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Opportunities for wideband wavelength conversion in foundry-compatible silicon waveguides covered with graphene

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Abstract—We numerically examine the opportunities for wideband wavelength conversion through four-wave mixing (FWM) in a foundry-compatible 220 nm-thick silicon-on-insulator (SOI) waveguide covered with the highly nonlinear two-dimensional material of graphene. As a case study, we consider a foundry-compatible SOI waveguide shaped as a double spiral and covered with two separate graphene sheets, which are covered in turn by two solid polymer electrolyte gates. When combining sub-watt level pump powers with a short waveguide length of only a few hundreds of microns, ‘perfectly phase-matched’ conversion with significant efficiencies close to −20 dB can be obtained over a more than 40 THz-wide signal band adjacent to the pump frequency. The tunability of the graphene properties using the electrolyte top gates, it is also possible to obtain ‘quasi-phase matched’ FWM conversion through a periodic sign reversal of the graphene third-order nonlinearity along the waveguide. Conversion efficiencies exceeding −30 dB can be achieved over a 3.4 THz-wide signal band that is situated as much as 58 THz away from the pump frequency. Finally, the tunability of the graphene also allows for switching the converter from the perfectly phase-matched to the quasi-phase-matched operation mode.

Index Terms—Integrated optics, nonlinear wave propagation, phase matching, silicon on insulator technology, thin film devices, waveguide components

I. INTRODUCTION

ONLINEAR optical wavelength conversion, a research area which emerged together with the invention of the laser in 1960, continues to be widely investigated today with a broad range of application possibilities in spectroscopy, metrology, optical telecommunications, and a host of other fields [1]. Over the past decade much attention has been paid to the development of waveguide-based nonlinear optical wavelength converters that can be embedded in photonic integrated circuits [2]-[17]. The process most often employed in these devices is the third-order nonlinear interaction of Kerr four-wave mixing (FWM), which entails the conversion of a signal input frequency νs to an idler output frequency νi in the presence of a pump at frequency νp, with νp = νs = νi = νg. Various waveguide converters have already been demonstrated, several of which have been implemented in the silicon-on-insulator (SOI) material platform allowing for strong light confinement and high-precision manufacturing using the mature CMOS technology [1].

When targeting efficient FWM conversion over a wide range of signal wavelengths, wideband phase matching is required. This is quite challenging in SOI waveguides because of silicon’s strong material dispersion [4]. One approach to tackle this problem is to engineer the cross-sectional dimensions, and hence the dispersion, of SOI waveguides to establish (almost) perfect phase matching (PPM) over a wide continuous signal frequency band adjacent to the pump frequency. Turner and co-workers succeeded in designing an SOI waveguide exhibiting dispersion-engineered PPM over a broad pump-to-signal bandwidth of 52 THz in the near-infrared telecom domain [11]. However, the waveguide had to be made as long as 1.5 cm to attain significant conversion efficiencies around −20 dB at a pump input power of 110 mW. Such waveguide lengths result in large device footprints of minimally 10000μm². The required length could be somewhat reduced to sub-cm-values when suppressing the free carrier absorption in the waveguide by means of carrier-extracting p-i-n diodes. Nevertheless, the particular SOI waveguide developed by Turner et al. [11] exhibits another drawback: Its highly customized cross-section does not comply with the fabrication constraints of multi-project-wafer-oriented silicon photonics foundries, which employ a standard waveguide thickness of 220 nm [18]. Since the use of such foundries can pave the way to the large-volume fabrication of integrated photonic components at low cost [1], adhering to these foundry standards is a natural strategy to exploit the full potential of silicon photonic devices.

