We investigate the opportunities of Raman lasers based on integrated single-crystal diamond ring resonators. We model continuous-wave (CW) Raman lasing action while taking into account the lasing directionality, the linear and nonlinear losses, and the coupling of the fields between the bus and ring sections of racetrack-shaped diamond ring resonators. Besides designing the ring resonators for a short-wavelength infrared (SWIR) and an ultraviolet (UV) Raman laser, we also design diamond gratings to couple light in and out of the resonators. Using our Raman lasing model, we determine the lasing directionality, pump threshold, and lasing efficiency of the considered SWIR and UV devices. We find that both can yield efficient CW operation with SWIR and UV lasing slope efficiencies of 33% and 65%, respectively. These results showcase the potential of integrated diamond ring Raman lasers for producing wavelengths that are challenging to generate with other types of integrated lasers.

1. INTRODUCTION

Integrated silicon Raman lasers have been extensively investigated over the past decade, in both standing-wave and ring-cavity configurations [1–8]. The demonstration of the first continuous-wave (CW) integrated silicon Raman laser in 2005 marked a significant milestone in silicon photonics [1]. Silicon is a material that is of extreme importance not only to the electronic processor industry but also to that of photonics. Silicon photonic devices have the advantage that they can be fabricated using the high-precision CMOS fabrication technologies while also offering possibilities for integration with silicon-based electronic components. Furthermore, the large refractive-index difference in silicon-on-insulator (SOI) structures establishes a strong confinement of the light, which enables an extensive component miniaturization and facilitates triggering nonlinear optical effects in these structures. However, silicon suffers from some important disadvantages. With a bandgap wavelength of \( \lambda_g \approx 1100 \text{ nm} \), silicon is a semiconductor material that is not transparent in the visible (VIS) and ultraviolet (UV) wavelength regions (Table 1), making silicon-based Raman lasers for these spectral regions impossible. Another downside of this semiconductor material is the deteriorating effect of its nonlinear optical losses, i.e., two-photon absorption (TPA) and TPA-induced free-carrier absorption (FCA), occurring at wavelengths below 220 nm. Even multi-photon absorption and its consequent FCA were experimentally found to have a significantly deteriorating influence on the operation of silicon devices [9]. The latter is especially true when using long laser pulses (>10 ps) and in particular CW light. Therefore, despite silicon’s high Raman nonlinearity, near-infrared silicon Raman lasers are quite inefficient, and CW operation of these lasers is only possible if a carrier-extracting PIN diode is embedded around the silicon waveguides to minimize the FCA losses [1]. Because of these losses strong heat dissipation also occurs. The dissipated heat inside a silicon cavity, combined with strong thermo-optic effects, can cause cavity resonances to shift making the stable operation of a silicon Raman laser challenging in practice [10].

To overcome the main limitations of silicon-based Raman lasers, other nonlinear waveguide materials need to be considered. A very promising semiconductor material is diamond. Recently, diamond is increasingly being researched for use in integrated photonics thanks to the commercial availability of synthetic single-crystal diamond wafers (closely resembling natural diamond) of sufficient optical quality and considerable area of the order of 10 mm² [11]. A single-crystal diamond
possesses the same cubic crystalline structure as silicon. Although its refractive index of $n = 2.39$ at $\lambda = 1550\,\text{nm}$ is lower than that of silicon (Table 1), it still is high enough to ensure proper light confinement inside a diamond waveguide. Diamond features a Raman gain coefficient ($g_R = 13.5\,\text{cm/GW}$ at $\lambda = 1030\,\text{nm}$) that is of the same order as that of silicon (Table 1). Diamond has a very broad optical-transparency window due to its relatively large indirect bandgap ($E_g = 5.5\,\text{eV}$ [12]) and hence is transparent not only in the infrared (IR) wavelength region but also in the VIS and UV regions (Table 1). As such, diamond Raman lasers offer the possibility to generate UV light. Also because of the large bandgap, TPA and FCA only occur for light at UV wavelengths below 440 nm (Table 1). The Raman shift of diamond ($\Omega_0 = 2\pi \times 40 \times 10^{12}\,\text{rad/s}$) is substantially larger than that of silicon (Table 1), enabling Raman lasing at a wavelength widely separated from the pump wavelength. This is particularly interesting when targeting lasing in the application-rich but still rather exotic short-wavelength IR (SWIR) domain around 2 μm since in diamond this could be obtained using commonly used pump wavelengths around 1.55 μm [13].

The fabrication techniques of diamond waveguides are similar to those used for silicon waveguides [14]. Furthermore, diamond is, like silicon, CMOS compatible. Finally, the thermal conductivity of diamond, approximately 2000 W/(m·K) [15], is much larger than that of silicon so that a better heat distribution can be obtained in the former material.

Several Raman lasers based on free-space cavities containing a bulk diamond crystal and requiring high threshold powers (of the order of W–kW) have already been demonstrated for various wavelength domains ranging from the UV to the mid-IR [23–30]. However, according to our knowledge, the demonstrations of on-chip diamond Raman lasers are to date only in the exploratory phase [31]. An earlier theoretical study showed the potential of integrated diamond Raman lasers with a standing-wave cavity configuration [11]. However, rather than using a standing-wave cavity realized by dielectric coatings on the waveguide facets, it would be more advantageous to have a monolithic diamond Raman laser based on a ring cavity [2,31]. Furthermore, integrated diamond ring Raman lasers can be made more efficient than their standing-wave counterparts due to the strong nonreciprocity of the Raman gain (see Section 2.B) in highly confining sub-micron waveguides, which generally results in unidirectional lasing so that the entire output power exits the laser at only one port [32]. Therefore, in this paper we study Raman lasing in monolithic integrated diamond ring resonators.

When considering the SWIR spectral domain around 2 μm, which is useful for, e.g., on-chip biosensing, spectroscopy of biomolecules, detection of atmospheric gases, and food sorting, we find that the number of available integrated lasers generating radiation at these wavelengths is very limited [13]. Furthermore, when looking at the UV domain, which is useful for, e.g., forensic analysis, laser engraving, optical storage, and on-chip water purification [13], we see that integrated lasers emitting UV light have, according to our knowledge, not yet been demonstrated. That is why we aim at exploiting the Raman effect in on-chip resonators made of synthetic diamond to realize SWIR and UV integrated lasers. To this end we design and model in this paper two CW Raman lasers based on integrated diamond ring resonators: one device emitting SWIR light, the other UV light. For the SWIR laser we consider a pump beam and resulting Stokes lasing beam (see Section 2.A) at $\lambda = 1550\,\text{nm}$ and $\lambda = 1950\,\text{nm}$, respectively. There are no nonlinear losses (TPA and FCA) in diamond at all in the case of an IR pump and Stokes beam. At the same time, the Raman gain is smaller than that of UV Raman lasers since the Raman gain coefficient $g_R$ scales approximately with frequency (see Section 4.A). The laser operating in the UV is designed for a pump beam at $\lambda = 266\,\text{nm}$ and a Stokes lasing beam at $\lambda = 276\,\text{nm}$. The UV pump beam can be produced by frequency doubling (second-harmonic generation) of laser light at the prevalent wavelength of 532 nm. For the above UV pump and Stokes wavelengths, diamond suffers from nonlinear losses but also exhibits a strong Raman gain. As a practical remark we note that the UV pump laser at 266 nm can, in the presence of oxygen, cause etching of the diamond-waveguide surface due to the decay of two-photon excited excitons [24,33]. As oxygen is required to initiate this etching process, operating the Raman laser in vacuum or in an $\text{N}_2$-purged chamber atmosphere would be sufficient to inhibit the etching mechanism.

