Series and Parallel Elastic Actuation: 
Impact of Natural Dynamics on Power and Energy 
Consumption

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\textbf{Abstract}

This paper provides a detailed analysis of the power and mechanical/electrical energy consumption of Series Elastic Actuators (SEAs) and Parallel Elastic Actuators (PEAs). The study is done by imposing a sinusoidal motion to a pendulum load, such that the natural dynamics automatically present itself in the power and energy consumption. This allows to link the actuators' dynamics to their loss mechanisms, revealing interesting characteristics of series and parallel elastic elements in actuator designs. Simulations demonstrate that the SEA and PEA allow to decrease both peak power and energy consumption, provided that the stiffness of their elastic element is tuned properly. For the SEA, both are minimized by tuning the elastic element to the antiresonance frequency of the actuator. For the PEA, peak power is minimal at the link's resonance frequency, but the optimal stiffness for minimal electrical energy consumption cannot be determined by a theoretical resonance and needs to be calculated using a complete system model. If these guidelines are followed, both types of elastic actuators can provide significant energetic benefits at high frequencies. This was confirmed by experiments, which demonstrated energy reductions of up to 78% (SEA) and 20% (PEA) compared to rigid actuators.

\textbf{Keywords:} Compliant actuators; Energy efficiency; Dynamics; Series Elastic Actuators; Parallel Elastic Actuators

\section{Introduction}

Recent developments in robotics have aimed to bring robots to our homes and working environments, focusing on human-robot interaction instead of care-
fully separating the robot’s task space from the user. The requirement of close
human-robot interaction results in an increasing relevance of soft or elastic robot
designs [10, 14, 19], which can provide human safety through reacting by de-
formation in case of contact [10, 14]. Further, elastic designs yield benefits
in robotic motion assistance [1, 11] and rehabilitation [15]. Besides improving
human-robot interaction, elastic actuator designs can increase the energy
efficiency of mobile (assistive) devices by adapting actuator elasticity to the op-
erating state [29], e.g., by matching the natural behavior of the system with
the trajectory frequency [3, 4, 27, 30].

Motivated by these promising examples, various concepts for actuators with
fixed or variable elasticity have been proposed in recent years [28]. Many of
those incorporate a series elastic element as a compliant coupling between drive
and link in order to enable safe human-robot interaction. Such compliant cou-
plings help to reduce the risk of injuries, since the robotic structure can deform
upon impact with humans. This concept, which is generally known as Series
Elastic Actuation (SEA), was introduced in the middle of the 1990s [18, 22].
While the mechanical stiffness of the implementation in [22] is fixed, the stiff-
ness of the concept from [18] can be varied. The actuator presented in this work
can therefore also be considered as one of the first Variable Stiffness Actuators
(VSAs) [28].

Regardless of stiffness variation capabilities, the configuration of elastic ele-
ments and motors has a significant impact on the dynamics and energy efficiency
of elastic actuators. As mentioned before, most contemporary concepts utilize
an elastic element in series with the actuator. Yet, parallel elastic actuators
(PEA) or actuators combining serial and parallel elastic elements can also yield
advantageous power/energy characteristics [9, 16, 17, 21] if the parallel spring
is tuned considering task-specific requirements, e.g., by setting the appropriate
equilibrium angle. Further, the analysis of dynamic properties like inertial or
gravitational effects in elastic actuators shows that those have distinct influence
on the natural dynamics [3, 2]. Hence, a precise characterization and model-
ing is crucial to exploit natural dynamics by design and control. However, the
dynamic interaction between motor and load is not sufficiently considered for
dimensioning in many cases, e.g., [13, 22, 23, 26, 29, 33]. Analyzing natural
dynamics of series elastic actuators considering the interaction of motor and
load shows that natural and antiresonance modes can be exploited [3] and lead
to significant decreases in energy consumption [2]. In conclusion, there seems
to be a demand for a detailed comparison of the SEA and PEA concepts with
respect to natural dynamics and power/energy consumption.

This paper compares the natural dynamics and power/energy characteristics
of rigid actuation (RA), PEA, and SEA. To investigate mechanical and
electrical energy, the dynamics of the whole system comprising load, actuator,
kinematics/gear boxes and electronics are considered, including energy regen-
eration as suggested in [2, 32]. In this regard, the paper differs from the work
done by Grimmer et al. [9], which only compared the SEA and PEA in terms
of mechanical peak power and mechanical energy consumption and, hence, did
not take motor and gearbox properties into account. Furthermore, the work of
Grimmer et al. presents a large set of simulations of a very specific application: a prosthetic ankle actuator at different walking and running speeds. In this work, the aim is to find the relationships between the dynamics of the actuators and their power and energy consumption in order to provide insight into the specific properties of series and parallel elastic elements. As such, the simulations and experiments apply to a very general case which allows to identify the natural dynamics of the actuator: a sinusoidal trajectory applied to a 1-DOF link.

The paper is structured as follows. Section 2 describes the investigated actuator types, corresponding models, and their dynamics. A power and energy analysis based on simulations is given in Section 3 to identify favorable operation modes and compare the different concepts at variable operating frequencies and stiffnesses. Experimental investigations with the test setup from [32] are used to evaluate the simulation results in a real system and shown in Section 4. Finally, the results of the paper are discussed in Section 5 and summarized in Section 6.

2. Actuator types and their dynamics

Figure 1 presents schematics of the three studied actuator topologies, moving a one degree of freedom pendulum with a mass $M$ and a length $l$ (the distance between the rotation axis and the center of mass). Combined actuator and gearbox inertia is denoted as $J_m + J_{tr}$. As seen in Figures 1a and 1b, the load inertia is $J_l$ and angular positions of the pendulum are equal to those at the gearbox output in the RA and PEA cases. Pendulum motion corresponds to the reduced motor motion $\theta = n^{-1}\theta_m$, where $\theta$ and $\theta_m$ are the positions of the
output and motor, respectively, and \( n \) is the gear ratio. The frontal view of the pendulum given in Figure 1d defines the direction of \( \theta \) as well as its maximum and minimum values \( \pm \theta_{max} \). Motor torque \( T_m \), as defined in Figure 1a-c, is the sum of the torque available at the motor shaft and the torque required to accelerate the rotor inertia \( J_m \).

The stiffness of the parallel elasticity in the PEA (Figure 1b) is given by \( k_p \). Considering a SEA with series stiffness \( k_s \) (Figure 1c), inertias \( J_{l1} \) and \( J_{l2} \) are separated by the elastic element, and as a result, the positions of pendulum \( \theta \) and gearbox output \( n^{-1} \theta_m \) differ.