Moving from the customized SOI waveguide as discussed above [11] to foundry-compatible 220 nm-thick SOI waveguides does not have a strong influence on the required propagation length for efficient conversion, but it inevitably reduces the achievable PPM bandwidth [8]. However, it is possible to extend the wavelength range of phase-matched FWM outside the PPM bandwidth using the concept of quasi-phase-matching (QPM). When employing QPM for a given set of pump, signal and idler wavelengths, the phase mismatch for the considered set of wavelengths is periodically compensated by varying the waveguide’s optical properties along the propagation path. Different schemes for QPM of FWM processes in silicon waveguides were recently introduced, including QPM based on an adiabatic variation of the dispersion along an SOI waveguide [15] - [17], and ‘automatic’ QPM where the
anisotropy of silicon’s Kerr tensor is exploited to induce a spatially varying Kerr nonlinearity for Transverse Electric (TE) modes propagating along a uniform SOI ring or spiral [12]-[13]. These QPM schemes can be implemented while also allowing PPM conversion around the pump frequency [13] - [16], albeit with a small yet distinguishable penalty in PPM conversion efficiency. This penalty is induced by the spatial waveguide variations required for QPM, yielding less efficient PPM conversion than possible with uniform waveguide properties optimized for PPM all along the propagation path. More importantly, the efficiency of these QPM conversion schemes is less than optimal in that none of them achieve the ‘ideal’, most effective scenario where the third-order FWM nonlinearity periodically reverses sign along the waveguide, in analogy with periodically poled second-order nonlinear media [19]. As such, the existing QPM schemes for FWM processes in silicon waveguides are in fact not optimal when targeting wideband yet efficient wavelength conversion in foundry-compatible 220 nm-thick SOI waveguides with preferably a short propagation length.

In this paper, we numerically examine the opportunities for wideband wavelength conversion in a foundry-compatible 220 nm-thick SOI waveguide combined with the highly nonlinear two-dimensional material of graphene. When limiting the waveguide length to a few hundreds of microns to enable a small device footprint, we find that the deposition of a graphene top layer on the SOI waveguide leads to a significant conversion efficiency improvement in the PPM regime. Furthermore, the graphene deposition enables a new scheme for establishing QPM. Indeed, because of the tunability of the graphene properties through control of its chemical potential \( \mu \), it is possible to periodically reverse the sign of the effective FWM nonlinearity experienced by the TE waveguide modes along their propagation path, enabling for the first time QPM of FWM processes employing the ‘ideal’ and most effective scenario. The tunability of the top layer also allows for reconfiguration between the PPM and QPM conversion modes, so that both can be realized within the same waveguide without a penalty in PPM conversion efficiency. As a case study, we numerically simulate the wavelength conversion performance of a spiral-shaped graphene-covered foundry-compatible SOI waveguide in both the PPM and QPM regimes, and compare it with the performance attainable in the bare SOI waveguide.

This paper is organized as follows: In Section II, we address the basic concepts of PPM and QPM wavelength conversion in SOI waveguides covered with graphene. In Section III, we discuss the design of a spiral-shaped graphene-covered foundry-compatible SOI waveguide allowing both PPM and QPM conversion, and evaluate its performance in the PPM regime (Section III.A) and the QPM regime (Section III.B). Finally, we conclude in Section IV.

II. BASIC CONCEPTS OF PPM AND QPM WAVELENGTH CONVERSION IN GRAPHENE-COVERED SOI WAVEGUIDES

The spatial evolution of the pump, signal and idler fields at frequencies \( \nu_{p/s/i} \) or wavelengths \( \lambda_{p/s/i} \) in a graphene-covered SOI waveguide converter is governed by nonlinear propagation equations that describe Kerr-nonlinear interactions, including not only the FWM conversion process but also Kerr-induced phase modulation. We exclude Raman-nonlinear interactions since we can assume that the FWM transitions are detuned far away from the Raman resonances of graphene [20]. Furthermore, we focus on (quasi-) continuous-wave operation in the strong-pump approximation [21], and assume all fields to be TE polarized. Thus, the polarization of the fields is within the plane of the graphene top layer; having out-of-plane field components would not be beneficial for the FWM efficiency, since the Kerr tensor elements of graphene are considered negligible for the polarization direction perpendicular to the graphene sheet [22]. Finally, although a graphene-covered SOI waveguide is a heterogeneous medium, we will describe the fields in the same way as those in a homogeneous waveguide and introduce effective waveguide parameters (see Appendix) to take into account the medium’s hybrid structure. As such, we can use the following set of generic equations for describing the steady-state spatial evolution of the slowly varying pump, signal, and idler field amplitudes \( A_p(\zeta), A_s(\zeta), A_i(\zeta) \) [12], [21], [23]:

\[
\frac{\partial A_p}{\partial \zeta} = i \bar{\gamma}_p |A_p|^2 A_p - \alpha_p A_p - \left( \frac{\omega_p}{\omega_p} \right)^2 \left( \frac{\sigma_{FCA}}{2} - i \frac{\omega_p}{c} \sigma_{FCI} \right) \bar{N} A_p 
\]

\[
\frac{\partial A_s}{\partial \zeta} = i \bar{\gamma}_C |A_p|^2 A_s + i \bar{\gamma}_M A_p A_s^* e^{-i \Delta k \zeta} - \alpha_s A_s - \left( \frac{\omega_s}{\omega_s} \right)^2 \left( \frac{\sigma_{FCA}}{2} - i \frac{\omega_s}{c} \sigma_{FCI} \right) \bar{N} A_s 
\]

\[
\frac{\partial A_i}{\partial \zeta} = i \bar{\gamma}_C |A_p|^2 A_i + i \bar{\gamma}_M A_p A_i^* e^{-i \Delta k \zeta} - \alpha_i A_i - \left( \frac{\omega_i}{\omega_i} \right)^2 \left( \frac{\sigma_{FCA}}{2} - i \frac{\omega_i}{c} \sigma_{FCI} \right) \bar{N} A_i 
\]

where \( \zeta \) is the spatial coordinate along the waveguide, and where \( \bar{N}_{p/s/i}(\zeta) \) are normalized such that \( |A_{p/s/i}(\zeta)|^2 \) corresponds to power. The FWM terms are those that contain both the linear phase mismatch \( \Delta k = -2\kappa_p + k_s + k_i \) (with \( k_{p/s/i} \) being the pump, signal and idler wave numbers), and the effective coefficients \( \bar{\gamma}_M/M_2 = \bar{\gamma}_{M1/Si/M2/Si} + \bar{\gamma}_{M1/g/M2/g} \) comprising contributions from the SOI waveguide and the graphene sheet. In Eq. (1) the effective coefficient \( \bar{\gamma}_S \) contains \( \bar{\gamma}_{S,SL} \) accounting for self-phase modulation and two-photon absorption at the pump wavelength in the SOI waveguide, and also comprises \( \bar{\gamma}_{S,GR} \) capturing the corresponding phenomena in the graphene layer. In Eqs. (2)-(3) the effective coefficients \( \bar{\gamma}_{C1/C2} \) consist of \( \bar{\gamma}_{C1/Si/C2/Si} \) covering cross-phase modulation and cross-two-photon absorption in the SOI waveguide for pump/signal and pump/idler photons, respectively, and also comprise \( \bar{\gamma}_{C1/g/C2/g} \) representing the equivalent effects in the graphene. The factors \( \alpha_{p/s/i} = \alpha_{p/Si/Si/Si} + \alpha_{p/g/s/g/i} \) account for the linear losses in the SOI waveguide and the graphene sheet. The formulas for all these effective parameters can be found in Appendix. We remark that they all are function
of the spatial coordinate $\zeta$, as is required for QPM operation and will be discussed in detail in the next section. The last term in Eqs. (1)-(3) represents free-carrier effects with $\sigma_{\text{FCA}}$ and $\sigma_{\text{FCI}}$ coefficients quantifying the efficiency of free-carrier absorption and free-carrier index change, respectively [9], and with $\omega_p/\gamma_i = 2\pi|\gamma_i|/\gamma_p$ and $\omega_p = 2\pi c/(1550 \text{ nm})$ where $c$ indicates the speed of light. The factor $N$ in this term is the effective free-carrier density in the waveguide:

$$N = \frac{\tau_{\text{eff}}}{2h\nu_p A} \left[ \text{Im}(2\gamma_{\text{S,Si}}) + r_D \text{Im}(2\gamma_{\text{S,g}}) \right]|A_p|^4 + \frac{\tau_{\text{eff}} r_D}{h\nu_p A} \times \left( 2\alpha_{p,g}|A_p|^2 + 2\alpha_{s,g}|A_s|^2/\nu_s + 2\alpha_{s,g}|A_i|^2/\nu_i \right)$$