This paper is organized as follows: in Section 2 we introduce the Raman-lasing equations describing the process of stimulated Stokes Raman scattering (SSRS) in integrated single-crystal diamond ring Raman lasers, as shown in Fig. 1, with the inclusion of propagation losses. Considering the influence of the waveguide geometry on SSRS, and in particular the lasing directionality, we also introduce effective areas for SSRS. Moreover, we specify the mathematical formalism for the evanescent coupling of light between the bus and ring sections of the diamond ring resonator. In Section 3 we first determine the waveguide dimensions cross-sectional dimensions and then design the evanescent light coupling region between the bus and the ring. We also design grating couplers to inject the pump light coming from, e.g., an optical fiber into the bus and to extract the Stokes output from the bus. In Section 4, to calculate the Raman lasing efficiency in the designed integrated ring resonators, we numerically solve the Raman lasing equations in these resonators and discuss the lasing directionality, lasing threshold, and lasing efficiency. Finally, we draw conclusions in Section 5.

### 2. Theory of Raman Lasing in Integrated Diamond Ring Resonators

#### A. Raman Lasing Equations

A Raman laser is based on SSRS, a nonlinear optical process in which a pump beam at frequency $\omega_p$ (generated by a pump

<table>
<thead>
<tr>
<th>Material Constants</th>
<th>Silicon</th>
<th>Diamond</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>3.48</td>
<td>2.39</td>
</tr>
<tr>
<td>$\lambda_p$ (nm)</td>
<td>1100</td>
<td>220</td>
</tr>
<tr>
<td>$g_R$ (cm/GW)</td>
<td>20</td>
<td>13.5</td>
</tr>
<tr>
<td>$\Omega_0$ (10^{12} rad/s)</td>
<td>$2\pi \times 15.6$</td>
<td>$2\pi \times 40$</td>
</tr>
</tbody>
</table>

*Value evaluated at 1030 nm.*
The effective Raman gains for co-propagating (\(\omega_p = \omega_s = \Omega\)) inside a Raman medium (i.e., diamond in this case). At Raman resonance the frequency offset \(\Omega = \Omega_0\) corresponds with a material energy level \(\hbar \Omega_0\) of the Raman medium, and consequently the strength of SSRS is maximal [34].

We assume the (quasi-)CW pump and (quasi-)CW Stokes waves inside a waveguide can be described by \(x\) (propagation coordinate) dependent electric fields whose spectra are concentrated in narrow frequency bands around \(\omega_p\) and \(\omega_s\), respectively. \(A_\pm^p(x)\) is the slowly varying envelope of the forward-propagating (\(+\)), respectively, backward-propagating (\(-\)) electric field at the pump frequency \(\omega_p\). Similarly, \(A_\pm^s(x)\) is the slowly varying envelope of the forward-propagating (\(+\)), respectively, backward-propagating (\(-\)) electric field at the Stokes frequency \(\omega_s\). These envelopes are normalized in such a way that the square of their magnitudes equals the optical power at their respective frequencies. The Raman lasing equations, describing the energy transfer between the pump and Stokes fields, are given by the following set of coupled nonlinear differential equations [7,8,34]:

\[
\frac{\partial}{\partial z} A_\pm^p(x) = \pm \frac{1}{2} \frac{\partial \omega_p}{\partial x} [G_\pm^p|A_\pm^p|^2 + G_\pm^s|A_\pm^s|^2] A_\pm^p - \gamma^\pm A_\pm^p, \\
\frac{\partial}{\partial z} A_\pm^s(x) = \pm \frac{1}{2} \frac{\partial \omega_s}{\partial x} [G_\pm^p|A_\pm^p|^2 + G_\pm^s|A_\pm^s|^2] A_\pm^s - \gamma^\pm A_\pm^s.
\]  
(1a)

(1b)

The effective Raman gains for co-propagating (\(+\)) and counter-propagating (\(-\)) pump and Stokes beams are defined as

\[
G_\pm^\text{eff}(\omega) = \frac{g}{A_\pm^\text{eff}} H_R(\omega_p - \omega),
\]  
(2)

and incorporate the Raman spectral function \(H_R(\Omega)\), the effective Raman gain \(g\) and the effective areas \(A_\pm^\text{eff}\) for SSRS. The complex-valued Raman spectral function [34]

\[
H_R(\Omega) = \frac{\Gamma_R \Omega \Omega_0}{(\Omega_0^2 - \Omega^2)^2 + \frac{\Gamma_R^2}{2} \Omega^2} - j \frac{\Gamma_R \Omega_0 (\Omega_0^2 - \Omega^2)}{(\Omega_0^2 - \Omega^2)^2 + \frac{\Gamma_R^2}{2} \Omega^2}
\]  
(3)

models the dependence of the Raman susceptibility on the pump-Stokes frequency offset \(\Omega\). At perfect Raman resonance \((\Omega = \Omega_0)\), \(H_R(\Omega_0) = 1\). The real part of \(H_R(\Omega)\) determines the spectral shape of the Raman gain, and has in the case \(\Omega \approx \Omega_0\) (near Raman resonance) a Lorentzian distribution centered around the frequency \(\Omega_0\) with a FWHM (full width at half-maximum) of \(2\Gamma_\Omega\). For diamond materials \(\Gamma_\Omega \approx 2\pi \times 36 \times 10^9 \text{ rad/s} [35]\). The (peak) Raman gain coefficient \(g_\text{p}\) quantifies the strength of the SSRS process for a given Raman material. However, considering the finite spectral linewidth \(\Delta \omega_p\) of the pump laser source, the effective Raman gain coefficient

\[
g = \frac{g_\text{p}}{1 + \Delta \omega_p / 2\pi}
\]  
(4)

should be employed instead of the peak coefficient [4]. The effective areas \(A_\pm^\text{eff}\) for SSRS are defined in Section 2.B. In Section 2.C we treat the propagation losses \(\gamma^\pm\) included in Eq. (1). Finally, we remark that Eq. (1) takes into account the depletion of the pump wave due to SSRS.

**B. Effective Areas for SSRS**

We study diamond waveguides that are nominally oriented along the [011] direction—i.e., this is the direction of light propagation—and that are fabricated on a (100) wafer. This is an often used configuration because of cleaving convenience [36]. In Fig. 1 we define a waveguide coordinate system \((x, y, z)\) such that the transverse electric (TE) polarization corresponds predominantly to polarization along the \(y\) axis (i.e., in the plane of the wafer), while transverse magnetic (TM) polarization corresponds predominantly to polarization along the \(x\) axis (i.e., perpendicular to the plane of the wafer). The propagation coordinate thus is \(z\). For practical reasons we only study the case in which the pump and Stokes beams have the same polarization state. It can then be verified that for the considered diamond waveguide orientation the SSRS gain is maximal in the case of a TE co-polarized pump and Stokes beams and minimal in the case of a TM co-polarized pump and Stokes beams so that TM co-polarized modes do not lead to efficient lasing action [8,37]. This is a consequence of the symmetry of diamond’s Raman susceptibility tensor and thus explains why both modes, the pump and Stokes modes, should be TE polarized.
The effective areas \( A_{\text{eff}}^\pm \) for SSRS, introduced in Eq. (1), through the effective Raman gains, account for the influence of the waveguide geometry, i.e., the overlap between, on the one hand, the pump and Stokes mode fields, and, on the other hand, the nonlinear Raman susceptibility tensor of the waveguide material. Considering only (fundamental) single-mode operation of the pump and Stokes beams in the diamond waveguide and working at Raman resonance, the effective areas for the contribution to SSRS of co-propagating (+) and counter-propagating (–) pump and Stokes beams are, respectively [8,37,38],

