimensionless parameter $\beta$. In case of long focal length and/or small focal spot, the input laser beam should be sufficiently large in order to get a large $\beta$ value for a satisfying shaping quality. Therefore additional beam expansions might be needed. In this work, we propose a different approach with two plano-aspheric lenses that allows to control both irradiance and phase at focus. The two lenses are designed by an extended ray mapping technique combined with a rigorous backward wave propagation method, so that diffraction effects around laser focus can be implemented in a reliable way. With the developed approach, the shaping quality is guaranteed without the possible need for extra beam expanders, which makes the system more compact. The advantage of our design approach is demonstrated in direct comparison with the conventional Fourier approach for the same design example to transform a Gaussian beam to have a circular flat-top irradiance pattern.

Keywords: Laser beam shaping, Geometrical optical design, aspherical lens design, lens design, wave propagation

1. INTRODUCTION

Laser beam shaping at focus, often referred to as focal beam shaping (FBS), is widely used in many laser applications, such as laser material processing, medical operations, optical processing, optical data storage, laser printing, and laser research.\textsuperscript{1} FBS means that the laser beam is focused to a small area to have a sufficiently high power density, and in the meantime is shaped to obtain a desired irradiance pattern (often a flat-top) at the focal plane to achieve optimum system performance.

There are various techniques developed to shape laser beams at focus. The shaping can be divided into two steps such as Focal \textsuperscript{2}shaper\textsuperscript{2} with the first step to generate the collimated beam with shaped irradiance and the second step to focus the shaped collimated beam to the Fourier plane of the lens. More often, it is one-step shaping using the configuration illustrated in Fig. 1.\textsuperscript{3} The input beam is a collimated single-mode laser beam with a Gaussian irradiance distribution. The phase element can be either refractive or diffractive optics which modifies the phase of the beam. The transform element is the lens that generates the Fourier transform of the optical field with the modified phase at its focal plane. The phase of the phase element is defined so that the irradiance at the focal plane is as desired. As this approach is based on the Fourier transforming properties of lenses, we will refer to it as “Fourier approach” throughout this paper.

There is an inherent limitation for the Fourier approach. The quality of the irradiance pattern that can be obtained strongly depends on the dimensionless parameter $\beta$:\textsuperscript{3–8}

$$\beta = \frac{CR_o}{\lambda f},$$

(1)

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where $C$ is a constant, $r_o$ is the radius of the input laser beam, $R_o$ is the radius of the output beam at the focal plane, $\lambda$ is the design wavelength, and $f$ is the focal length of the Fourier transform lens or the distance from the FBS system to the focal plane. In literature, $C$ has been chosen to be either $2\pi^5$ or $2\sqrt{2\pi}^{1,4,6-8}$ for calculation convenience. In the following discussions, we have chosen $C = 2\sqrt{2\pi}$ as mostly used in the case of shaping a Gaussian beam to have the flat-top profile at focus.

The impact of $\beta$ on the shaping quality for the circular flat-top pattern at focus is illustrated in Fig. 2. The red solid line is the cross section of the targeted pattern described by a step function. The dotted lines show results for different $\beta$ values by changing the radius of the input laser beam. These simulation results have been obtained using the unified optical modeling software “VirtualLab”, and they agree very well with the reported figures in literature.$^{3,7,8}$ With a decreasing $\beta$ value, deviations from the ideal flat-top shape begin to appear. Depending on what might be considered as still sufficiently close to a flat-top distribution, a threshold value for $\beta$ can be defined, for example $\beta = 4$. For $\beta$ values smaller than this threshold, the beam shaping system will not deliver acceptable results.

![Figure 2](http://proceedings.spiedigitallibrary.org/)

Figure 2. The cross section profiles for the circular flat-top pattern with sharp edges at focus: The solid line is the design target and the dashed lines are design results by the Fourier approach. $\beta$ has to be large enough to obtain the satisfying profile sufficiently close to the target.

In practice, the laser wavelength $\lambda$, the spot radius at focus $R_o$, and the working distance or the focal length $f$ are typically determined by the considered applications. In case of a too small $\beta$ value, the only possibility is to expand the initial laser beam so that the newly obtained $\beta$ will be large enough to provide the irradiance profile sufficiently close to the target. A typical solution is shown in Fig. 3 where a two-lens telescope has been added to expand the input laser beam. However, these additional optical elements makes the complete system
less compact with more elements to be aligned. It should be stressed that for targeted irradiance distributions other than the flat-top, the threshold value of $\beta$ can be significantly higher than 4 to still obtain satisfactory beam shaping results.\(^3\)

![Figure 3](image)

Figure 3. To obtain satisfying shaping quality for small $\beta$ values, a practical solution to enlarge $\beta$ is to add a two-lens telescope to expand the input laser beam.