with $h$ Planck’s constant. The first term at the right-hand side of Eq. (4) represents the free-carrier generation induced by two-photon absorption in the SOI waveguide and the corresponding absorption contribution in the graphene top layer, and the second term indicates the free-carrier generation induced by one-photon absorption in the graphene layer only. The graphene contribution to these two terms has been expressed in a rather phenomenological way since not all photons absorbed in the graphene sheet give rise to the creation of free carriers and instead can contribute to, amongst others, intra-band transitions. The factor $\tau_{\text{eff}}$ in Eq. (4) indicates the effective free-carrier lifetime. Because of the short free-carrier lifetime in graphene, only the graphene-generated free carriers that diffuse into the silicon waveguide will effectively contribute to the free-carrier effects, as shown earlier [24]. As such, in our numerical simulations we are allowed to employ the values for $\tau_{\text{eff}}$, $\sigma_{\text{FCA}}$ and $\sigma_{\text{FCI}}$ of the bare SOI waveguide, and incorporate the contribution from the graphene-generated free carriers by including a graphene-to-SOI diffusion ratio $r$ in Eq. (4). This ratio quantifies the fraction of the free carriers generated in the graphene that diffuses into the SOI waveguide. Finally, the factor $A'$ in Eq. (4) represents the waveguide cross-sectional area over which the free carriers are distributed.

Turning now to the FWM terms in Eqs. (1)-(3), both the linear phase mismatch $\Delta k = -2k_p + k_s + k_i$ and the nonlinear phase mismatch contribution, function of $\text{Re}(\gamma_S)$ and the pump power $P_p$, need to be added up to obtain the full phase mismatch $\Delta k_{\text{total}}$. Using a Taylor series expansion for the linear contribution, $\Delta k_{\text{total}}$ can be expressed as

$$\Delta k_{\text{total}} = \beta_2 \Delta\omega_{\text{ps}}^2 + (1/12)\beta_4 \Delta\omega_{\text{ps}}^4 + 2\text{Re}(\gamma_S) P_p$$

with $\beta_i$ representing the $i$-th order dispersion at the pump wavelength and with $\Delta\omega_{\text{ps}} = 2\pi|\nu_s - \nu_p|$. Taking into account that the second-order dispersion term generally dominates over the fourth-order dispersion term, one can engineer $\beta_2$ to obtain PPM (i.e. $\Delta k_{\text{total}} \approx 0$) within a certain operation bandwidth in the vicinity of the pump frequency $\nu_p$. As the evolution of the idler power $|A_i|^2$ with position $\zeta$ depends on the phase mismatch along the expression \( \partial |A_i|^2 / \partial \zeta \propto \cos(-\Delta k_{\text{total}} \zeta) \), the idler experiences a continuous power growth in the PPM regime along the waveguide, and no phase-mismatch-induced transition from idler growth to idler depletion can occur.

Outside the PPM operation bandwidth, the phase mismatch $\Delta k_{\text{total}}$ can deviate significantly from zero due to the fourth-order dispersion term in Eq. (5), inducing a change in the fields’ phase relation so that the term $\cos(-\Delta k_{\text{total}} \zeta)$ determining the idler power evolution will periodically evolve along the waveguide from $\cos(0) = 1$ to $\cos(\pi) = -1$ and back. As a result, there will be a reversal of the conversion process with idler photons being annihilated, deteriorating the net idler growth. The latter can be overcome by employing QPM, so that for a discrete set of signal and idler frequencies very far away from the pump frequency one can also attain an efficient idler growth. Generally speaking, QPM aims at periodically compensating the phase-mismatch-induced change in the fields’ phase relation while avoiding the detrimental reversal of the conversion process, so that an overall efficient growth of the idler power is achieved [12]. Ideally, in the sections where the sign of $\cos(-\Delta k_{\text{total}} \zeta)$ has changed, one should also have a sign reversal in the Kerr nonlinearity, so that both sign changes annihilate each other and the idler continues to grow.