\[
A_{\text{eff}}^+ = \frac{4Z_0^2 N_p N_s}{n_p n_s} \left[ \int \text{d}A \times e_{i;x}(e_{p;y}^2 - e_{s;z}^2) + e_{s;y}(e_{p;z}^2 + e_{s;x}^2) \\
- e_{s;x}(e_{p;x}^2 - e_{s;z}^2) + 2(e_{s;x} e_{s;y} e_{p;x} e_{p;y}) \\
- e_{s;y} e_{s;x} e_{p;x} e_{p;z} - e_{s;x} e_{s;y} e_{p;z} e_{p;x} \right]^{-1}, \tag{5a}
\]

\[
A_{\text{eff}}^- = \frac{4Z_0^2 N_p N_s}{n_p n_s} \left[ \int \text{d}A \times e_{i;x}(e_{p;y}^2 - e_{s;z}^2) + e_{s;y}(e_{p;z}^2 + e_{s;x}^2) \\
- e_{s;x}(e_{p;x}^2 - e_{s;z}^2) + 2(e_{s;x} e_{s;y} e_{p;x} e_{p;y}) \\
- e_{s;y} e_{s;x} e_{p;x} e_{p;z} - e_{s;x} e_{s;y} e_{p;z} e_{p;x} \right]^{-1}, \tag{5b}
\]

where the integration is performed over the cross-sectional area \( A_0 \) of the diamond waveguide, \( Z_0 = c \mu_0 \) is the free-space impedance, and \( n_{[p,s]} \) are the refractive indices of diamond at, respectively, the pump and Stokes wavelengths. The mode normalizations for, respectively, the pump and Stokes waves are

\[
N_{[p,s]} = \frac{1}{2} \int |e_{[p,s]}(x,y) \times h_{[p,s]}^*(x,y)| \cdot \text{d}A. \tag{6}
\]

The electric- and magnetic-mode fields of the linear lossless diamond waveguides—at both the pump and Stokes frequencies—were represented in Eqs. (5) and (6) as

\[
e_{i}(x,y) = e_{i;x}(x,y) \hat{x} + e_{i;y}(x,y) \hat{y} + e_{i;z}(x,y) \hat{z}, \tag{7}
\]

\[
h_{i}(x,y) = h_{i;x}(x,y) \hat{x} + h_{i;y}(x,y) \hat{y} + h_{i;z}(x,y) \hat{z}, \tag{8}
\]

with \( i \in \{p,s\} \) and \( \hat{x}, \hat{y}, \hat{z} \) being the unit vectors in, respectively, the \( x, y, \) and \( z \) directions. In writing Eq. (5) we have used the common convention that the mode fields are normalized such that the TE and magnetic-mode fields, \( e_{i;[x,y]} \) and \( h_{i;[x,y]} \), are purely real, and consequently the longitudinal electric and magnetic mode-field components, \( e_{i;x} \) and \( h_{i;x} \), are purely imaginary [39].

Although Eq. (5) was originally developed for silicon waveguides, it applies to single-crystal diamond waveguides as well because of the same cubic crystal structure of both materials. When comparing Eqs. (5a) and (5b) we notice that if the pump and Stokes mode fields both contain longitudinal components \( e_{p;x} \neq 0 \) and \( e_{s;x} \neq 0 \), \( A_{\text{eff}}^- \) will be smaller than \( A_{\text{eff}}^+ \), so that a nonreciprocity exists in the effective Raman gain between the cases of co- and counter-propagating beams. This means that for a Raman laser that is pumped at one waveguide facet and that exhibits nonzero longitudinal field components, the gain asymmetry will favor the amplification of the Stokes wave propagating in the direction opposite to the pump wave, and as such will lead to efficient unidirectional lasing. The longitudinal components of the mode fields can be particularly strong in highly confining sub-micron waveguides, as we will consider in this paper [8,38].

C. Propagation Losses in Diamond Waveguides

Now we treat the propagation losses in the diamond waveguides. The propagation losses \( \gamma_{p[s]} \) in Eq. (1) contain three contributions [7]:

\[
\gamma_{p} = \frac{\alpha_p}{2} + \gamma_{p,(\text{TPA})} + \gamma_{p,(\text{FCA})} \tag{9a}
\]

\[
\gamma_{s} = \frac{\alpha_s}{2} + \gamma_{s,(\text{TPA})} + \gamma_{s,(\text{FCA})} \tag{9b}
\]

where \( \alpha_{[p,s]} \) are the linear waveguide losses, \( \gamma_{[p,s],(\text{TPA})} \) represent nonlinear TPA losses characterized by the frequency-dependent TPA coefficient \( \beta \), and \( \gamma_{[p,s],(\text{FCA})} \) stand for TPA-induced FCA losses characterized by the FCA efficiency coefficient \( \phi \) and the effective charge-carrier lifetime \( \bar{\tau} \).

As mentioned in Section 1, TPA only takes place in diamond for light at wavelengths below 440 nm (Table 1). The contributions of TPA to the propagation losses are [8]

\[
\gamma_{p,(\text{TPA})} = \frac{1}{2} \frac{\beta_{pp}}{A_{\text{eff}}^p} \left( |A_p|^2 + 2|A_{s}^p|^2 \right) \\
+ \frac{1}{2} \frac{\beta_{ps}}{A_{\text{eff}}^p} \left( |A_p|^2 + 2|A_{s}^s|^2 \right), \tag{10a}
\]

\[
\gamma_{s,(\text{TPA})} = \frac{1}{2} \frac{\beta_{sp}}{A_{\text{eff}}^s} \left( |A_p|^2 + 2|A_{s}^s|^2 \right) \\
+ \frac{1}{2} \frac{\beta_{ss}}{A_{\text{eff}}^s} \left( |A_p|^2 + 2|A_{s}^p|^2 \right), \tag{10b}
\]

where \( \beta_{[pp,sp]} \) are the degenerate-TPA coefficients for two pump photons and two Stokes photons, respectively. We assume that we can set \( \beta_{[pp,ss]} = \beta \). Also the nondegenerate-TPA coefficient \( \beta_p \) for a pump and a Stokes photon is approximated here by \( \beta \). We do distinguish the effective areas for degenerate TPA of pump \((i = p)\) and Stokes \((i = s)\) photons, and for nondegenerate TPA \((sp)\). These areas, are, respectively, given by [8]

\[
A_{\text{eff}}^i = \frac{4Z_0^2 N_i^2}{\mu_0} \left[ \int \text{d}A \right]^{-1}, \tag{11a}
\]

\[
A_{\text{eff}}^p = \frac{4Z_0^2 N_p N_p}{n_p n_p} \left[ \frac{1}{3} \int \text{d}A \right]^{-1}, \tag{11b}
\]

where the integration is performed over the cross-sectional area \( A_0 \) of the diamond waveguide.