2.1. Rigid actuation

Considering the topology given in Figure 1a and friction effects, the system’s equations of motion are given by

\[
T_m = (J_m + J_{tr})n \ddot{\theta} + \frac{C}{n} T_{load} \tag{1}
\]

where \( T_{load} \) is defined as

\[
T_{load} = J_l \ddot{\theta} + T_c \text{sign} \left( \dot{\theta} \right) + \nu_l \dot{\theta} + Mgl \sin \theta \tag{2}
\]

The first term on the right side of Eq. (1) represents the inertial torque due to rotating components in the gearbox and motor. \( T_{load} \) represents the torque due to the motion of the pendulum load. Essentially, it includes the gravitational and inertial pendulum loads as well as Coulomb and viscous friction, characterized by their respective coefficients \( T_{c,l} \) and \( \nu_l \). The term is scaled by the gear ratio \( n \) and the gearbox efficiency function \( C \):

\[
C = \begin{cases} 
1/\eta_{tr} & \text{(load driven by motor)} \\
\eta_{tr} & \text{(motor driven by load)} 
\end{cases} \tag{3}
\]

which models the behavior of a gearbox in a dynamic system, including the effects of power flow reversal [8]. For RA and PEA, \( T_{load} \dot{\theta} \) designates whether the motor is driving the load or vice versa. If \( T_{load} \dot{\theta} \geq 0 \), the power through the gearbox is positive and the load is driven by the motor, so \( C = 1/\eta_{tr} \). Conversely, if \( T_{load} \dot{\theta} < 0 \), the motor is driven by the load and \( C = \eta_{tr} \).

As shown in [3, 32], the natural behavior of actuation systems is crucial for achieving energy efficient operation. At resonance frequency, a system with one degree of freedom requires nearly no torque\(^1\) to perform an oscillating motion, regardless of the desired amplitude. Consequently, an actuator which operates near resonance can potentially be designed to incorporate smaller motors and will consume only a small amount of power.

The resonance frequency is calculated from the equations of motion of the linearized, frictionless system. Rewriting Eq. (1) as such, and transforming it to the frequency domain, we find

\(^1\)The motor still needs to deliver some torque to compensate for the energy losses.
The resonance frequencies can then be found by imposing $T_m(\theta) = 0$ and solving for $\omega$. This results in

$$\omega_{\text{rs,RA}} = \pm \sqrt{\frac{Mgl}{J_l + n^2(J_m + J_{tr})}}$$

(5)

There is only one resonance frequency, since the system given by Eq. (4) is of second order. We can, however, find more resonance frequencies by looking at different subsystems of the actuator. As discussed in [32], another resonance frequency is of importance in rigid actuators. It is retrieved from the torque on the gearbox shaft, given by Eq. (2). After linearizing and removing friction terms, we find

$$T^*_{\text{load}} = J_l \ddot{\theta} + Mgl\theta$$

(6)

which yields the resonance frequency of the link subsystem

$$\omega_{\text{rl,RA}} = \pm \sqrt{\frac{Mgl}{J_l}}$$

(7)

2.2. Parallel Elastic Actuation

The equilibrium angle of the Parallel Elastic Actuator’s parallel spring can be an important design parameter when it comes to energy efficiency [6]. In this pendulum setup, however, torques and angles are symmetric for trajectories that have no offset, making an equilibrium angle of 0° the obvious choice. Consequently, the required motor torque can be calculated with

$$T_m = (J_m + J_{tr})n\ddot{\theta} + C \left[ J_l \ddot{\theta} + T_{c,l}\text{sign} \left( \dot{\theta} \right) + \nu_l\dot{\theta} + Mgl\sin \theta + k_p\theta \right]$$

(8)

This equation is identical to that of a stiff actuator (Eq. (1)), except for the additional spring term $k_p\theta$, in which $k_p$ is the spring constant of the parallel spring. The single resonance frequency of this second-order system is:

$$\omega_{\text{rs,PEA}} = \pm \sqrt{\frac{Mgl + k_p}{J_l + n^2(J_m + J_{tr})}}$$

(9)

Similarly, we can associate a resonance frequency with the torque at the gearbox shaft:

$$\omega_{\text{rl,PEA}} = \pm \sqrt{\frac{Mgl + k_p}{J_l}}$$

(10)

These formulas demonstrate that parallel stiffness can be used to modify the pendulum’s natural frequency. Note that, if $k_p = 0$, we retrieve the same resonance frequency as that of the stiff system (given by Eq. (5)).
2.3. Series Elastic Actuation

Due to the decoupling of motor and gearbox inertia, the equations of motion of the Series Elastic Actuator are more sophisticated than the ones presented previously. The spring introduces a relationship between the output angle and the motor angle which depends on the load torque. Defining the load torque \( T_{load,SEA} \) of the SEA as

\[
T_{load,SEA} = J_{l2} \ddot{\theta} + \nu_2 \dot{\theta} + T_{c,l2} \text{sign}(\dot{\theta}) + Mgl \sin \theta
\]

where \( \nu_2 \) and \( T_{c,l2} \) represent the viscous and Coulomb friction coefficients of output shaft, the relationship between the motor angle \( \theta_m \) and output angle \( \theta \) can be written as

\[
\theta_m = n \left( \frac{T_{load,SEA}}{k_s} + \theta \right)
\]

The equations of motion are given by

\[
\begin{cases}
T_{load,SEA} - k_s \left( \frac{\dot{\theta}_m}{n} - \dot{\theta} \right) = 0 \\
(nJ_m + nJ_{tr} + J_{l1}) \ddot{\theta}_m + \frac{c_s}{n} \left[ k_s \left( \frac{\dot{\theta}_m}{n} - \theta \right) + T_{c,l1} \text{sgn}(\dot{\theta}_m) + \nu_1 \ddot{\theta}_m \right] = T_m
\end{cases}
\]

Here, \( T_{c,l1} \) and \( \nu_1 \) are the Coulomb and viscous friction coefficients of the shaft connected to the gearbox output. The first equation represents the dynamics of the link, the second those of the motor. The gearbox efficiency function \( C \) is still defined as in Eq. 3, but here, the case where the motor is driving the load is designated by

\[
\dot{\theta}_m \left( T_m - (J_m + J_{tr})\ddot{\theta}_m \right) \geq 0
\]

meaning that the power through the gearbox is positive.

