Using a polynomial decoupling algorithm for state-space identification of a Bouc-Wen system

Alireza Fakhrizadeh Esfahani
afakhriz@vub.ac.be

Philippe Dreesen
Philippe.Dreesen@vub.ac.be

Johan Schoukens
Johan.Schoukens@vub.ac.be

Koen Tiels
Koen.Tiels@vub.ac.be

Vrije Universiteit Brussel, Dept. ELEC

1 Introduction
The polynomial nonlinear state space (PNLSS) approach [1] is a powerful tool for modeling nonlinear systems. A PNLSS model consists of a discrete-time linear state space model, extended with polynomials in the state and the output equation:

\[ x(t+1) = Ax(t) + Bu(t) + E\zeta(t) \] (1)
\[ y(t) = Cx(t) + Du(t) + F\eta(t) \] (2)

where \( \zeta(x(t), u(t)) \) and \( \eta(x(t), u(t)) \) are both vectors with monomials in the states \( x(t) \) and the inputs \( u(t) \). The matrices \( E \) and \( F \) contain the polynomial coefficients. The PNLSS model is very flexible as it can capture many different types of nonlinear behavior, such as nonlinear feedback and hysteresis. This flexibility generally comes at the cost of a large number of parameters. Increasing the order of the polynomials for example leads to a combinatorial increase of the number of parameters due to the multivariate nature of the polynomials \( E\zeta(x(t), u(t)) \) and \( F\eta(x(t), u(t)) \). In this study, the PNLSS approach is used to model a Bouc-Wen hysteretic system. The multivariate polynomials \( E\zeta(x(t), u(t)) \) and \( F\eta(x(t), u(t)) \) are decoupled using the method in [2]. Like this, the nonlinearity in the PNLSS model is described in terms of univariate polynomials for which increasing their order is not so parameter expensive.

2 Methodology
The simulated data is extracted from the Bouc-Wen model with the equations:

\[ m\ddot{y} + c\dot{y} + ky + z(y, \dot{y}) = u(t) \] (3)
\[ \dot{z} = y - \alpha |y| \dot{y} - \beta |\dot{y}| \] (4)

where \( m \) is the mass, \( c \) is the damping coefficient, \( y(t) \) is the measured output and \( u(t) \) is the input force which is a random phase multisine. The parameters \( \alpha \) and \( \beta \) are chosen equal to \( 5 \times 10^4 \) and 0.8 respectively. In a first step, we estimate the best linear approximation (BLA) [3] of the system. A linear state-space model estimated on the BLA serves as an initial guess for the PNLSS model in (1) and (2), which is optimized using a Levenberg-Marquardt approach. In a second step, the multivariate polynomials \( E\zeta(t) \) and \( F\eta(t) \) are decoupled using the decomposition method in [2]:

\[ E\zeta(x(t), u(t)) \approx W_g V^T \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \] (5)
\[ F\eta(x(t), u(t)) \approx W_g V^T \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \] (6)

where \( V \) transforms the states and inputs in new variables \( \xi = V^T \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \). The function \( g \) is a collection of univariate polynomials \( g_i(\xi) \) for \( i = 1, 2, \ldots, r \) that act as basis functions for the decoupled state-space model. The matrices \( W_g \) contain the corresponding basis function coefficients.

3 Results
The Bouc-Wen model is excited with a random phase multisine of 6.813 N in the standard deviation. The results for the PNLSS modelling is plotted in Figure 1 for the validation data.

4 Conclusion
A PNLSS model can capture the behavior of a Bouc-Wen system. A decoupled PNLSS model reaches a similar accuracy, but has less than two third of the number of parameters. The order of the polynomials in the decoupled model can also be increased without blowing up the number of parameters, as it is the case for the full PNLSS model.

Acknowledgement
This work has been supported by the ERC Advanced grant SNLSD, under contract 320378.

References