As a result of TPA, FCA will occur and is quantified by the FCA efficiency coefficient \( \phi \) and by the effective charge-carrier lifetime \( \bar{\tau} \), which represents the average lifetime of the TPA-generated electrons and holes in the waveguide. The contribution of FCA to the propagation losses equals [8]
\[ \gamma^\pm_{A\text{FCA}} = \pm 2 \left( \frac{\pi c}{\omega_j} \right)^2 \beta N_{\text{eff}}, \quad (12a) \]

\[ \gamma^\pm_{\beta\text{FCA}} = \pm 2 \left( \frac{\pi c}{\omega_j} \right)^2 \beta N_{\text{eff}}. \quad (12b) \]

The effective charge-carrier density \( N_{\text{eff}} \) in the CW regime is

\[ N_{\text{eff}} = \frac{\pi \tau_{\text{eff}}}{\hbar \beta^2} F_{\text{TPA}}, \quad (13) \]

where the factor \( F_{\text{TPA}} \) is defined as

\[ F_{\text{TPA}} = \frac{\beta_{pp}}{\alpha \omega_{\text{FCA}}} \left( |A_j^+|^4 + |A_j^-|^4 + 4 |A_j^0|^2 |A_j^2|^2 \right) \]

\[ + \frac{\beta_{pp}}{\alpha \omega_{\text{FCA}}} \left( |A_{j+}^+|^4 + |A_{j-}^-|^4 + 4 |A_{j-}^0|^2 |A_{j+}^2|^2 \right) \]

\[ + \frac{4 \beta_{pp}}{\alpha \omega_{\text{FCA}}} \left( |A_j^+|^2 + |A_j^-|^2 \right) \left( |A_j^0|^2 + |A_j^2|^2 \right). \quad (14) \]

As we will show later on, despite the occurrence of TPA and FCA losses at UV wavelengths, the linear propagation losses due to waveguide surface roughness will have the largest impact on the efficiency of the UV diamond Raman laser.

**D. Ring Resonator Coupling**

In this section we describe the coupling of the pump (\( i = p \)) and Stokes (\( i = s \)) fields between the straight and ring sections of ring resonators (see Fig. 1). To this aim we first segment the straight bus waveguide into two sections, named bus 1 (denoted by \( b_1 \)) and bus 2 (denoted by \( b_2 \)), both of distance \( L_b/2 \) so that the total length of the bus waveguide is \( L_b \). The ring is indicated with subindex \( r \) and has a length \( L \). The coupling of light in the bus-ring coupling region between, on the one hand, \( A_{i,(a)}(0) \), the forward-propagating field in bus 2 at round trip number \( (a) \), and \( A_{i,(a-1)}^+(L) \), the forward-propagating fields in bus 1 and in the ring at round-trip number \( (a - 1) \), and on the other hand between \( A_{i,(a)}^+(0) \), the forward-propagating field in the ring at round-trip number \( (a) \), and \( A_{i,(a-1)}^+(L)/2 \) and \( A_{i,(a-1)}^+(L) \), the forward-propagating fields in bus 1 and in the ring at round-trip number \( (a - 1) \), is described by [7]

\[ \begin{pmatrix} A_{i,(a)}^+(0) \\ A_{i,(a)}^-(0) \end{pmatrix} = \begin{pmatrix} \sigma_i & jk_i \\ jk_i & \sigma_i \end{pmatrix} \begin{pmatrix} A_{i,(a-1)}^+(L)/2 \\ A_{i,(a-1)}^+(L) \end{pmatrix} e^{jk_iL}. \quad (15a) \]

The self-coupling coefficient \( \sigma_i \) and the cross-coupling coefficient \( k_i \) are assumed to be real valued and satisfy \( \sigma_i^2 + k_i^2 = 1 \) in the absence of coupler losses. The factor \( e^{jk_iL} \), with \( k_i = 2 \pi n_i / \lambda_i \), accounts for the phase evolution of the fields after one roundtrip in the ring, and is equal to 1 if ring resonance is assumed. Analogously, we can write for the backward-propagating (-) fields

\[ \begin{pmatrix} A_{i,(a)}^-(L)/2 \\ A_{i,(a)}^-(L) \end{pmatrix} = \begin{pmatrix} \sigma_i & jk_i \\ jk_i & \sigma_i \end{pmatrix} \begin{pmatrix} A_{i,(a-1)}^-+(0) \\ A_{i,(a-1)}^+(0) \end{pmatrix} e^{jk_iL}. \quad (15b) \]

Critical coupling of light between the bus and the ring, considering only a forward-propagating pump field, occurs when the cross-coupling coefficient \( |k_i|^2 \) for powers is exactly equal to the relative power losses in the ring at steady state

\[ \Phi_p^{+,-} = \frac{|A_{p,(a)}(0)|^2 - |A_{p,(a)}^+(L)|^2}{|A_{p,(a)}(0)|^2}. \quad (16) \]

so that the condition becomes \( |k_i|^2 = \Phi_p^{+,-} \). In Eq. (16), \( \alpha \to \infty \) indicates the steady-state regime. The same condition holds for a backward-propagating pump field since \( \Phi_p^{+,-} = \Phi_p^{+,-} \). In the case of critical coupling, the pump-power buildup in the ring is maximal since no pump energy is found in bus 2. The condition for critical coupling can also be derived by expressing mathematically that no pump power should be present in bus 2, i.e., the pump fields that couple from bus 1 and the ring into bus 2 must interfere destructively. As a concluding remark, we mention that no condition for critical coupling can be derived for the Stokes beam because this beam is, as opposed to the pump, not depleted along the ring but instead gains power so that \( \Phi_p^{+,-} < 0 \). As a result, no real-valued \( k_i \) can be determined for which \( |k_i|^2 = \Phi_p^{+,-} \).

**3. DESIGN OF INTEGRATED DIAMOND RING RAMAN LASERS**

**A. Design of Integrated Diamond Ring Resonators**

We start with the design of the diamond-waveguide cross section. The diamond-waveguide layer is situated on top of a 2 \( \mu \)m buried oxide; the latter is obtained by oxidizing a silicon substrate. The material platform is very similar to SOI, except that the top layer is made of single-crystal diamond instead of silicon. We study diamond-strip waveguide geometries, which can be fabricated with e-beam lithographic or UV photolithographic techniques [40]. Only TE modes are considered for the given waveguide orientation as TM modes correspond to a minimal Raman susceptibility and therefore do not experience efficient SSRS (see Section 2.B). In designing the waveguide cross sections we aim at single-mode operation at the pump and Stokes frequencies at well-confined pump and Stokes waveguide modes and at practically feasible cross-sectional dimensions. The e-beam lithographic techniques used nowadays are capable of fabricating diamond waveguides with dimensions as small as 100 nm [40].

With the aid of Numerical MODE Solutions [41], a commercial eigenmode solver in which diamond is implemented through its Sellmeier coefficients found in [17], we select for the IR waveguide a cross-sectional width of \( W = 0.80 \mu \)m and a height of \( H = 0.50 \mu \)m (see Fig. 2). We find at the pump wavelength \( \lambda_p = 1550 \) nm a modal area of 0.35 \( \mu m^2 \) and a power confinement factor of the mode of 89.0%. We also find at the Stokes wavelength \( \lambda_s = 1950 \) nm a modal area of 0.45 \( \mu m^2 \) and a power confinement factor of 79.2%. For the UV diamond waveguide we take \( W = 0.12 \mu \)m and \( H = 0.10 \mu \)m (see Fig. 3). This waveguide has a modal area of 0.0092 \( \mu m^2 \), and a power confinement factor of the mode of 93.7% at the pump wavelength \( \lambda_p = 266 \) nm, and of 0.0095 \( \mu m^2 \) and 92.8% at the Stokes wavelength \( \lambda_s = 276 \) nm. The small modal areas combined with the relatively high confinement factors will yield high effective Raman gains (see Section 4.A).