Removing all friction terms and linearizing, we can combine both equations of (13) to

\[
(nJ_m + nJ_{tr} + J_{l1}) \ddot{\theta}_m + \frac{1}{n} \left( J_{l2} \ddot{\theta} + Mgl \theta \right) = T_m
\]

Replacing \( \ddot{\theta}_m \) in this equation by the second derivative of Eq. (12) and transforming it to the frequency domain, we find a transfer function

\[
G(\omega) = \frac{\theta}{T_m} = \frac{k_s}{c_4 \omega^4 + c_2 \omega^2 + c_0}
\]

with coefficients

\[
\begin{align*}
c_4 &= (nJ_m + nJ_{tr} + J_{l1})J_{l2} \\
c_2 &= -(nJ_m + nJ_{tr} + J_{l1})(Mgl + k_s) - \frac{k_s}{n} J_{l2} \\
c_0 &= k_s \frac{1}{n} Mgl
\end{align*}
\]
The poles of this fourth-order transfer function correspond to the two resonance frequencies $\omega_{r1,SEA}$ and $\omega_{r2,SEA}$ of the system. They are given by

$$\omega_{r1/2,SEA} = \pm \left( -c_2 \pm \sqrt{c_2^2 - 4c_4c_0} \right)^{1/2}$$  \hspace{1cm} (17)

Alternatively, by rewriting Eq. (14) as a function of $\theta_m$ and transforming it to the frequency domain, we find the transfer function

$$H(\omega) = \frac{\theta_m}{T_m} = -\frac{nJ_{l2}\omega^2 + n (Mgl + k_s)}{c_4\omega^4 + c_2\omega^2 + c_0}$$  \hspace{1cm} (18)

with coefficients $c_4$, $c_2$ and $c_0$ as defined in Eq. (16). The zeros of this transfer function,

$$\omega_{a,SEA} = \sqrt{\frac{k_s + Mgl}{J_{l2}}}$$  \hspace{1cm} (19)

correspond to the antiresonance frequencies of the system, at which the output can oscillate while the input (motor) is standing still. Since mechanical power is proportional to speed, antiresonance can be expected to be an advantageous operating point, just like resonance. Note the similarity between Eq. (19) and Eq. (10), the only difference being the exclusion of the gearbox shaft’s inertia in Eq. (19). Since $J_{l2}$ is bigger than $J_{l1}$ in most practical systems, $J_l \approx J_{l2}$ and thus $\omega_{a,SEA} \approx \omega_{r1,PEA}$ for identical spring stiffnesses $k_s$ and $k_p$.

2.4. Motor model

A crucial part of motor-based actuation systems is undoubtedly the motor itself. Irrespective of its type, motor data sheets usually only specify a maximum efficiency. However, when a motor is operated at variable load and/or varying speed, motor efficiency can drop far below this value [32, 7]. This is why in this paper, a DC motor model is used to calculate load- and speed-dependent motor losses.

2.4.1. Motor equations

The electrical power consumption $P_{elec}$ is calculated as follows:

$$P_{elec} = UI$$  \hspace{1cm} (20)

In this equation, the motor voltage $U$ and current $I$ can be calculated by using the common DC motor model, provided that the motor torque $T_m$ and the motor shaft speed $\dot{\theta}_m$ are known:

$$\begin{cases} I = \frac{T_m + \nu_m \dot{\theta}_m}{k_t} \\ U = L \frac{dI}{dt} + RI + k_b \dot{\theta}_m \end{cases}$$  \hspace{1cm} (21)

In this equation, $R$ is the winding resistance and $L$ is the terminal inductance. $k_t$ and $k_b$ represent the torque and speed constants of the motor and have
equal values. All these parameters are readily available on the manufacturer’s datasheet. The losses due to motor bearing friction, friction of brushes, etc. are represented by a viscous damping term \( \nu_m \dot{\theta}_m \). Generally, the value for the motor’s viscous damping coefficient \( \nu_m \) is not specified on the motor datasheet. However, by assuming that the consumed current at the no-load speed \( \omega_{nl} \) is exactly the no-load current \( I_{nl} \), the following approximation can be found:

\[
\nu_m = \frac{k_t \cdot I_{nl}}{\omega_{nl}}
\]

The minimum voltage can therefore be expected to coincide with the antiresonance frequency.

Regarding current, the \( \nu_m \dot{\theta}_m \) term in Eq. (21) can be significant when low torques are commanded. Throughout the rest of the operating region, however, \( T_m \) will dominate. Consequently, the dynamics of motor current \( (\dot{\theta}/I_m) \) will roughly correspond to that of motor torque \( (\dot{\theta}/T_m) \), and resonance frequencies will lead to a reduction in motor current.

2.4.2. Influence on natural system dynamics

The DC motor also introduces electrical dynamics to the system. Before discussing these dynamics, some simplifications can be made to facilitate the analysis. Regarding voltage, the motor’s terminal inductance \( L \) is generally several orders of magnitude smaller than its winding resistance \( R \), meaning that the \( L \frac{dI}{dt} \) term can be neglected unless the application requires extreme variations in current. Furthermore, the term \( RI \) is a few orders of magnitude smaller than the back-EMF \( k_b \dot{\theta}_m \), except at very low speeds. Consequently, for the majority of the motor’s operating region, \( U = k_b \dot{\theta}_m \), so the dynamics of motor voltage \( U \) are similar to that of motor speed \( \dot{\theta}_m \), except for an additional gain and an additional pole at \( \omega = 0 \). The minimum voltage can therefore be expected to coincide with the antiresonance frequency.

Regarding current, the \( \nu_m \dot{\theta}_m \) term in Eq. (21) can be significant when low torques are commanded. Throughout the rest of the operating region, however, \( T_m \) will dominate. Consequently, the dynamics of motor current \( (\dot{\theta}/I_m) \) will roughly correspond to that of motor torque \( (\dot{\theta}/T_m) \), and resonance frequencies will lead to a reduction in motor current.