In this work, we present a different approach that uses two plano-aspheric lenses to control both irradiance and phase at laser focus. This novel design strategy and method does not rely on the Fourier transforming property of lenses, and is explained in Sec. 2. In Sec. 3, the developed design method is applied for a design task to transform a input Gaussian beam to a circular flat-top profile at focus with a small initial $\beta$ value. In contrast to the Fourier approach, our design method still does not require additional elements to expand the input laser beam. Conclusion is drawn in the end.

2. DIRECT DESIGN APPROACH OF LASER BEAM SHAPING AT FOCUS

The designed two-lens focal beam shaping system is schematically drawn in Fig. 4. In literature, the existing direct design methods to control only the irradiance at focus are based on either geometrical optics\(^3\) or diffraction theory.\(^5\) We have introduced the intermediate plane at the exit of the shaping optics to combine both geometrical optics and wave optics, which allows to control both irradiance and phase at the focal plane.

![Figure 4](image)

Figure 4. Schematic drawing of the focal beam shaping system with two plano-aspheric lenses used in the developed design approach: An arbitrary ray (in red) intersects the input plane at radius $r$, goes through the intermediate plane at radius $R$, and then arrives at the focal plane at a distance $f$ from the output plane.

The design procedure can be divided into two steps. First, the required optical field at the focus is propagated backward rigorously to the intermediate plane using the method of the angular spectrum of plane wave.\(^9\) Then, the field information at the intermediate plane is translated to ray information, so that the ray mapping technique\(^10\) can be extended to design the two lenses.
The optical field at the focal plane is defined to have the irradiance distribution as desired and the constant phase to have a virtual focus. Then this predefined optical field is propagated back to the intermediate plane in free space using the angular spectrum of plane wave method. Suppose the radial coordinate at the intermediate plane is \( R \), the obtained field information includes the irradiance \( I(R) \) and the phase \( \Phi(R) \). For calculation convenience, \( I(R) \) is normalized so that the total power is 1 W, and \( \Phi(0) \) equals to 0 by taking the point on optical axis as the reference. In our case, we have performed the backward propagation in “VirtualLab” by using the Spectrum of Plane Wave (SPW) operator. The field at the focal plane can be described either by analytical functions or discrete data points. The field information that can be extracted is numerical data.

The obtained field information at the intermediate plane can now be translated to the ray information needed to design the lenses. In Fig. 4, the path of an arbitrary ray is highlighted in red. For each ray, the necessary information at the intermediate plane is the ray position \( R \), the ray direction angle \( c(R) \), and the Optical Path Length (OPL) difference with respect to the on-axis ray \( OPD(R) \). Due to the rotational symmetry of the considered optical system, it is sufficient to consider only \( R \geq 0 \). Then the direction angle is calculated as:

\[
c(R) = \arcsin(-\frac{\Phi'(R)}{k}),
\]

where \( k \) is the wave number. The OPL difference is calculated as:

\[
OPD(R) = \frac{\Phi(R)}{k}.
\]

As the ray information both at the input plane and at the intermediate plane is known, the next step is to design two lenses to map rays between these two planes. Design of double freeform surfaces for laser beam shaping has been investigated over the years. However, detailed design procedures have been reported mainly for collimated laser beam shaping with plane wavefronts.\(^{10-19}\) There is work\(^{20}\) which can be used to generate a freeform wavefront, however, the irradiance is defined in the case at a plane close to the wavefront rather than on the wavefront itself. In our case, we have defined both the irradiance and the phase at the same plane, so that the design has a good accuracy even when the beam is strongly focused.

Figure 4 introduces all the necessary parameters for the optical design process. For the first lens, the radial coordinates at both surfaces are the same as \( r \). For the second lens, the first surface has the radial coordinate \( H \) while the second surface’s radial coordinate is \( R \). In order to make the design adaptable to both magnifying and demagnifying systems, the signs of certain parameters should be taken into account. The sag values \( z \) and \( Z \) of two aspherical surfaces are defined positive if the corresponding surface point is at the right side of the on-axis point, otherwise the value is negative. The slope values \( v \) and \( V \) are defined as

\[
v = \tan(\theta_1), \quad V = \tan(\theta_2),
\]

where \( \theta_1 \) and \( \theta_2 \) are positive when the surface normal is below the forward horizon as plotted. Otherwise they have negative values. Similar definitions apply for the angle \( a \) and \( c \). For the angle \( b \), the sign definition is in the opposite way for calculation convenience.

The direct numerical design procedure to design two lenses is explained step by step in the followings:

- **Step 1**: To define the clear aperture \( r_{max} \) and \( R_{max} \) for both lenses. We have selected the clear aperture \( r_{max} \) to be \( 3r_o \) to avoid truncation effects,\(^{4}\) and then \( R_{max} \) is calculated based on the same energy passing through the system.