Next we design the evanescent coupling regions of the race-track-shaped diamond ring resonators (see Fig. 1). We examine racetrack-shaped ring resonators because this ring shape takes
less space on a chip compared to circular-shaped resonators. This is especially true if the ring length \( L \) is relatively large and allows us to implement an evanescent waveguide coupler section where the bus waveguide and ring waveguide run parallel to each other over a given coupler length, providing an additional degree of freedom in coupler design. The coupling occurs between the straight bus waveguide and the straight section of the ring resonator. The spacing between both parallel waveguides, i.e., the coupler spacing \( d_c \) and the longitudinal distance of the coupling region of the waveguides, denoted by the coupler length \( L_c \), is to be found such that a given fraction of the light in the first waveguide couples to the second one through evanescent leakage. The fraction of optical power that should couple to the second waveguide is quantified by the power cross-coupling coefficient \( \kappa_2 \) for the pump beam and \( \kappa_2 \) for the Stokes beam (see Section 2.D), and their desired values are found from lasing simulations performed in Section 4.B (see Table 7). Initially, the first two lower-order TE mode fields of the combined structure consisting of the two adjacent diamond waveguides—these modes are called the coupled modes—are calculated in Lumerical MODE Solutions \([41]\) for different coupler spacings, \( d_c \), and at the pump and Stokes wavelengths, \( \lambda_{p,s} \). The coupled modes are characterized by effective refractive indices, \( n_{\text{eff}}^{(1)}(d_c, \lambda_{p}) \) and \( n_{\text{eff}}^{(2)}(d_c, \lambda_{p}) \), for, respectively, the first and the second mode. The difference of these effective indices for a given \( d_c \) is noted as

\[
\Delta n_{\text{eff}}(d_c, \lambda_{p,s}) = n_{\text{eff}}^{(1)}(d_c, \lambda_{p}) - n_{\text{eff}}^{(2)}(d_c, \lambda_{p,s}). \tag{17}
\]

Assuming all the optical power \( P_0 \) is initially (at \( z_c = 0 \)) in the first waveguide, the power \( P_2 \) in the second waveguide at the distance \( z_c = L_c \) can be calculated from \([42]\)

\[
P_2(L_c, d_c, \lambda_{p,s}) = P_0 \sin^2 \left( \frac{\pi L_c \Delta n_{\text{eff}}}{\lambda_{p,s}} \right). \tag{18}
\]

For the power-coupling ratios we demand that

\[
\frac{P_2(L_c, d_c, \lambda_{p,s})}{P_0} \equiv \kappa_2^2. \tag{19}
\]

Now Eq. (18) can be solved for \( L_c \) while satisfying Eq. (19), to find

\[
L_c(d_c, \lambda_{p,s}) = \lambda_{p,s} |\pi \Delta n_{\text{eff}}| \sin^{-1}(\kappa_2). \tag{20}
\]

The procedure to design the coupling region such that a specified \( \kappa_2 \) and \( \kappa_2 \) is achieved can be summarized as follows.

1. We choose a given coupler spacing \( d_c \) and through mode simulations we obtain \( \Delta n_{\text{eff}} \) at \( \lambda_{p,s} \).
2. From Eq. (20) we analytically calculate the optimal lengths \( L_c \) to achieve the desired coupling ratios for the pump and the Stokes power.
3. If the optimal lengths \( L_c \) for pump and Stokes coupling coincide, the design is completed. In the other case, another value of \( d_c \) is chosen and points 1 and 2 are repeated.

In the case of the IR evanescent waveguide coupler, we find by iteration that when \( d_c = 218 \) nm there is a coupler length \( L_c = 127.0 \) μm for which both IR beams (pump and Stokes) couple between the waveguides with the desired ratios specified in Section 4.B. The (semi-)analytical design is verified numerically by reconstructing in Lumerical MODE Solutions [41] the coupler TE mode fields as these propagate in the evanescent waveguide coupler. For this we employ the unidirectional eigenmode expansion method of the software. The resulting (normalized) mode-field magnitudes in the two waveguides, as a function of the coupler length \( L_c \), are shown in Fig. 4(a) at the pump frequency and in Fig. 4(b) at the Stokes frequency, confirming the (semi-)analytically calculated evanescent waveguide coupler parameters. The required and achieved coupling coefficients, as well as the optimal evanescent waveguide coupler parameters for the IR region, are summarized in Table 2.

For a coupler spacing of \( d_c = 62 \) nm, there is a coupler length \( L_c = 139.3 \) μm at which the UV pump and UV Stokes beams couple between the waveguides with the desired ratios specified in Section 4.B. After numerically verifying the coupler design in Lumerical MODE Solutions [41], we conclude from Fig. 5(a) (propagating TE pump-mode fields) and Fig. 5(b) (propagating TE Stokes-mode fields) that the designed evanescent waveguide coupler performs as expected. The optimal UV evanescent waveguide coupler parameters are summarized in Table 3.

### Table 2. Optimal Parameters of the IR Evanescent Waveguide Coupler (Required Values from Table 7)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Required</th>
<th>Achieved</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa^2_p )</td>
<td>%</td>
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<td>4.30</td>
</tr>
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<tr>
<td>( d_c )</td>
<td>nm</td>
<td>–</td>
<td>218</td>
</tr>
<tr>
<td>( L_c )</td>
<td>μm</td>
<td>–</td>
<td>127.0</td>
</tr>
</tbody>
</table>

B. Design of Diamond Grating Couplers

For the integrated diamond ring Raman lasers to operate, a pump beam must be injected into and the Stokes output must be extracted from the bus waveguide of the ring resonators. Typically, the beams are guided to and from the bus waveguide through optical fibers. One approach is to employ edge coupling with on-chip (inverted) tapers. The drawback of this technique is that it requires the deposition of, e.g., polymers aligned with the diamond waveguides and as such does not provide a monolithic solution [31,43]. Instead, we will design grating

![Fig. 4. Normalized magnitudes of the propagating (a) TE pump and (b) TE Stokes mode fields in the evanescent coupling region of the IR diamond ring resonator (at the center transversal position \( x = H/2 \) in the waveguides). Left in the figure the modes in the bus section are depicted; right in the figure the modes in the ring section.](image1)

![Fig. 5. Normalized magnitudes of the propagating (a) TE pump and (b) TE Stokes mode fields in the evanescent coupling region of the UV diamond ring resonator (at the center transversal position \( x = H/2 \) in the waveguides). Left in the figure the modes in the bus section are depicted; right in the figure the modes in the ring section.](image2)
couplers to realize the pump injection and Stokes extraction because such grating couplers can be fabricated fully monolithically in the diamond material and also rule out the necessity of aligning polymer depositions with the diamond waveguides [44,45]. Although the performance of IR inverted tapers could be better than IR grating couplers, the polymers required for edge coupling could introduce additional absorption losses in the UV region because of their narrower transparency window as compared to diamond. Other advantages of using grating couplers instead of inverted tapers are the high-alignment tolerance and the opportunities for easy packaging and wafer-scale testing of the integrated diamond chips due to the out-of-plane coupling [46]. Therefore, we choose to consistently couple the light in and out of both the SWIR and UV devices with grating couplers.