3. Power and energy analysis

As in previous works by Vanderborght et al. [29], Beckerele et al. [3] and Verstraten et al. [32], a pendulum setup is used to evaluate the performance of the actuator. The actuator, which is identical to the one in [32], can be mechanically modified to a RA, a SEA or a PEA. It imposes a sinusoidal trajectory with variable frequency around an equilibrium position \( \theta = 0^\circ \), which corresponds to a vertical line. The amplitude of the motion is 30°, which is significantly larger.
Table 1: Pendulum properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gearbox shaft inertia $J_{t_1}$</td>
<td>1.31e-4 kgm$^2$</td>
</tr>
<tr>
<td>Output shaft inertia $J_{t_2}$</td>
<td>1.57e-1 kgm$^2$</td>
</tr>
<tr>
<td>Mass $M$</td>
<td>1.85 kg</td>
</tr>
<tr>
<td>Distance from rotation axis to COG $l$</td>
<td>0.241 m</td>
</tr>
<tr>
<td>Coulomb friction coefficient $T_{c,t_1}$</td>
<td>0.049 Nm</td>
</tr>
<tr>
<td>Coulomb friction coefficient $T_{c,t_2}$</td>
<td>0.064 Nm</td>
</tr>
<tr>
<td>Damping coefficient $\nu_{t_1}$</td>
<td>0.044 Nms/rad</td>
</tr>
<tr>
<td>Damping coefficient $\nu_{t_2}$</td>
<td>0.079 Nms/rad</td>
</tr>
</tbody>
</table>

Table 2: Motor and gearbox properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal power $P_{\text{nom}}$</td>
<td>80 W</td>
</tr>
<tr>
<td>Torque constant $k_t$</td>
<td>23.4 mNm/A</td>
</tr>
<tr>
<td>Speed constant $k_b$</td>
<td>408 rpm/V</td>
</tr>
<tr>
<td>Motor efficiency $\eta_{\text{max}}$</td>
<td>87.8%</td>
</tr>
<tr>
<td>Terminal resistance $R$</td>
<td>0.212 $\Omega$</td>
</tr>
<tr>
<td>Terminal inductance $L$</td>
<td>0.0774 mH</td>
</tr>
<tr>
<td>No-load current $I_{nl}$</td>
<td>177 mA</td>
</tr>
<tr>
<td>No-load speed $\omega_{nl}$</td>
<td>7200 rpm</td>
</tr>
<tr>
<td>Motor inertia $J_m$</td>
<td>102 gcm$^2$</td>
</tr>
<tr>
<td>Gear ratio $n$</td>
<td>338/3</td>
</tr>
<tr>
<td>Gearbox inertia $J_{t_r}$</td>
<td>5e-7 kg m$^2$</td>
</tr>
<tr>
<td>Gearbox efficiency $\eta_{t_r}$</td>
<td>72%</td>
</tr>
</tbody>
</table>

The physical properties of the pendulum, derived from a CAD model, are listed in Table 1. The properties of the motor and gearbox can be found in Table 2. Note that, for RA and PEA, the Coulomb friction coefficients and damping coefficients are lumped together because of the rigid connection between the load shaft and gearbox shaft:

\[
\begin{align*}
T_{c,t} &= T_{c,t_1} + T_{c,t_2} \\
\nu_t &= \nu_{t_1} + \nu_{t_2}
\end{align*}
\]

The same pendulum setup was also used in previous papers on the modeling and energy consumption of rigid actuators [32, 31]. For the comparison between SEA and PEA presented in this paper, the amplitude $\theta_{\text{max}}$ is restricted to 30° because of limitations on the maximum extension of the springs on the physical setup, something which is especially critical for the PEA. This way, the SEA and the PEA can impose the same motion to the load without reaching the physical limits of the system, such that a fair comparison between both is obtained.

The simulations are based on the equations established in Section 2. The imposed trajectory is assumed to be tracked perfectly by both actuators, i.e. the angle $\theta$ can be replaced by the time-dependent function

\[
\theta = \theta_{\text{max}} \sin (\omega t)
\]
Figure 2: Mechanical peak power for PEA (left) and SEA (right), for varying swinging frequencies and spring stiffnesses. Minimal mechanical peak power for the PEA approximately corresponds to the system’s resonance frequency $\omega_{rs,PEA}$. For the SEA, the minimum occurs at the antiresonance frequency $\omega_{a,SEA}$, but a decrease is also observed at the resonance frequency $\omega_{r1,SEA}$.

### 3.1. Mechanical peak power

Figure 2 shows mechanical peak power for the PEA (left) and SEA (right), as a function of frequency and spring stiffness. The power is calculated as the product of motor speed $\dot{\theta}_m$ and motor torque $T_m$ (taking into account the inertia at the motor itself).

Apart from the obvious minimum at $\omega = 0$, the PEA only demonstrates one minimum in the peak power plot, occurring at the resonance frequency $\omega_{rs,PEA}$. For the SEA, there are two minima: one at the first resonance frequency $\omega_{r1,SEA}$ (local minimum, less distinct) and one at antiresonance $\omega_{a,SEA}$ (global minimum, very distinct). The second resonance frequency $\omega_{r2,SEA}$ does not lead to a clear minimum, which is in line with the results obtained with another test rig in [2]. The plots thus suggest that the optimal operating points in terms of mechanical peak power lie at the SEA’s antiresonance frequency $\omega_{a,SEA}$ and at the PEA’s resonance frequency $\omega_{rs,PEA}$. As seen in Figure 2, the PEA requires higher stiffnesses than the SEA in order to operate at its optimal frequency, confirming the results from [3].

For $k_p \to 0$ and $k_s \to \infty$, we find that $\omega_{rs,PEA}$ and $\omega_{r1,SEA}$ both converge to the same resonance frequency of 3.9 rad/s, which corresponds to the rigid system’s resonance frequency $\omega_{rs,RA}$ given by Eq. (5). The introduction of a series spring hardly affects this resonance frequency, except at very low stiffnesses. However, such compliant springs may compromise the operation of the SEA due to the large spring extensions required, possibly demanding excessive speeds from the motor. In the PEA, on the other hand, the presence of the spring moves the resonance frequency to higher values. This can be exploited in the design of the PEA to decrease peak power. Note that, the stiffer the parallel spring is, the lower the decrease in mechanical peak power at resonance will be. This is because the mechanical power is the product of torque and speed, of which only the former is decreased at resonance, and the latter in-
creases with frequency. As seen in Figure 2, frequency is positively related to stiffness at resonance, and therefore energy consumption increases with stiffness as well. Conversely, in the SEA at antiresonance, the speed is decreased instead of the torque. As a result, the mechanical peak power is largely unaffected by an increase in frequency, and low peak powers can still be achieved at high frequencies.

Finally, comparing the theoretical resonance and antiresonance lines to the actual minima of the plot, one observes that optimal peak power occurs at a slightly lower stiffness than expected for PEA. Conversely, for the SEA, optimal stiffness is slightly higher than predicted by the $\omega_a$ line. These deviations are due to the combination of nonlinearities and friction.

3.2. Energy consumption

Essentially, the power is the sum of two contributions $P_{\text{load}}$ and $P_{\text{loss}}$:

$$P = P_{\text{load}} + P_{\text{loss}}$$

where $P_{\text{load}}$ represents the power consumed by the load (i.e. the power consumed by the lossless system) and $P_{\text{loss}}$ the power losses. The latter can be attributed to bearing and gearbox friction (mechanical losses) and the motor’s winding resistance (electrical losses). Throughout most of the motion, the power consumed by the load will be the dominant factor; as a result, power is strongly related to the motion and the properties of the output link.