As mentioned, the different effective coefficients in Eqs. (1)-(3) take into account the contributions from both the SOI waveguide and the graphene top layer. The impact of the latter on both the loss parameters and the nonlinear parameters can be very strong. Experimental investigations with isolated graphene at photon energies above the one-photon absorption onset (i.e. $h\nu > 2 [\mu eV]$) have revealed extremely high values for the nonlinearity of the two-dimensional material [26] - [28]. Our recent calculations indicate that, when the chemical potential $\mu$ is tuned such that the photon energy is just below the onset of one-photon absorption, the nonlinearities also become very strong due to the presence of a resonance peak [22], [29]. This is not surprising, since conventional semiconductors also exhibit strong nonlinearity just below the threshold for single photon absorption [30]. This implies that the strong nonlinearity enhancement reported in [27] upon graphene deposition on a silicon waveguide can be combined with low linear absorption losses, favoring efficient PPM conversion. In addition, when moving from the one-photon to the two-photon absorption threshold a sign change occurs in the graphene nonlinearities, which is also in line with the behavior of the nonlinearities of direct-bandgap semiconductors [30]. For fixed photon energies this sign change can be controlled by changing $\mu$. Hence, by spatially varying the chemical potential $\mu$ of a graphene layer on top of an SOI waveguide using, e.g., locally deposited electrolyte gates, it should be possible to establish QPM conversion along the ‘ideal’ scheme where the nonlinearity periodically reverses sign along the propagation path. What is more, because of the voltage-tunability of graphene, one is also able to switch from the PPM conversion mode with a uniform, maximum

\[^1\]As shown earlier [16], [25], the idler evolution is not merely determined by $\Delta k$ as one might expect from Eq. (5), but by $\Delta k_{\text{total}}$ in which also the phase modulation effects are incorporated.
nonlinearity all along the waveguide, to the QPM conversion mode with a periodic sign reversal of the nonlinearity along the propagation path.

To numerically investigate the attainable conversion efficiency in these different regimes, we consider as a case study a foundry-compatible 220 nm-thick SOI waveguide converter shaped as a double spiral and covered with graphene. As shown in Figs. 1(a)-(b), the upper and lower spiral halves are covered with two separate graphene sheets with a small interspacing. These sheets are covered in turn with solid polymer electrolyte gates [31], indicated as the green-colored, dark-shaded area (‘gate 1’) and red-colored, light-shaded area (‘gate 2’). These two electrolyte gates are also separated by a small spacing. On top of each gate and on its underlying graphene sheet electrical contacts are placed, across which a voltage is applied to tune the graphene properties. By applying either the same or different bias voltages to the two gates, the optical fields propagating in the spiral waveguide experience either a constant high graphene nonlinearity along the spiral path as desired for PPM operation, or a graphene nonlinearity with periodically changing sign along the spiral as required for QPM operation. Neither PPM nor QPM operation will be affected by the spacing between the two graphene sheets as long as it is kept below a few micron, which is very well feasible using, e.g., photolithographic graphene patterning [32]. The particular converter design sketched in Fig. 1 offers several practical advantages: While the use of a spiral waveguide enables a small device footprint, the large-area gate patterning on top is less prone to fabrication errors than when depositing individual gates on each spiral section separately. As well, the solid polymer electrolyte gates made of, e.g., LiClO$_4$ and Poly(ethylene) Oxide (PEO) allow tuning the chemical potential of graphene to very high values using only low voltages (e.g., $|\mu| > 0.8$ eV using a voltage of only 3 V [31]), and can nowadays be patterned with submicron accuracy [33].

Another important asset of the spiral design is that in the QPM regime the periodicity of the positive nonlinear and negative nonlinear waveguide sections is chirped. As is the case for broadband second-harmonic generation in chirped periodically poled nonlinear media [19], we find here that this converter design allows QPM operation not just for one discrete signal wavelength far away from the pump wavelength, but for a continuous band of signal wavelengths, hence enabling truly wideband conversion.

In what follows we will determine the design parameters of the targeted graphene-covered SOI waveguide converter. To find the optimal values for the graphene chemical potential $|\mu|$, we will use graphs generated with our earlier presented theory [22], [29] for both the linear and nonlinear conductivities of graphene. This way we can determine at which $|\mu|$-values we have both low linear absorption and strong nonlinear effects. Our theoretical curves for the linear conductivity are in line with experimental data and with the widely used Kubo-formalism (see, e.g., Supplementary Information of [27]), so that we can directly implement these theoretical data in our numerical simulations for the converter. In contrast, the theoretical curves for the nonlinear conductivity are systematically lower than what has been experimentally observed.

