Common-denominator modelling for stability analysis of electronic circuits

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Abstract—In the local stability analysis of a circuit, the poles of the linearised circuit are investigated. If any of the circuit poles has a positive real part, the found solution for the circuit is unstable. The poles of the linearised circuit are usually obtained by fitting rational models on data samples of the Frequency Response Functions of the linearised circuit. Classically, a single impedance function is modelled to determine the poles. In this paper, it is shown that a common-denominator modelling of multiple impedance functions together can increase the robustness of the rational modelling procedure for stability analysis.

Index Terms—Stability analysis, pole-zero identification, common-denominator, Vector Fitting.

Pole-zero identification is becoming a common technique to perform the local stability analysis of electronic circuits [1]–[4]. If any of the poles of the linearised circuit has a positive real part, the circuit solution is unstable. The poles of the linearisation are determined by fitting rational functions on frequency responses (FRFs) of the linearised circuit. The required FRFs can be obtained by a small-signal simulation where the transfer functions from small-signal current sources to voltages at nodes in the circuit are calculated (see Figure 1).

All FRFs in a circuit share the same poles (common-denominator), but they have a different set of zeros. Pole-zero cancellations can occur, which obscure the stability analysis result. To reduce the risk of running into such an unfortunate pole-zero cancellation, multiple FRFs of the circuit are considered. The identification routine is then applied to all obtained impedances separately [4].

Several identification algorithms allow estimating rational models with a common-denominator for multiple FRFs [5]–[8] modelled together. In this paper, the use of common-denominator modelling for stability analysis is investigated. First, the local stability analysis of the DC solution is discussed in Section I. It is shown that using the common-denominator identification algorithm does not really improve the robustness of the stability analysis of simple circuits. In Section II, the local stability of a large-signal orbit is considered. In this case, the common-denominator approach can improve the robustness of the stability analysis.

I. STABILITY ANALYSIS AROUND DC

To determine the stability of the DC solution of a circuit, the circuit-under-test is first terminated in its load and source impedances $Z_l$ and $Z_s$. Then $n$ nodes in the circuit are excited by small-signal current sources\(^1\) that are given a small-signal amplitude $I_i$ one-by-one during $n$ AC simulations (see Figure 1). During the simulations, $m$ different voltages $V_i$ in the circuit are evaluated at a set of frequencies $f = [f_1, ..., f_{N_{freq}}]$. The $m$ evaluated voltages of the $i^{th}$ AC simulation are gathered in a vector and divided by the applied current to obtain $m$ FRFs

$$Z_i(\omega) = \frac{1}{I_i(\omega)} \left[ V_1(\omega) \cdots V_m(\omega) \right]$$ (1)

where $\omega = 2\pi f$. The FRFs of the different AC simulations are combined in a vector $Z(\omega)$

$$Z(\omega) = \left[ Z_1(\omega) \cdots Z_m(\omega) \right]$$ (2)

All FRFs in $Z$ share a common denominator. Written in a pole-residue format, a rational model can be expressed as

$$\tilde{Z}(s) = \sum_{p=1}^{N} \frac{R_p}{s - a_p} + D$$ (3)

starting from the frequency-domain data samples $Z(\omega)$, where $s$ is the Laplace variable and $N$ is the order of the model. The poles $a_p$, residues $R_p$ and direct term $D$ are the parameters to be estimated. In this paper, the Vector Fitting technique is

\(^1\)Series voltage excitations can also be used, but are more difficult to add to a netlist as they introduce an extra node.
used to estimate the rational model [5]–[7]. The Vector Fitting technique is a well-established approach for the modelling of simulated or measured frequency responses by means of rational functions in a pole-residue form. It is a highly robust and efficient method that is able to accurately model systems with both smooth and resonant frequency responses over wide frequency bands. The robustness of the method is mainly due to the use of partial fraction bases instead of polynomials and to the relocation of the poles of these partial fraction bases in successive iterations. The cost function minimized by the Vector Fitting algorithm to find the poles $\mathbf{p}$, residues $\mathbf{R}$, and direct term $\mathbf{D}$ based on the data $\mathbf{Z}$ is

$$
\sum_{q=1}^{N_{freq}} \left\| \mathbf{Z}(j\omega_q) - \tilde{\mathbf{Z}}(j\omega_q) \right\|_2^2
$$

Selecting the correct order of the model is critical in the pole-zero based stability analysis [9]. With a too low order, critical poles can be missed, and all other estimated poles are moved from their correct location in order to compensate for the missing poles. A too high order on the other hand introduces poles that are compensated by a zero. These 'rogue poles' can end up anywhere in the complex plane, possibly resulting in a wrong answer for the stability analysis.