By definition, energy is calculated as the integral of power\(^2\):

$$E = \int P \, dt$$

$$= \int P_{\text{load}} \, dt + \int P_{\text{loss}} \, dt$$

$$= E_{\text{load}} + E_{\text{loss}}$$

In the specific case studied in this paper, $E_{\text{load}}$ equals zero because the imposed motion is a cyclic motion applied to a conservative force field. Consequently, the total energy consumption $E$ is equal to the system's energy losses $E_{\text{loss}}$. In contrast to power, energy consumption will therefore be dictated by the energy losses rather than the motion of the load itself. Of course, most losses being closely related to torque, energy losses can also be expected to be somehow related to the output link and its motion. Nevertheless, there are some differences between the peak power of SEAs and PEAs and their — mechanical or electrical — energy consumption. Those differences are the subject of this section.

\(^2\)Note that, by calculating the energy as the integral of power, it is implicitly assumed that negative power can be regenerated. While dynamic applications require a 4-quadrant controller, which inherently has the ability to regenerate power, battery protection circuits may still limit the amount of negative power that can be sent back to the battery. The consequences for the energy consumption are discussed in [32].
3.2.1. Mechanical energy consumption

Figure 3 shows mechanical energy consumption for the PEA (left) and SEA (right), as a function of frequency and spring stiffness. Mechanical energy is calculated as the integral of the mechanical power. As in section 3.1, mechanical power is the product of speed and torque at the motor (including the torque due to the acceleration of the motor’s own inertia). Consequently, it is the mechanical energy required to complete one entire pendulum cycle at a specific frequency.

For the PEA, mechanical energy is minimal at link resonance $\omega_{rl,PEA}$ (i.e. resonance without motor inertias). This is a significant difference with the results in mechanical peak power, where the minimum occurred at motor resonance $\omega_{rs,PEA}$. At $\omega_{rs,PEA}$, motor torque is reduced, leading to minimal (electrical) motor losses. However, as explained in [32], gearbox torque is nonzero at this frequency because of the inertia of the motor, which adds an inertial torque. Gearbox losses, which are proportional to the torque going through it, are instead minimized at the resonance frequency of the output link $\omega_{rl,PEA}$. The minimum in mechanical energy consumption, which is dominated by the gearbox losses, therefore lies at $\omega_{rl,PEA}$ instead of $\omega_{rs,PEA}$.

For the SEA, the global minimum still occurs at antiresonance. The second resonance frequency, again, does not lead to a reduction in mechanical energy consumption. A local minimum is present at about 5 rad/s, a slightly higher frequency than the first resonance frequency $\omega_{rl,SEA}$. The reasons for this shift are the same as those for the PEA: for minimal mechanical power consumption, gearbox losses need to be minimized, which is the case at $\omega_{rl,RA}$ instead of $\omega_{rl,SEA}$ (minimal motor losses).

As was the case for mechanical power, we observe that, if the SEA is operated at antiresonance, energy consumption is low regardless of the frequency. For the PEA, higher frequencies demand higher mechanical energy input, because speed
Figure 4: Electrical energy consumption per cycle for PEA (left) and SEA (right), for varying swinging frequencies and spring stiffnesses. Minimal electrical energy consumption for the PEA occurs somewhere in between the resonance frequencies $\omega_{r,s,PEA}$ and $\omega_{r,l,PEA}$. The SEA has a distinct minimum at an antiresonance $\omega_{a,SEA}$.

is not reduced by the parallel spring. Also note that, as explained in Section 2.3, $\omega_{r,l,PEA}$ and $\omega_{a,SEA}$ are nearly identical. Hence, for minimizing the mechanical energy consumption, there is almost no difference between the optimal spring stiffness of the SEA and PEA.

3.2.2. Electrical energy consumption

Figure 4 shows electrical energy consumption for the PEA (left) and SEA (right), as a function of frequency and spring stiffness. Electrical energy is calculated as the integral of electrical power, which is the product of motor voltage and current. As predicted in Section 2.4, the plots are very similar to those for the mechanical energy, but the results are more pronounced due to the addition of the Joule and gearbox losses, which are both very dependent on torque.

For the PEA (left of Figure 4), minimum electrical energy consumption occurs in between gearbox resonance $\omega_{r,l,PEA}$ (gearbox losses minimized) and motor resonance $\omega_{r,s,PEA}$ (Joule losses minimized). Neither of both lines provides a good approximation of the optimum by itself. In this case, the minimum is closer to gearbox resonance, but this observation cannot be generalized to any actuator system. The location of the actual minimum depends entirely on the system’s losses. If most losses are due to the gearbox, the minimum will be closer to $\omega_{r,l,PEA}$; if motor losses dominate, the minimum will be closer to $\omega_{r,s,PEA}$. Therefore, the optimal stiffness needs to be calculated based on the full equations of the system, including friction factors, gearbox efficiency and a motor model.

For the SEA (right of Figure 4), antiresonance can serve as a good approximation to calculate the optimal spring stiffness. There is a clear minimum in the electrical energy consumption, which does not deviate much from the theoretical antiresonance line. Energy consumption is also lowered at a frequency slightly higher than resonance, but the reduction is not as pronounced as at
an tiresonance.

Again, we notice that the PEA's electrical energy consumption rises at higher frequencies due to the increased speed required from the motor. The SEA maintains low energies up to higher frequencies. However, unlike in the mechanical power and energy plots, electrical energy of the SEA also increases at higher frequencies. This is due to the proximity of the resonance frequency $\omega_{r,2,SEA}$ which reduces motor torque, and the link's resonance frequency $\omega_{r,1,RA}$, at which gearbox torque is minimized. As $\omega_{r,2,SEA}$, $\omega_{r,0,SEA}$ and $\omega_{r,1,RA}$ approach each other at low stiffnesses, a simultaneous reduction of torque and speed occurs, and energy consumption will drop to rather low values. At higher stiffnesses, the SEA still profits from the decrease in speed, but the torque-reducing influence of $\omega_{r,2,SEA}$ and $\omega_{r,1,RA}$ is less strong, leading to an increased energy consumption.

In conclusion, minimum mechanical peak power, minimum mechanical energy and minimum electrical energy all occur at different frequencies for the PEA. This makes the calculation of its optimal spring stiffness a difficult task, which is highly dependent on the model being used. For the SEA, the calculation is a lot simpler, since in good approximation all minima can be traced down to the antiresonance frequency. Moreover, in this set of simulations, the SEA at antiresonance outperforms the PEA in terms of peak power and, especially, energy consumption.