- **Step 2**: To map \( r \) to \( R \). Supposing there is no energy loss in the beam shaping system, the radial coordinates \( r \) and \( R \) can be mapped to each other based on the theory of energy conservation:

\[
2\pi \int_0^r xI_{in}(x)dx = 2\pi \int_0^R xI(x)dx,
\]
where the total power at the input plane is also normalized. According to our previous work,\(^{19}\) the radial coordinates \(r\) and \(R\) should both be sampled by equal distant bins as:

\[
\begin{align*}
  r_i &= 0, \frac{r_{\text{max}}}{p}, 2\frac{r_{\text{max}}}{p}, \ldots, r_{\text{max}}, \\
  R_i &= 0, \frac{R_{\text{max}}}{q}, 2\frac{R_{\text{max}}}{q}, \ldots, R_{\text{max}},
\end{align*}
\]

where \(i\) is the counting index, \(p\) and \(q\) are the number of bins for \(r\) and \(R\) respectively.

- **Step 3:** To write the first-order differential equation for the sag of the first aspherical surface \(z\) in the format:

\[
\frac{dz}{dr} = v(r, z).
\]

It is straightforward that the slope \(v\) can be calculated in the triangle L-M-S by:

\[
v = \frac{S}{nM - L},
\]

where \((L, M, S)\) depends only on \((r, R, a, z, Z)\). From Step 2, we have \(R\) dependent on \(r\) as \(R(r)\). Next we would derive \(a(r)\) and \(Z(r)\) so that we could have the slope \(v\) dependent on \((r, z)\) only. We could directly have some parameters dependent on \(R\) only. \(\Phi(R)\) is obtained from backward propagation. The refraction angle at the last refractive surface \(c(R)\) is given by Eq. (2). The angle \(a(R)\) can then be calculated based on the Snell’s Law on the plane surface of the second lens. As \(R\) is a function of \(r\), these parameters can be also written as \(\Phi(r)\), \(c(r)\) and \(a(r)\). The only parameter left to be calculated as a function of \((r, z)\) is the sag of the second surface \(Z\), which can be found by calculating the OPD between any arbitrary ray and the center ray and substituting it to Eq. (3). The OPL from the input plane to the intermediate plane should be calculated. For the center ray on axis,

\[
\text{OPL}_0 = nt_1 + d + nt_2.
\]

For any other ray,

\[
\text{OPL} = N(t_1 + z) + M + n\left(\frac{t_2 - Z}{\cos(a)}\right).
\]

As the OPL difference at the input plane is 0, the OPL difference from the two equations above is the same as that in Eq. (3). So that

\[
\frac{\Phi}{k} = n(z - t_2 + t_2 - Z \frac{1}{\cos(a)}) + M - d.
\]

Substituting \(R(r), \Phi(r), a(r), M(r, z, Z)\) into the Eq. (11), we can derive \(Z\) is a function of \((r, z)\). Hence we can have the slope \(v\) as \(v(r, z)\) to have the expected differential equation in Eq. (7).

- **Step 4:** To calculate the sag \(z\) and the slope \(v\) for the first aspherical surface. Eq. (11) is solved for \(r_i\) to have \(z_i\). We have chosen the Ordinary Differential Equation (ODE) solver “ode45” in “Matlab” using the Runge-Kutta methods. The slope of the first surface \(v_i\) is calculated according to the function \(v(r, z)\) in the last step.

- **Step 5:** Similarly to Step 3, the following equation is written to calculate the sag of the second aspheric surface \(Z\):

\[
\frac{dZ}{dR} = V(R, Z),
\]

according to the mapping \(r(R)\), the Snell’s Law on different refracting surfaces and Eq. (11).

- **Step 6:** To calculate the sag \(Z\) and the slope \(V\) for the second aspherical surface. The sag \(Z_i\) is calculated by solving Eq. (12), and the slope \(V_i\) is calculated according to the function \(V(R, Z)\) in the last step.
• Step 7: It should be noticed that the radial coordinate of the second aspheric surface is not $R$ but $H$ which can be calculated by

$$H = (t_2 - Z)\tan(a) + R.$$  \hspace{1cm} (13)

Following Eq. (13), $H_t$ and $H_{max}$ can be calculated from $R_t$ and $R_{max}$.

In the end, we have the full information to describe the two aspheric surfaces with $(r_i, v_i, z_i)$ for the aspheric surface of the first lens and $(H_i, V_i, Z_i)$ for the aspheric surface of the second lens.

3. DESIGN EXAMPLE TO HAVE A CIRCULAR FLAT-TOP AT FOCUS

Although our developed approach in Sec. 2 is not limited to a flat-top profile at focus, we have selected this design example as it is the most commonly used design example in case of the Fourier approach.