To facilitate the grating design and fabrication, we only consider binary grating couplers that are fully etched until the silica layer (see Fig. 1), and solely characterized by the grating period $\Lambda$ and the grating duty cycle $d_c/\Lambda$, where $L_g$ is the longitudinal length of one grating line. Our aim for the grating coupler design is to achieve a pump-power transmission of $\xi_p > 20\%$ and a Stokes-power transmission of $\xi_s > 1\%$. This choice for a high pump-power transmission is motivated by the knowledge that nonlinear processes (like SSRS) require high pump irradiances. Nevertheless, another choice for the minimal power transmissions could also be considered. We calculate the transmissions $\xi_{p/s}$ at the pump and Stokes wavelengths through 2D (height $x$ and longitudinal length $z$) grating coupler simulations in Lumerical FDTD Solutions [47], a commercial finite-difference time-domain (FDTD) Maxwell equations solver. In the simulations we fix the angle between the vertical axis ($x$) and the fiber to $\phi_f = 10^\circ$, as this is a commonly used positioning angle for fibers mounted above grating couplers.

For the IR grating we first find the optimal parameters for coupling only the pump or the Stokes radiation (see Table 4). Since the IR pump and Stokes wavelengths are relatively far from each other, the optimized grating couplers at these wavelengths are quite different. In other words, the pump-optimized grating coupler performs poorly for Stokes light (for which unacceptable low transmissions $\xi_s < 1\%$ are obtained), and vice versa. Therefore, we alter the pump-optimized grating to increase its transmission at the Stokes wavelength in the following way.

1. Since the Stokes transmission can be improved by raising $\Lambda$ [48], we first slightly increase the pump-optimized grating period $\Lambda$.

2. For the new value of $\Lambda$, the transmission $\xi_p$ at the pump wavelength is plotted as a function of the grating duty cycle, and the duty cycle is slightly varied around its value that maximizes $\xi_p$ (in steps of 1%) to optimize the transmission $\xi_s$ at the Stokes wavelength while still maintaining $\xi_p > 20\%$.

3. If the Stokes transmission cannot be made $\xi_s > 1\%$, the grating period $\Lambda$ is further increased in point 1.

The parameters of the resulting grating coupler optimized for both pump and Stokes light are also listed in Table 4 (last column). The sensitivity of the transmission of the final IR grating with respect to the design parameters is shown graphically in Fig. 6(a) (for the grating period) and Fig. 6(b) (for the grating duty cycle). From these two graphs it is clear that the final grating coupler is mainly optimized for the pump wavelength. At the Stokes wavelength the transmission reaches only a local maximum. We point out that the IR grating could be made more efficient at both (distantly separated) pump and Stokes wavelengths by considering more advanced coupler structures including partially etched grating profiles [49], nonuniform gratings (with varying duty cycle along the grating length) [49], chirped gratings (with varying period along the grating length) [50] and apodized gratings (with varying duty cycle and period along the grating length) [51,52]. Nevertheless, since the number of demonstrated diamond-grating couplers...
is quite limited to date [44,45] and because of the increased design and fabrication complexity of these advanced grating structures, we have chosen to stick to basic fully etched gratings without chirping nor apodization at this stage.

Also for the UV grating we first determine the optimal parameters for only pump or Stokes coupling (see Table 5). The UV pump and Stokes wavelengths are relatively closely spaced so that the pump- and Stokes-optimized gratings do not differ much that, and consequently, the maximal grating transmissions at both wavelengths are achieved with approximately the same parameter values. As a result, the pump-optimized grating coupler is more than efficient enough (ξ > 1%) at the Stokes wavelength so there is no need to tune its transmission sensitivity of the final UV coupler with respect to the grating parameters in Fig. 6(c) (grating period) and Fig. 6(d) (grating duty cycle). These graphs confirm that the pump-optimized grating coupler also yields an efficient coupling at the Stokes wavelength.

**4. MODELING OF RAMAN LASING IN INTEGRATED DIAMOND RING RESONATORS**

A. Raman Lasing Modeling Strategy

For the actual Raman lasing simulations we consider a CW pump input injected at the left-hand side of bus 1 in Fig. 1. Since we consider ring resonators, the laser output is either the Stokes forward power, coming out of bus 2, or the Stokes backward power, coming out of bus 1. Bidirectional lasing, exhibiting forward and backward output powers, is not likely, as was shown in [7]. Two factors determine the lasing directionality:

- The nonreciprocity in effective areas of the Raman gain, $A_{\text{eff}}^+$ and $A_{\text{eff}}^-$ [see Eq. (2)]: when $A_{\text{eff}}^+ > A_{\text{eff}}^-$, backward lasing is much more likely to occur as the backward-propagating Stokes beam experiences more gain than the forward-propagating Stokes beam.

- The nonlinear waveguide losses of diamond in the UV region: for pump-coupled input powers $|A_{\text{fin}}^{+,\text{in}}|^2$ larger than a certain rollover point $|A_{\text{fin}}^{+,\text{cl}}|^2$, increasing $|A_{\text{fin}}^{+,\text{in}}|^2$ results in a decreased net gain, as the nonlinear losses grow more rapidly with increasing pump power than does the pump-induced Raman gain [5]. Even if one would have $A_{\text{eff}}^+ = A_{\text{eff}}^-$, it can be shown [7] that the laser operates in the forward direction when $|A_{\text{fin}}^{+,\text{in}}|^2 < |A_{\text{fin}}^{+,\text{cl}}|^2$ and in the backward direction when $|A_{\text{fin}}^{+,\text{in}}|^2 > |A_{\text{fin}}^{+,\text{cl}}|^2$.

The values of the various parameters of the Raman lasing simulations, as well as the corresponding sections in which these have been introduced, are listed in Table 6. Concerning the Raman gain coefficient, we remark that far away from diamond’s absorption resonances $g_R$ scales approximately linearly with (pump) frequency [53] so that at wavelength $\lambda_1$, $g_R(\lambda_1)$ can be calculated from $g_R(\lambda_2)$ at wavelength $\lambda_2$ as

$$g_R(\lambda_1) \approx \lambda_1 A_{\text{eff}}(\lambda_1).$$

Using Eq. (21) and the Raman gain coefficient in Table 1 at $\lambda = 1030$ nm, we obtain for the SWIR Raman laser a value $g_R \approx 9.0$ cm/GW. However, since the UV Raman laser operates relatively close to diamond’s absorption resonances, we do not rely on Eq. (21) for determining its gain value, and instead we take for $g_R$ of the UV laser the experimentally determined value 100.0 cm/GW for the 266 nm pumped bulk-diamond Raman laser of [24]. In Eq. (4) we assume the pump FWHM, $\Delta \omega_p$, to be 10 times smaller than $\Gamma_R$, which is a realistic assumption for CW pump lasers. The effective areas for SSRS, $A_{\text{eff}}^{\pm}$ are calculated from the diamond waveguide modes (obtained in Section 3.A) according to Eq. (5). For the UV laser, the effective areas for TPA, $A_{\text{eff}}^{\lambda_1}$, and $A_{\text{eff}}^{\lambda_2}$, are also calculated from the waveguide modes using Eq. (11). The values of these effective areas lie close to that of $A_{\text{eff}}$, and therefore we set $A_{\text{eff}}^{\lambda_1} \equiv A_{\text{eff}}^{\lambda_2}$. For the linear waveguide losses $\alpha_{\text{lin}}^r$ of the SWIR Raman laser we adopt 1 dB/cm from [53]. We remark that this loss value is still conservative since much lower losses of the order of 0.34 dB/cm have already been realized experimentally at telecom wavelengths [43]. To estimate the linear waveguide losses of the UV waveguide, we scale the losses of the IR waveguide using the model introduced by Payne and Lacey [54]. This model, providing a rigorous upper limit of the scattering losses in waveguides, is commonly used to calculate the radiation losses arising from scattering by surface roughness of the waveguide walls [45,55,56]. Taking into account the IR and UV waveguide geometry and waveguide mode data, the model predicts the UV linear losses to be two orders of magnitude larger than the losses in the IR. Consequently, we choose