Fig. 1. Basic concept of the graphene-covered spiral-shaped SOI waveguide converter with (a) PPM operation and (b) QPM operation. The upper and lower spiral halves are covered with two separate graphene sheets. These graphene sheets are covered in turn with separate solid polymer electrolyte gates, indicated as the green-colored, dark-shaded and red-colored, light-shaded areas. On top of each gate and on its underlying graphene sheet electrical contacts are placed, across which a voltage is applied to tune the graphene properties. In the case of PPM operation, different voltages are applied to both gates so that the optical fields propagating in the spiral experience a constant graphene nonlinearity along the spiral path (see right-hand side of figure (a)). In the case of QPM operation, the same voltage is applied to both gates so that the optical fields propagating in the spiral experience a spatially varying graphene nonlinearity along the spiral path (see right-hand side of figure (b)).
[22], [29], so we take them only as a qualitative guide. We use them only to estimate at which values of $|\mu|$ with low linear absorption we can expect nonlinearities as strong as those observed experimentally at $|\mu|$-values where the linear absorption was high. We then take these values of $|\mu|$ with small linear absorption as our working point, and implement the experimentally determined values of the nonlinearity in our numerical simulations.

III. DESIGN AND MODELING RESULTS FOR THE SPIRAL-SHAPED FOUNDRY-COMPATIBLE SOI WAVEGUIDE CONVERTER COVERED WITH GRAPHENE

A. PPM conversion

We first investigate the optimal graphene properties for PPM conversion. The two gates in Fig. 1 should be subjected to the same bias voltage $U_{PPM}$, or, in other words, all graphene sections should be tuned to the same chemical potential $\mu_{PPM}$. This chemical potential should be chosen such that the corresponding graphene nonlinearity is high and the linear absorption loss is low. As PPM conversion occurs between pump, signal and idler wavelengths that lie relatively close to each other, the nonlinearity $\sigma^{(3)}(-\omega_s, \omega_p, \omega_i)$ for FWM between the pump, signal and idler can be approximated by the nonlinearity for self-phase modulation at the pump wavelength. When assuming a telecom pump wavelength $\lambda_p$ of 1625 nm (this corresponds to a photon energy of 0.763 eV) and a conservative carrier scattering rate of 33 meV for both inter- and intra-band scattering in graphene, our earlier presented theory [22], [29] predicts that the graphene linear conductivity $\sigma^{(1)}(\omega_p)$ and its third-order conductivity for self-phase modulation $\text{Im}(\sigma^{(3)}(-\omega_p, \omega_p, \omega_p))$ will vary with the chemical potential as shown in Figs. 2 and 3. We point out that the relation between these conductivities and their corresponding bulk susceptibilities is given by $\chi = \sigma / (-i\omega\varepsilon_0d_c)$ with $\varepsilon_0$ the dielectric permittivity and $d_c = 0.3$ nm the effective thickness of graphene. Fig. 2 for the linear conductivity shows that, at $|\mu| > 0.763$ eV, for which the considered pump photon energy is below the one-photon absorption threshold, the one-photon absorption loss quantified by $\text{Re}(\sigma^{(1)})$ decreases from the universal conductivity $\sigma_0$ to values far below $\sigma_0$. In this region one no longer has absorption associated with interband transitions in the graphene, but the intraband transitions can still induce losses. As such, $\text{Re}(\sigma^{(1)})$ will not entirely vanish, as has also been verified experimentally [34], [35]. Regarding graphene’s nonlinearity, to our knowledge only experimental data for $|\mu| \leq 0.26$ eV are available. For example, Gu and co-workers [27] experimentally quantified the nonlinearity of a graphene sample with $|\mu| = 0.26$ eV for narrow-band FWM in the near-infrared telecom domain, and they obtained a Kerr nonlinear index $n_2 = 10^{-13}$ m$^2$/W and a two-photon absorption coefficient $\beta = 3000 \times 10^{-11}$ m/W. Following the approach in [36] and taking into account that at $|\mu| = 0.26$ eV the linear conductivity $\sigma^{(1)} \approx \sigma_0$ yields a complex refractive index, we find that the corresponding non-linear susceptibility is given by $\chi^{(3)}(-\omega_p, \omega_p, \omega_p) = (3.39 + i3.46) \times 10^{-15}$ m$^2$/V$^2$. This yields a nonlinear conductivity of $\sigma^{(3)}(-\omega_p, \omega_p, \omega_p) = (1.84 - i1.80) \sigma_0 \times 10^{-16}$ m$^2$/V$^2$.