### 3.3. Comparison with rigid actuation

In this section, we will compare the energy consumption of the PEA and the SEA to that of rigid actuation. The relative difference is calculated as

$$e = \frac{E - E_{RA}}{E_{RA}}$$

where $E$ stands for the energy consumption of the PEA or SEA, depending on which type of actuation is considered for the comparison. The results for different frequencies and stiffnesses are shown in Figure 5.

In this case study, elastic elements do not yield energetic advantages below 5 rad/s. For the PEA, the most distinct reduction in energy consumption is focused around $\omega_{r,1,PEA}$. The favorable region includes this frequency completely, while $\omega_{r,s,PEA}$ does not yield a reduction in energy consumption at frequencies close to $\omega_{r,1,RA}$. As expected, antiresonance marks the greatest energy reduction for the SEA. Reduced energy consumption covers a certain area around this frequency. Hence, tuning for antiresonance is advantageous, even if the model does not fit the real system perfectly. The resonances lie outside the beneficial areas for the SEA and should therefore be avoided [2]. In fact, the resonance frequency $\omega_{r,2,SEA}$ approximately matches the upper boundary of energetic advantage of the SEA. The lowest frequency at which an energetic advantage is achieved with series springs is $\omega_{r,1,RA} = 5.3$ rad/s; for any frequency below this value, RA will perform better. Note that, at $k_p = 0$, the PEA transforms into a system equivalent to RA, hence $e = 0$. Consequently, in contrast to the SEA, there is no practical lower boundary on the spring stiffness of the favorable region for the PEA.
Figure 5: Relative difference between energy consumption of the rigid actuator compared to the elastic actuator. Left: rigid-PEA. Right: rigid-SEA. The black lines denote the combinations of frequencies and stiffnesses at which RA and PEA (RA and SEA) perform equally well. The highest relative differences are situated around $\omega_{rl,PEA}$ (PEA) and $\omega_{a,SEA}$ (SEA). At any frequency below $\omega_{rl,RA} = 5.3 \text{ rad/s}$, RA performs best. Note that RA corresponds to the cases of zero parallel stiffness ($k_p = 0 \text{ Nm/rad}$ in the plot to the left) or infinite series stiffness ($k_s \to \infty$ in the plot to the right).

(a) PEA  
(b) SEA

Figure 6: Setups used for experiments. 1. Motor-gearbox, 2. Gearbox shaft encoder, 3. Torque sensor, 4. Springs, 5. Output shaft encoder

4. Experimental evaluation

In order to evaluate the conclusions from the power and energy analysis, the electrical energy plots were obtained experimentally from a physical setup matching the properties of the simulated system. In this section, the physical test set-up is described along with the corresponding control algorithms. Subsequently, the experimental results are presented, discussed and compared to the simulations.

4.1. Test setup

The tests are performed on a set-up that corresponds to the pendulum presented in Section 2. Its parameters are given in Table 1 and are identical to those that are used for the simulations in Section 3. The actuator consists of a
80 W Maxon DCX35L motor and a Maxon GPX42 338:3 planetary gearbox (1), of which the parameters are listed in Table 2. The compliant elements are implemented by means of two antagonistic tension springs (4), which are mounted on one of both sides of the pendulum to yield a PEA (Figure 6a) or a SEA (Figure 6b). Additionally, the springs can be replaced to change the stiffness of the actuator in discrete steps between the experimental trials. The resulting rotational stiffness values can be varied between 2 Nm/rad and 10 Nm/rad for the SEA and between 0.3 Nm/rad and 2.3 Nm/rad for the PEA. By removing the parallel springs and making the series connection rigid, the RA case can be investigated as well.

A torque sensor (3) and an encoder (2) are placed on the gearbox shaft to measure the mechanical energy consumption. The torque sensor is an ETH Messtechnik DRBK torque transducer (range 20 Nm, accuracy 0.5%) while positions are acquired using a US Digital E6 series optical encoder with 2000 counts per turn. To measure the output shaft angle of the SEA, another unit of this encoder type (5) is placed on a measurement shaft, which is connected to the load shaft with a pulley (1:1 ratio). The inertial properties of the torque sensor (136 gcm²) and the encoder wheels (0.073 gcm²) are included in the simulation, despite their insignificance with respect to the total inertia of the system. To assess electrical energy consumption, the voltage at the motor terminals and the motor current are measured. To sense current, an Allegro ACS712 current sensor with a range from -5 A to 5 A, a total output error of 1.5%, and a resistance of 1.428 mΩ is used. All sensors and the resistance of the cables in between the motor and the controller (0.228 Ω) are considered in the system model in order to obtain an appropriate comparison of simulations and experiments.

The sensory data is acquired with a National Instruments sbRIO 9626 board, which is also used to implement the control algorithms. For every measurement, at least ten pendulum periods are recorded after start-up transients receded. The measurement is then decomposed into separate sine periods, which are averaged with respect to one another in order to reduce noise and other non-reproducible effects.

### 4.2. Control algorithm

As demonstrated in Section 2, the equations of motion for the actuated pendulum are nonlinear. For small angles, the equations for the stiff actuator and the PEA can easily be linearized, and a PID controller can deliver a satisfactory control performance. For the SEA, however, the missing collocation due to the
elastic coupling has to be taken into account [3], and the resulting fourth-order
dynamics can no longer be controlled by a simple PID system. Because of its
ability to handle these nonlinearities and collocation issues, the model-based
control strategy of feedback linearization [25] provides a suitable solution for
this type of problem. This control strategy, which will be employed for all
actuator types, results in the controller architecture sketched in Figure 7.

The control law for the PEA (and RA) compensates the second-order non-
linear dynamics by applying the motor current:

\[ I_{m,PEA} = \frac{1}{k_t} \left[ \left( n J_m + n J_tr + \frac{J_l}{n} \right) y_{PEA} + \frac{1}{n} Mgl \sin(\theta) + \frac{1}{n} k_p \theta \right] \]  

(28)

It consists of a feedforward term that is based on Eq. (8) in combination with
feedback motion control by the auxiliary input \( y_{PEA} \). This is chosen to be

\[ y_{PEA} = \ddot{\theta}_d + P (\theta_d - \theta) + D (\dot{\theta}_d - \dot{\theta}) \]  

(29)

where \( P \) and \( D \) are the proportional and differential control parameters [24] and
\( \dot{\theta}_d, \ddot{\theta}_d, \theta_d \) are the desired angular accelerations, velocities, and positions of the
pendulum.