<table>
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<tr>
<th>Parameter</th>
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<th>Pump/Stokes</th>
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<table>
<thead>
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<td>276</td>
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<td>cm/GW</td>
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<td>100.0</td>
</tr>
<tr>
<td>$g_R$</td>
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<td>10$^9$ rad/s</td>
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<td>$2\pi \times 36$</td>
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<tr>
<td>$\Delta \omega_p$</td>
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<td>10$^9$ rad/s</td>
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<tr>
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<td>1</td>
</tr>
<tr>
<td>$\phi_p$</td>
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<tr>
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<td>4.A</td>
<td>mW</td>
<td>20.0</td>
<td>340.0</td>
</tr>
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</table>
\[ \alpha_{p,s} = 100 \text{ dB/cm}. \] We point out that since the model of Payne and Lacey provides an upper limit for the losses, the UV loss value of 100 dB/cm is expected to be, like the IR loss of 1 dB/cm, a rather conservative estimation. Considering the continuous advancement of lithographic fabrication techniques, considerable lower UV waveguide losses of a few 10 s of dB/cm should soon come within range. As for the nonlinear losses, we are reminded that the SWIR Raman laser does not exhibit nonlinear losses while the UV laser does (see Section 1). The TPA-coefficients \( \beta_{p,s,n,p} \) are taken to be 1 cm/GW \([57]\), and the FCA efficiency coefficient is \( \phi = 6.0 \cdot 10^{-10} \) \([58]\). We overestimate the effective charge-carrier lifetime by choosing \( \tau_{\text{eff}} = 0.5 \text{ ns} \) \([53]\), which is of the same order of magnitude as the effective charge-carrier lifetime of the bulk chemical-vapor-deposition grown diamonds \([59]\). The total racetrack length \( L \), and as such also the racetrack bend radius \( R \), is selected to make sure ring resonance (and thus cavity enhancement) is obtained at both the pump and Stokes wavelengths, i.e. \([60]\),
\[ k_{p,s} = \frac{2\pi}{L}, \] (22)
where \( k_{p,s} \) are the wavenumbers of the pump and Stokes waves and \( m_{p,s} \) the mode numbers (which are positive integers). The frequency of the pump beam can be tuned to coincide with a cavity mode. As for the Stokes frequency, the free spectral range (FSR) of the ring cavity is chosen small enough to make sure at least one cavity mode falls inside the FWHM \( 2\Gamma_R \) of the Raman gain. The FSR (in frequency units) of the ring resonator is given by
\[ \Delta f_{\text{FSR}} = \frac{c}{nL}. \] (23)
For a total racetrack length \( L = 1885 \text{ m} \), the FSR is 63.6 GHz (when approximating the refractive index \( n \) by 2.5), while the Raman-gain FWHM is 72 GHz so that both the pump and Stokes can be at ring resonance. We assume \( R \) to be 30 m and \( L_b = 1000 \text{ m} \). Finally, we choose a realistic pump input power (coupled in the resonator) of \( |A_{p,0}^+|^2 = 20.0 \text{ mW} \) for the SWIR device. For the UV device we set the pump input power to a much higher value of \( |A_{p,0}^+|^2 = 340.0 \text{ mW} \) due to the much higher linear waveguide losses. We remark that to achieve these power values in the resonator, \( |A_{p,0}^+|^2 = 66.7 \text{ mW} \) (for the SWIR laser) or \( |A_{p,0}^+|^2 = 1.0 \text{ W} \) (for the UV laser) has to be injected in the grating coupler at the left-hand side of Fig. 1. We point out that when using a less conservative UV scattering loss value of a few 10 s of dB/cm, a coupled pump power of about 100 mW would be sufficient for the UV laser.

To carry out the actual lasing simulations we numerically solve the Raman lasing equations for both the forward- and backward-propagating waves, Eq. (1), in each section of the ring resonator (bus 1, bus 2, and the ring). When solving the lasing equations, the evanescent light coupling between the ring and bus sections is taken into account according to Eq. (15). We remark that the SSRS process is initiated by spontaneous Raman scattering. This is modeled in the simulations by initializing the Stokes waves with a small value (Stokes seed) at the first round trip (\( \tau = 1 \)). Since the waveguide orientation is not constant but varies along the ring, we also need to take into account the anisotropic nature of diamond’s Raman-susceptibility tensor. Consequently, the effective Raman gains in Eq. (2) have to be modified to
\[ G_{\text{eff}}(\omega_s) = \frac{g}{A_{\text{eff}}^2} \times \rho, \] (24)
when assuming resonant SSRS, i.e., \( \omega_p - \omega_s = \Omega_0 \), and thus \( H_R(\Omega_0) = 1 \). The gain variations in the bent sections of the ring are described by \( \rho = \cos^2(2\theta) \) \([7]\), where \( \theta \) is defined in Fig. 1. Since the straight waveguide sections are assumed to be oriented along the [011] direction, the gain only varies to lower values in the bent sections of the racetrack-shaped ring and is maximal (\( \rho = 1 \)) in its straight sections that constitute the largest part of the ring. To solve the lasing equations, taking the aforementioned remarks into consideration, we use the iterative resonator method (IRM) of \([5,7]\). The IRM enables us to solve numerically the lasing equations inside the racetrack-shaped ring resonator at every round trip of the propagating pump and Stokes waves until steady-state operation is reached.

**B. Raman Lasing Modeling Results**

Using the IRM, we first satisfy the condition for critical coupling (Section 2.D) of the pump wave to maximize the power buildup in the ring by determining the optimal pump cross-coupling coefficient \( \kappa_p \), i.e., by taking the \( \kappa_p^2 \) value that is equal to the relative pump-power losses in the ring in steady state. Since for the SWIR Raman laser the only propagation losses are linear, the relative pump-power losses \( \Phi_p^{<r} \) in the ring for a forward-propagating wave [see Eq. (16)] do not depend on \( \kappa_p \) and can thus readily be calculated. The pump power \( |A_{p,\infty}^+(L)|^2 \), in absence of a Stokes beam, is attenuated over a distance \( L \) in the ring as
\[ |A_{p,\infty}^{+r}(L)|^2 = |A_{p,\infty}^{+r}(0)|^2 \times 10^{-\alpha_p L/10}, \] (25)
so that \( \Phi_p^{+r} \) is found to be
\[ \Phi_p^{+r} = 1 - 10^{-\alpha_p L/10} \approx 4.25\%. \] (26)
Critical coupling of the pump beam is thus obtained when \( \kappa_p^2 = \Phi_p^{+r} \approx 4.25\% \) [see also Fig. 7(a)]. Since the UV Raman laser suffers from nonlinear losses, the relative pump-power losses in the ring depend on \( \kappa_p \). By calculating for the UV device, the relative pump-power losses for several values of \( \kappa_p^2 \) with the aid of the IRM lasing simulations [see Fig. 7(b)], the optimal value \( \kappa_p^2 = 98.77\% \) is selected for critical coupling. We also note from the nearly flat curve in Fig. 7(b) that the high-linear losses dominate the nonlinear losses for the regarded pump input power. The larger optimal value of the pump cross-coupling coefficient, compared to its value for the SWIR laser, is mainly due to the higher linear losses and, to a lesser extent, due to the nonzero nonlinear losses of the UV laser.