The model order selection is easy in the case of lumped circuit. This is demonstrated in the following example. If distributed elements, like transmission lines, are present in the circuit, a perfect rational approximation of $\mathbf{Z}$ cannot be obtained. In [9], [10], an automatic order selection method is described that can be used when distributed elements are present in the circuit. Using a common-denominator model in this method could increase its robustness, but it is considered outside of the scope of this paper.

A. Example: LNA

As an example, a low noise amplifier (LNA) from Infineon [11] is used (see Figure 2). The circuit is stable, so any instability found is an artefact of a wrong pole estimation. The small-signal current source is connected to the collector of the transistor. The AC simulations are performed on a logarithmic frequency grid between 1 MHz and 100 GHz. The transition frequency $f_T$ of the transistor used in the LNA is 45 GHz, so the simulated frequency span covers the full range where the transistor can possibly generate oscillations.

![Fig. 2. The LNA under study is built with the BFP520 bipolar transistor of Infineon.](image)

![Fig. 3. Sweeping the model order and looking at the model error allows selecting the correct order for the model. The dashed line indicates the error for the common-denominator model, while the full line indicates the error obtained while modelling a single impedance function. Stable models are indicated with green circles, while unstable models are indicated with red crosses.](image)

The poles of the circuit are estimated twice. First only one FRF is given to the Vector Fitting algorithm. Then a common-denominator model is estimated using 6 different FRFs of the circuit.

The order of the model is determined with a simple scan. The root-mean-square (rms) error of both estimated models as a function of the model order is plotted in Figure 3. In both cases, the rms error of the models reaches numerical precision for a model order of 22 with a clear accuracy jump and a stable model is obtained. Below and above the correct order, several falsely unstable models are obtained.

This result indicates that the common-denominator approach does not yield a significant improvement with respect to the pole-residue modelling from a single impedance FRF concerning stability analysis results. Of course, the latter result depends on the circuit and on the chosen node. The benefit of common-denominator modelling becomes more clear in the case of stability analysis of large-signal orbits.

II. Stability of a Large-Signal Orbit

Analysing the stability of a large-signal orbit is performed in a similar simulation set-up as in the DC case [12]. The circuit is now excited at its input by a large-signal tone (see Figure 1) and the circuit response to that tone is calculated with a Harmonic Balance simulation. Then the circuit is linearised around the found solution in a mixer-like simulation. $n$ small-signal current sources are applied to the circuit and the response at $m$ voltages in the circuit is determined.

In classic mixer simulations, the response of the linearised system to complex exponents

$$
i_q(t) = I_k e^{-j2\pi f_q t}
$$

is calculated, where $f_q$ spans a certain range. Due to mixing with the large-signal at a frequency $f_0$, the voltages contain contributions at frequencies $f \pm k f_0$ with integer $k$. The frequency response from the current excitation frequencies $f$ to the voltage frequencies $f + k f_0$ is referred to as Harmonic Transfer Function number $k$ (HTF$_k$) [13], [14]. The HTFs
from current source $i$ to the different voltages are stored in a vector $Z_k^{[i]}$

$$Z_k^{[i]}(\omega) = \frac{1}{T_i(\omega)} \left[ V_1(\omega + k\omega_0) \cdots V_m(\omega + k\omega_0) \right]$$

(6)

Again, the results of the different mixer-like simulations around the same orbit can be combined in a vector

$$Z_k(\omega) = \begin{bmatrix} Z_k^{[1]}(\omega) & \cdots & Z_k^{[n]}(\omega) \end{bmatrix}$$

(7)

Similar formulas can be used for $Z_{-k}$. The mixer simulation works with complex excitation signals, so the resulting HTFs require rational models with complex coefficients for the modelling. To allow approximation by rational models with real coefficients, the HTFs are transformed into their equivalent $\hat{Z}_k$, which use sine and cosine basis functions, instead of complex exponentials

$$\hat{Z}_k(\omega) = \frac{1}{2} [Z_k(\omega) + Z_{-k}(\omega)]$$

(8)

$$\hat{Z}_{-k}(\omega) = \frac{1}{2} [Z_k(\omega) - Z_{-k}(\omega)]$$

(9)

2$K + 1$ FRFs are combined in one large vector

$$Z(\omega) = \begin{bmatrix} \hat{Z}_{-K}(\omega) & \cdots & Z_0(\omega) & \cdots & \hat{Z}_K(\omega) \end{bmatrix}$$

(10)

which can be fed to the identification algorithm.