To tackle the fourth order dynamics of the SEA presented in Eq. (14), the
control law is extended to

\[ I_{m,SEA} = \frac{J_{\text{drive}}}{k_t} \left( J_{12} y_{SEA} + \frac{Mgl(\cos \theta \ddot{\theta} - \sin \theta \dddot{\theta})}{k_s} + \dot{\theta} \right) + \frac{1}{nk_t} \left( J_{12} \dddot{\theta} + Mgl \sin \theta \right) \]  

(30)

in which we defined the driveside inertia \( J_{\text{drive}} \) as

\[ J_{\text{drive}} = n J_m + n J_tr + J_l \]  

(31)

The auxiliary control input \( y_{SEA} \) in Eq. (30) is

\[ y_{SEA} = \dddot{\theta}_d + R_0 (\theta_d - \theta) + R_1 (\dot{\theta}_d - \dot{\theta}) + R_2 (\ddot{\theta}_d - \ddot{\theta}) + R_3 (\dddot{\theta}_d - \dddot{\theta}) \]  

(32)

as suggested in [25]. In this equation, the feedback control parameters \( R_0, R_1, R_2, R_3 \) are related to the errors on pendulum position, velocity, acceleration,
and jerk which define the system state. In both cases, these control parameters
are manually tuned for each investigated combination of frequency and stiffness.
This way, the best possible tracking for a sound comparison of power and energy
characteristics is assured.

4.3. Experimental results

The experimentally obtained energy consumption of PEA and SEA is pre-
sented in Figure 8. Due to limitations on spring extension and permissible speed
Figure 8: Electrical energy consumed by the PEA (left) and SEA (right) at different frequencies and stiffnesses. Measurements are indicated as green dots, while simulated energies are shown as a blue wiregrid. As predicted in the simulations, there is a wide region of low energy consumption for the PEA, whereas for the SEA, a distinct minimum exists around antiresonance. Note that the experiments for the RA correspond to the experimental results for the PEA at $k_p = 0 \text{ Nm/rad}$.

of the motor, the frequency range is limited to 7 rad/s for both actuators, and the stiffness is limited to 2.3 Nm/rad for the PEA and 10 Nm/rad for the SEA. Comparing the measured (green dots) and simulated (blue wiregrid) energy profiles shows that the consumption of PEA is slightly overestimated by the models. Yet, the model yields a good prediction of the global actuator behavior. In the SEA case, the experimental results show good accordance with the analytical ones, except for overestimating the consumed energy at combinations of low frequencies and low stiffness values. However, those combinations are not of interest for practical SEAs, because energy consumption is rather high at these operating points.

The results for the PEA (left of Figure 8) are generally in accordance with the minimum area found in the simulations, although the experiments do not allow to exactly trace the minimum to the resonance frequencies of either the system or the link due to the limited frequency resolution. Furthermore, the PEA suffers from possible inaccuracies due to the setting of the equilibrium angle, which is of no concern for the SEA. Nevertheless, the experiments confirm that the PEA can be operated in a favorable region by adjusting its stiffness.

The experimental energy profile of the SEA (right of Figure 8) shows that operation at antiresonance leads to minimum energy consumption. As observed in the simulation, the region of reduced energy consumption covers a certain area around antiresonance: lowest energy consumption values are 0.45 J for 1.4 Nm/rad and 0.70 J for 2 Nm/rad considering operation at 6 rad/s. For 7 rad/s, minimum values are 1.4 J for 2 Nm/rad and 0.57 J for 4 Nm/rad. Hence, the conclusion that tuning the SEA for antiresonance yields good results, even if the model is not ideal, is confirmed experimentally. Conversely, very little energetic benefit is obtained by matching the operating points to the resonance frequency. The first resonance frequency, $\omega_{r1,SEA}$, does not show a clear reduction of electrical energy consumption, as predicted analytically. The
The experimental energy requirements for frequencies ranging from 1 to 7 rad/s are also presented in Table 3. For the SEA and PEA, only one result is shown for every frequency. This result corresponds to the experiment with smallest energy consumption; the corresponding spring stiffness is mentioned between brackets. Consistent with our findings in section 3.3, at frequencies below the link’s resonance $\omega_{r,RA} = 5.3$ rad/s, the SEA performs worse than the RA. As a logical result, the optimal stiffness of the SEA is pushed up to high values in order to obtain a behavior which is as close as possible to the RA. Similarly, the PEA’s optimal stiffness at low frequencies is zero, such that the PEA becomes identical to the RA. In accordance with Figure (5)a in Section (3.3), Table 3 shows that 5 rad/s is the lowest frequency at which a parallel spring becomes beneficial.

Besides the increasing energy requirement with rising frequency above $\omega_{r,RA}$, it becomes distinct that the PEA always demands more energy than the RA operated at this specific point. This is consistent with our findings in Section 3.2.2. Still, the PEA can be beneficial if stiffness is modified to match varying trajectory frequencies, as can be seen from the results for 6 rad/s (same energy consumption) and 7 rad/s (energy consumption lowered by 20%). In contrast to the PEA, the SEA at antiresonance exhibits significantly lower energy requirements than the RA operated at resonance, with reductions up to 78% at 6 rad/s. Note that, both for SEA and PEA, the optimal stiffnesses in Table 3 depend on the discrete set of available springs for the test setup. Higher reductions

<table>
<thead>
<tr>
<th>Frequency</th>
<th>RA</th>
<th>PEA</th>
<th>SEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 rad/s</td>
<td>2.4 J</td>
<td>2.4 J</td>
<td>3.1 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(10 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>2 rad/s</td>
<td>1.8 J</td>
<td>1.8 J</td>
<td>2.6 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(8 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>3 rad/s</td>
<td>1.4 J</td>
<td>1.4 J</td>
<td>2.2 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(8 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>4 rad/s</td>
<td>1.5 J</td>
<td>1.5 J</td>
<td>1.9 J</td>
</tr>
<tr>
<td></td>
<td>(0 Nm/rad)</td>
<td>(8 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>5 rad/s</td>
<td>1.8 J</td>
<td>1.6 J</td>
<td>1.9 J</td>
</tr>
<tr>
<td></td>
<td>(0.3 Nm/rad)</td>
<td>(6 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>6 rad/s</td>
<td>2.0 J</td>
<td>2.0 J</td>
<td>0.45 J</td>
</tr>
<tr>
<td></td>
<td>(1.3 Nm/rad)</td>
<td>(1.4 Nm/rad)</td>
<td></td>
</tr>
<tr>
<td>7 rad/s</td>
<td>2.4 J</td>
<td>2.0 J</td>
<td>0.57 J</td>
</tr>
<tr>
<td></td>
<td>(2.3 Nm/rad)</td>
<td>(4 Nm/rad)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Measured electrical energy consumption $E_{elec}$ of RA, SEA and PEA for various frequencies. For SEA and PEA, the lowest value across all stiffnesses is shown. The corresponding spring stiffness is mentioned in brackets.
can most likely be obtained if the spring stiffness could be matched exactly to the theoretically optimal stiffness. Anyhow, these results give a clear indication that, for a 1-DOF link swinging above its resonance frequency, a properly tuned SEA is superior to the PEA in terms of energy consumption.