Next we fix the Stokes cross-coupling coefficient \( \kappa_s \) at its value that maximizes the laser’s output power for a pumped-coupled input power of 20.0 mW in the IR and of 340.0 mW in the UV. The IR lasing simulations show that for \( \kappa_s^2 = 6.46\% \) the backward-propagating Stokes output power is maximal [see Fig. 7(c)]. The reason for backward lasing is that \( A_{\text{eff}}^+ > A_{\text{eff}}^- \). We point out that backward lasing in ring
Raman lasers based on nanowires has already often been predicted [7,32,37,38]. To illustrate why at a certain Stokes cross-coupling coefficient the laser’s output power is maximal, we provide input-output power graphs for the optimal cross-coupling coefficient value, for a smaller and larger value, and for a smaller and larger value than this optimal value in Fig. 8(a). When increasing/decreasing the laser threshold power of 11.1 mW. When taking into account the grating coupler transmission, the corresponding net Stokes output power equals \( A_j \). The UV laser with \( \kappa_j = 0.17\% \) is the most efficient with \( |A_j^{\text{out}}|^2 = 2.6 \text{ mW} \), yielding \( |A_j^{\text{out}}|^2 / |A_j^{\text{in}}|^2 = 0.8\% \) and a slope efficiency of 65\%. The corresponding output power after transmission through the grating coupler is \( |A_j^{\text{out}}|^2 = 0.8 \text{ mW} \). The high slope efficiency is caused by the relatively high effective Raman gain induced by the high Raman-gain coefficient \( g_R \) at UV frequencies and by the small effective areas of the Raman gain in the UV waveguide. A pump threshold power of 334.9 mW is also observed. We remark that the spikes in Fig. 8(b) are caused by minor oscillatory behavior of the powers inside the resonator as a function of the number of propagation round trips. Finally we note that the pump threshold power becomes lower than 100 mW when using less conservative UV scattering losses of a few 10 s of dB/cm while still maintaining a high slope efficiency above 50\%.

An overview of the resulting optimal cross-coupling coefficients and laser output-power values (for IR and UV) can be found in Table 7.

The longitudinal distributions along the ring of the steady-state pump power and Stokes power are shown in Fig. 9 for the SWIR laser and in Fig. 10 for the UV laser. While the forward-propagating IR pump beam [Fig. 9(a)] decreases from left to right (increasing \( z \)) due to optical losses and depletion, the
backward-propagating IR Stokes beam [Fig. 9(b)] grows from right to left (decreasing \( z_r \)) due to SSRS. In Fig. 10(a) we notice the severe effect of depletion and the high optical losses of the UV waveguide on the forward-propagating UV pump wave. In the steady-state longitudinal distribution of the backward-propagating UV Stokes wave we can discern two regimes. In the first regime, at the right-hand side of Fig. 10(b) (large \( z_r \)), the pump wave is too weak to sufficiently amplify the Stokes beam and that is why in the region of large \( z_r \) the latter beam attenuates from right to left due to the strong losses. When the pump wave is sufficiently high in the second regime at the left-hand side of Fig. 10(b) (small \( z_r \)), SSRS causes the Stokes power to increase from right to left. The abrupt changes in the left and right parts of the Stokes graphs for both the SWIR and UV lasers are caused by the variation of the effective Raman gain [see Eq. (24)] in the bent sections of the racetrack-shaped ring resonator. Figures 9(b) and 10(b) illustrate that using a racetrack-shaped ring with a small bending radius \( R \) offers the additional advantage that the Stokes growth in the ring is hardly influenced by the anisotropic nature of diamond's Raman susceptibility as defined in Eq. (24).

### Table 7. Resulting Optimal Cross-Coupling Coefficients and Output Power Values from the Lasing Simulations in the IR and UV

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Value for IR</th>
<th>Value for UV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_s^2 )</td>
<td>%</td>
<td>4.25</td>
<td>98.77</td>
</tr>
<tr>
<td>( \kappa_p^2 )</td>
<td>%</td>
<td>6.46</td>
<td>0.17</td>
</tr>
<tr>
<td>( P_{\text{p;in}} )</td>
<td>mW</td>
<td>11.1</td>
<td>334.9</td>
</tr>
<tr>
<td>(</td>
<td>A_{s;out}^2</td>
<td>)</td>
<td>mW</td>
</tr>
<tr>
<td>(</td>
<td>A_{p;gr}^2</td>
<td>)</td>
<td>mW</td>
</tr>
</tbody>
</table>

5. CONCLUSION

We have thoroughly studied the suitability of integrated single-crystal diamond ring resonators for Raman lasing. In particular, we have considered grating coupled racetrack-shaped diamond ring resonators. Racetrack-shaped rings take less space on a chip compared to circular-shaped rings, especially if the ring length is relatively large, and allow us to implement an evanescent waveguide coupler section where the bus waveguide and ring waveguide run parallel to each other over a given coupler length, providing an additional degree of freedom in coupler design. We have consistently regarded grating couplers to guide the light beams into and out of the integrated devices because grating couplers, as opposed to inverted tapers, can be fabricated fully monolithically in the diamond material and also rule out the necessity of aligning polymer depositions with the diamond waveguides. We have carried out the design and modeling for the entire devices including the gratings.

CW Raman lasing in both the SWIR and the UV ranges has been considered. The Raman lasing simulations predict that both the SWIR and UV lasers operate in the backward direction because of the Raman gain non-reciprocity. For a pump coupled input power in bus 1 of 20.0 mW, the optimal SWIR laser configuration yields a lasing slope efficiency of 33% and a Stokes output power of 4.0 mW. For the optimized UV device and a pump coupled-input power of 340.0 mW, the lasing slope efficiency equals 65% and the Stokes output power equals 2.6 mW. Whereas this is a low output-power value for a pump coupled-input power of 340.0 mW, we note that this value could strongly increase when considering a less conservative scattering loss value for the UV laser. As such, efficient lasing is possible with integrated diamond ring Raman lasers.

![Fig. 9](image_url) Longitudinal distributions of the steady-state (a) pump forward and (b) Stokes backward power along the ring of the SWIR Raman laser.

![Fig. 10](image_url) Longitudinal distributions of the (a) pump forward and (b) Stokes backward power along the ring of the UV Raman laser.
including the grating coupler transmission, the net output power of the SWIR laser is 0.1 mW and of the UV laser 0.8 mW. We remark that the net output power of the SWIR device could be enhanced by using partially etched, nonuniform, chirped, and/or apodized gratings. Another practical way to increase the Stokes coupling efficiency is by directing the backward-propagating Stokes output beam through a directional coupler to another (basic) grating coupler that is optimized for only Stokes light transmission.

Whereas UV Raman lasing in silicon is not possible at all, we have thus shown that UV Raman lasing in a diamond ring resonator can potentially be effective. Moreover, as opposed to integrated silicon Raman lasers, no carrier-extracting PIN diodes have to be embedded around the waveguides of diamond Raman lasers to minimize the effects of FCA.

Although Raman lasers based on free-space cavities containing a bulk diamond crystal have already been experimentally demonstrated, the demonstrations of integrated diamond Raman lasers are to date only in the exploratory phase to our knowledge. We have shown that integrated diamond ring Raman lasers are very promising devices to produce laser light at wavelengths that are hard (or not possible) to generate with other types of integrated lasers.

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