All FRFs in $Z$ share a common denominator, so they can be modelled by a common-denominator rational model using Vector Fitting. Due to the time-varying nature of the linearised circuit, an infinite amount of copies of each pole in the system appear at vertical lines in the complex plane, spaced at multiples of the system pulsation $\omega_0$ [12, 14]. The correct model is described by

$$\hat{Z}(s) = \sum_{p=1}^{N} \sum_{k=-\infty}^{\infty} \frac{B_{p,k}r}{s - (a_p - k\omega_0)} + D$$

(11)

Estimating this rational model with the constraint on the location of the poles in the Laplace domain from FRF data is a very hard problem [14] and the estimation algorithms that can fit this model are not widespread. Obviously, the $k$ index in (11) can be truncated based on model accuracy criteria.

The alternative is to use a classic pole-zero identification tool with a very high order. This approach can achieve a high accuracy, but the location of the poles is not guaranteed to satisfy the strong constraint imposed by (11). Although it is sub-optimal, this approach will be used in this paper. It has been shown in literature that it can work [12].

For higher $k$, the amplitude of the elements in $\hat{Z}_k, \hat{Z}_{-k}$ decreases rapidly. In (10), $K$ should be chosen such that the FRF amplitude does not become too small. The large amplitude difference in the different $\hat{Z}_k, \hat{Z}_{-k}$ negatively affects the numerical performance of the pole-zero estimation algorithm. This can be solved by using weighting functions in the cost function of the identification algorithm. In the following, the inverse magnitude weighting function will be used. The weighted cost function minimized by Vector Fitting can be written as

$$\sum_{q=1}^{N_{fr, q}} \left\| \frac{Z(j\omega_q) - \tilde{Z}(j\omega_q)}{Z(j\omega_q)} \right\|^2_2$$

(12)

where /. indicates an element wise division.

A. Large-signal stability analysis on the LNA

The same LNA of the previous example is used, but now the stability around a single-tone excitation at 1.9 GHz is investigated. A Harmonic Balance simulation with an order of 10 is used to obtain the large-signal orbit and a small-signal current source is connected to the collector of the transistor. The small-signal excitation is swept in a mixer-like simulation from 10 MHz to 100 GHz on a logarithmic frequency grid. Some of the obtained $\hat{Z}_k$ are shown in Figure 4. The higher-order harmonic transfer functions contain a lot more information about the location of the repeated poles in the system, so including them in the modelling algorithm enhances its robustness towards stability analysis results.

It is clear from Figure 4 that $\hat{Z}_0$ does not contain high dynamics around the higher-order copies of the system poles. The higher-order $\hat{Z}_k$ do show strong resonances at the frequencies associated with the copies. Due to this behaviour, the common-denominator estimation with multiple $\hat{Z}_k, k = -3, -2, -1, 0, 1, 2, 3$ together allows a more robust stability analysis with respect to the classic estimation performed on $\hat{Z}_0$ alone (see Figure 5). The classic approach generates many false-positives concerning the stability analysis during the scanning of the order. Instead, the common-denominator approach is more robust in this case. Due to the structure of the model (11), a clear indication on the correct order cannot be obtained. The large difference visible in Figure 5 concerning the rms error of the two models is due to the modelling of multiple FRFs together and the weighting function used in the modelling step.
III. Conclusion

In this paper, we have shown that using common-denominator rational modeling for local stability analysis can increase the robustness of the whole procedure. In the stability analysis of the DC solution, no evident benefit was found. However, in the stability analysis of a large-signal orbit, the common-denominator approach shows some advantages which are worth the extra algorithmic complexity. An interesting aspect for future work is to study an identification algorithm that is able to satisfy the constraint on the location of the poles imposed by the model structure (11).

Acknowledgement

This work is sponsored by the Research Foundation Flanders (FWO-Vlaanderen), the Strategic Research Program of the VUB (SRP-19), the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen), the Belgian Federal Government (IUAP VII) and the ERC advanced grant SNLSID, under contract 320378.

F. Ferranti and E. Louarroudi are Post-Doctoral Research Fellows of FWO-Vlaanderen.

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