Although experimental data are available only at relatively small chemical potentials, the qualitative trends of the curve in Fig. 3 show that at $|\mu| = 0.46$ eV the self-phase modulation nonlinearity is expected to be as large as at $|\mu| = 0.26$ eV, allowing us to adopt the $\sigma^{(3)}$-value of Gu and co-workers [27], while the single-photon loss in Fig. 2 is much lower than at $|\mu| = 0.26$ eV ($\text{Re}(\sigma^{(1)}) = 0.155 \sigma_0$). Hence, tuning both gates 1 and 2 to a bias voltage at which the underlying graphene sheet acquires $|\mu_{PPM}| = 0.46$ eV is an appropriate working point for the targeted PPM conversion. A detailed overview of the graphene conductivities is provided in Table I. This table also specifies the contributions of the graphene layer to the effective parameters for the graphene-covered SOI converter with PPM operation (see formulas in Appendix).

We now choose an appropriate cross-sectional SOI waveguide geometry with which we can fulfill the PPM condition $\Delta_{\text{total}} \approx 0$. We do not want our choice to depend on the pump power, since we target efficient conversion at different
pump powers. An appropriate way to approach $\Delta k_{\text{total}} \approx 0$ regardless the pump power used is by choosing a waveguide geometry for which $\beta_2$ vanishes at $\lambda_p = 1625 \, \text{nm}$, making the linear phase mismatch zero in the vicinity of the pump wavelength. We consider a 220 nm-thick strip SOI waveguide on which an oxide cladding is deposited that afterwards is etched down to the silicon top surface. As such, a planarized geometry is created [37] (see Fig. 4) which facilitates the transfer of a graphene layer on top using, e.g., the electrochemical delamination technique [38]. We note that, since wideband FWM is targeted, the waveguide input and output should be provided with broadband edge couplers [39] rather than grating couplers. The graphene layer, patterned [32] to create two separate sheets on top of the two spiral halves, is covered by solid polymer PEO electrolyte [33] to enable gating. For reasons that will be explained in the next section, we assume the waveguide spiral is constructed with an average radius of 23 $\mu$m, which for an Archimedean spiral outline as shown in Fig. 1 yields a total propagation distance of 350 $\mu$m and a device footprint as small as 2200 $\mu$m². For a 220 nm-thick SOI waveguide with an average bending radius of 23 $\mu$m, numerical mode simulations with the graphene cover layer and PEO cladding included show that a waveguide width of 670 nm yields a close-to-zero $\beta_2$ or dispersion parameter $D$ for the TE mode at $\lambda_p = 1625 \, \text{nm}$, as shown in Fig. 4.

The chosen cross-sectional waveguide geometry yields a cross-sectional area $A' = 1.5 \times 10^{-13} \, \text{m}^2$, an effective free-carrier lifetime $\tau_{\text{eff}}$ around 0.5 ns [11], and a linear SOI propagation loss $\alpha_{\text{prop}}(1.55 \, \text{µm}) = 34.5 \, \text{m}^{-1}$ (the latter corresponds to a power loss of $3 \, \text{dB/cm}$). For the free-carrier absorption efficiency and index change efficiency of the SOI waveguide we use $\sigma_{\text{FCA}} = 1.45 \times 10^{-21} \, \text{m}^2$ and $\sigma_{\text{FCI}} = -5.3 \times 10^{-27} \, \text{m}^3$.