5. Discussion

In this section, we will dig deeper into how the topologies of SEAs and PEAs affect their energy consumption. As mentioned in the introduction of Section 3.2, the energy consumption of the studied system depends completely on the system’s energy losses. These losses can roughly be classified into three major categories:

- **Bearing losses**: Components that move relative to one another (typically bearings) are subject to friction, which causes a loss of energy. In this paper, this friction is modeled as a combination of Coulomb friction (constant torque with speed-dependent direction) and viscous friction (torque proportional to speed).

- **Gearbox losses**: Gearbox losses have several complex causes making them speed- and torque-dependent [20], but they can roughly be considered proportional to the torque through the gearbox. Generally, the efficiency of gearboxes decreases with increasing gear ratio. This is especially the case in multi-stage planetary gearboxes, where each stage causes a significant drop in gearbox efficiency.

- **Motor losses**: The motor model presented in Section 2.4 incorporates two types of losses: Joule losses ($RT^2$) due to the resistance of the windings, and a damping loss $\nu m \dot{\theta}_m$ which represents other unmodeled motor losses such as bearing friction, losses through the brushes, etc. The motor's damping coefficient $\nu_m$ being very small for most motors, the predominant loss mechanism in DC motors is Joule heating. Since Joule losses are proportional to current squared, and current is proportional to torque, the motor losses can be minimized by reducing the motor torque $T_m$. It is important to mention that this torque includes the motor’s inertial torque $J_m\dot{\theta}_m$. Since, as a rule of thumb, reflected motor inertia is roughly equal to the output inertia in well-sized systems [12], the motor’s inertial torque can be expected to provide a significant contribution to the total motor torque. For this reason, omitting it would give incorrect results in most cases.

As explained in Section 3, the optimal operation of PEA and SEA relies on very different principles. In PEA and RA, the resonance frequency can be exploited, reducing the torque on the motor and gearbox. Consequently, motor and gearbox losses are minimized, resulting in a more energy-efficient operation. SEA, on the other hand, relies on the exploitation of antiresonance to
decrease motor speed. Smaller motor speeds lead to a reduction in viscous friction losses, while gearbox and motor losses remain unaffected. Since, in the actuator system under study in this paper, the latter are generally responsible for the majority of the losses, one would expect the usage of a PEA to lead to better overall efficiencies. The simulations, however, reveal some additional considerations which need to be taken into account when comparing SEA to PEA. Firstly, as discussed in Section 3.2.2, gearbox and motor losses cannot be minimized simultaneously because they correspond to different resonance frequencies, and optimal stiffness for the PEA is a trade-off between both. Consequently, even at the optimal stiffness, a significant amount of energy will still be lost in the gearbox and motor. Secondly, the dynamics of the SEA demonstrate a very interesting feature: the antiresonance frequency $\omega_{a,SEA}$ (minimal motor speed), the resonance frequency $\omega_{r2,SEA}$ (minimal motor torque) and the link’s resonance frequency $\omega_{rl,RA}$ (minimal gearbox torque) approach each other for $k_s \to 0$. As a result, an SEA operated at antiresonance does not only take advantage of the reduced motor speed, it also enjoys a slight reduction of motor and gearbox torque. While the torque reduction is not as distinct as in the PEA – the PEA’s resonance frequencies $\omega_{r x,PEA}$ and $\omega_{rl,PEA}$ being much nearer to each other – the combination of reduced motor speed and motor torque provides an energetic advantage to the SEA which cannot be rivaled by the PEA. In other words, the favorable dynamics of the SEA allow it to outperform the PEA in terms of energy consumption.

6. Conclusion

In recent years, Series Elastic Actuators and Parallel Elastic Actuators have emerged as valid alternatives to rigid actuators in energy-efficient robotic designs. The choice between a SEA and PEA can be based on many practical criteria such as human-robot interaction safety, shock resistance, design effort and ease of control. In this paper, we compared both actuator types in terms of one of their most interesting properties: energy efficiency. As a basis for comparison, a task in which the actuators impose a sinusoidal motion to a pendulum was studied. This case study can be related directly to the natural dynamics of the actuators, allowing for a direct comparison between theory, simulations and experiments.

Consistent with the findings of earlier work [2], we found that SEAs should be tuned to antiresonance, regardless of whether peak power or energy consumption is considered. For PEAs, the optimal tuning is case-dependent. Minimum mechanical peak power occurs at the resonance frequency $\omega_{r x,PEA}$, whereas minimum mechanical energy consumption corresponds to the resonance frequency $\omega_{r1,PEA}$ of the gearbox subsystem. Minimum electrical energy consumption, finally, is not defined by any of these frequencies, but occurs somewhere in between depending on how the losses are distributed between gearbox and motor. Consequently, finding the energy-optimal stiffness of a PEA requires a detailed system model, whereas for the SEA, the antiresonance frequency already provides an excellent estimate.
Comparing the energy consumption of SEA and PEA to RA, energy can be reduced at frequencies above the link's resonance frequency $\omega_{r_{l,RA}}$. The highest reductions are situated around the antiresonance frequency for the SEA and around the resonance frequency $\omega_{r_{l,PEA}}$ for the PEA. In a resonance-tuned PEA, however, energy consumption rises with increasing stiffness; consequently, its energy consumption can never be lower than that of the rigid actuator ($k_p = 0$) at resonance. Conversely, the antiresonance-tuned SEA consistently demonstrates lower energy consumption than the rigid system at any frequency above $\omega_{r_{l,RA}}$. The drop is significant, with experimental results indicating gains in energy consumption of up to 78%.

In conclusion, the paper confirms that natural dynamics can be exploited to achieve efficient operation for RA, PEA, and SEA. Beyond the link's resonance frequency, PEA and SEA allow to decrease the energy consumption with respect to RA. In this respect, the SEA clearly outperforms the PEA thanks to its more favorable dynamics. At the highest measured frequency of 7 rad/s, we recorded energy reductions of up to 78% for the SEA, compared to 20% for the PEA. Of course, these results are based on a very specific and simple case study. Extending this case study to systems with more complex dynamics and more complex motions will be the subject of further research.

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