The input beam is a single mode He-Ne laser beam which has its waist at the input plane of the beam shaper, so the irradiance distribution is Gaussian, described by

$$I_{in}(r) = \frac{2}{\pi r_o^2} \exp\left(-2 \frac{r^2}{r_o^2}\right),$$  \hspace{1cm} (14)

where $r_o$ is the beam radius within which the encircled energy is 86%. The flat-top profile at the focal plane is chosen to have roll-off edges rather than the perfect steep ones to avoid strong ripples in the far-field diffraction pattern. Reference\(^{21}\) has summarized for laser beam shaping different flattened profiles, from which we have selected a commonly used one Fermi-Dirac (FD) profile described by

$$I_{FD}(R) = I_{FD}\{1 + \exp[\beta_{FD}(\frac{R}{R_o} - 1)]\}^{-1},$$  \hspace{1cm} (15)

where $I_{FD}$ is the constant to have normalized total power and $R_o$ is the characteristic radius where the irradiance is 0.5$I_{FD}$. $\beta_{FD}$ should be properly selected to ensure not only a flattened profile but also without too steep edges. We have chosen $\beta_{FD} = 16.25$ as proposed in literature.\(^{13, 21}\)

As is discussed in Sec. 1, the difficulty of obtaining the expected focal spot shape increases with reduced focal spot sizes and/or increased focal lengths, in case that the input beam is given and has a fixed size at the beam shaper. For our design, with the fixed input radius $r_o = 0.34mm$ and wavelength $\lambda = 632.8nm$, the output radius $R_o$ is selected rather small as 0.2mm and the focal length $f$ is chosen relatively long as 269.2mm. The resulting $\beta = 2$ for the Fourier approach means that additional beam expansion has to be used to ensure a good shaping quality of the generated flat-top profile.

In our design, the lenses are made of Poly(methyl methacrylate) (PMMA) which has a refractive index of 1.489. In order to ensure that the designed lenses are practical, the thickness all over the lens aperture should better be not smaller than 5mm, which can be controlled by specifying proper center thicknesses of the lenses $t_1$ and $t_2$. In addition, there is normally a upper limit for the surface slope values and a lower limit for sag values, depending on different fabrication techniques. Both the slope and sag values decrease by increasing the lens separation $d$. We have selected 5mm and 5.1mm respectively for $t_1$ and $t_2$ with $d = 200mm$. The simulated irradiance profile at the focal plane is given in Fig. 5(a). By having the input beam radius $r_o = 0.34mm$ directly at the input plane, the flat-top profile can be well maintained.

To compare our results, we have used the Fourier approach to do the same design task by using the solution shown in Fig. 3 to enlarge $\beta$. The telescope is formed by two ideal lenses with the first one negative and the second one positive. The phase element is assumed to be a plano-asppheric lens with very thin center thickness 1 $\mu$m, and the thickness of the lens varies to fit the required phase. The transform element is assumed to be an ideal lens with the required focal length $f$. The separation $d$ is also very small in the range of a few $\mu$m to hundreds of $\mu$m just enough to have no overlaps between two elements. The simulation results in Fig. 5(b) show that the external beam expansion has to be 16 times, increasing $\beta$ from 2 to 32, so that the achieved beam shaping quality starts to match the result obtained with our approach. This result underlines the necessity to use an extra beam expanding system in case that the initial $\beta$ value is not large enough.
4. CONCLUSION

Within the scope of this work, we have presented a new design strategy and method for focal beam shaping based on two plano-aspheric lenses that allows to control both irradiance and phase at focus. This additional phase control at laser focus is realized by combining both geometrical optics and wave optics: an approach that could also prove useful for other laser-based system designs with diffraction effects included. Unlike the Fourier approach, our approach does not require to use additional beam expanders to increase the otherwise too low $\beta$ value. For the example of transforming Gaussian to a circular flat-top pattern at focus, the initial $\beta = 2$ value had to be increased to 32 with a $16\times$ beam expander for the Fourier approach in order to achieve the shaping quality that is comparable to the result obtained for the two lenses used in our approach. Without using additional beam expansions, it does not only reduce the size of the system, but also use less optical elements to ease the alignment efforts.

Future work will focus on the generalization to achieve non-rotationally symmetric irradiance patterns at focus using freeform optics and on building a first prototype to verify the results experimentally.

ACKNOWLEDGMENTS

The work reported in this paper is mainly funded by a SBO Project grant 110070: eSHM with AM of the Agency for Innovation by Science and Technology (IWT). It is also supported by the Research Foundation - Flanders (FWO-Vlaanderen) that provides a post-doctoral grant to Fabian Duerr, and in part by the IAPBELSPO grant IAP P7-35 photonics@be, the Industrial Research Funding (IOF), FWO (G008413N), the MP1205, COST Action, the Methusalem and Hercules foundations and the OZR of the Vrije Universiteit Brussel (VUB).

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