For the mode simulations we used the commercial mode solver Lumerical. For silicon we adopted the refractive index data outlined in [40] for wavelengths above 1.2 $\mu$m. For SiO₂ we implemented the Sellmeier coefficients of [41]. The refractive index of the PEO electrolyte covering the graphene was assumed to be 1.45 [42]. The graphene layer was implemented as a thin, three-dimensional layer with a refractive index determined by its linear conductivity.

![Fig. 4. (Top) TE mode profile (black line) and (bottom) dispersion parameter D around 1625 nm for a 670 nm-wide, 220 nm-thick strip SOI waveguide with a bend radius of 23 $\mu$m and with oxide at the sides and with a graphene sheet featuring $|\mu| = 0.46 \, \text{eV}$ covered by a solid polymer PEO electrolyte on top of the waveguide.](image)

![Fig. 5. Signal-to-idler conversion efficiency in the PPM regime versus pump input power for: the graphene-covered converter (solid black line); the graphene-covered converter in the absence of linear absorption (solid grey line); the bare SOI waveguide converter (dashed black line); the bare SOI waveguide converter in case PPM and QPM operation are combined (dotted grey line). For all simulations the signal input power $P_{s,in}$ is taken to be 250 $\mu$W.](image)

[9]. Regarding the graphene-to-SOI diffusion rate we can derive from earlier reported experiments [24] that in the case of weakly doped graphene (i.e., with $|\mu| \lesssim 0$) deposited on an SOI waveguide $r_D$ is very small and below 0.01. Since we have worked with strongly doped graphene, the Shottky barrier between the graphene sheet and the intrinsic silicon underneath [43] will exceed that of [24] so that $r_D$ will most likely be even smaller than 0.01. Nevertheless, in our simulations we assume $r_D = 0.01$ to avoid understimating the graphene-induced free-carrier absorption and refractive index change. Finally, for the nonlinear parameters of the silicon waveguide we take a nonlinear refractive index $n_2 = 6 \times 10^{-18} \, \text{m}^2/\text{W}$ and a two-photon absorption coefficient $\beta = 0.5 \times 10^{-11} \, \text{m}/\text{W}$ [44], yielding $\chi^{(3)} = 2.56 \times 10^{-19} + 2.76 \times 10^{-20} \text{m}^2/\text{V}^2$.

To determine the attainable conversion efficiencies in the PPM regime for the considered graphene-covered SOI wave-
SOI waveguide converter considered earlier [11], a −20 dB conversion efficiency was achieved at a pump input power of only 110 mW, but the propagation length was more than 40 times longer and the footprint more than 4 times larger than for the graphene-covered foundry-compatible SOI waveguide converter considered here. The solid grey curve in Fig. 5 shows that, when totally neglecting the linear absorption loss of the graphene on top as is done in some studies (see, e.g., [46]), considerably higher performances exceeding −20 dB are predicted. However, even at our working point below the threshold for single-photon absorption induced by interband transitions, the graphene will still cause non-negligible loss associated with intraband transitions, as discussed earlier on and verified experimentally. The solid black curve in Fig. 5 properly accounts for the influence of this loss.

As shown in Fig. 6, our simulation results for the graphene-covered converter indicate a large PPM bandwidth. The −3dB-bandwidth is found to be 43 THz, which is not as large as the 52 THz bandwidth obtained for the extensively dispersion-engineered SOI waveguide considered earlier [11] but still represents a very significant value for a foundry-compatible waveguide with limited dispersion engineering possibilities. The difference between the two values is less than 20%. Hence, although graphene exhibits, like silicon, a strong material dispersion as shown by the imaginary part of the linear conductivity in Fig. 2, the graphene-covered SOI waveguide converter allows for broadband PPM operation.

### B. QPM conversion

For signal and idler wavelengths situated outside the PPM bandwidth, QPM conversion can be employed instead. As a case study, we consider QPM conversion between \( \lambda_s = 2370 \) nm (i.e. a signal photon energy of 0.523 eV) and \( \lambda_i = 1236.4 \) nm (i.e. an idler photon energy of 1.003 eV), while pumping at \( \lambda_p = 1625 \) nm. For the converter of Fig. 1 to operate in the QPM regime, gates 1 and 2 should induce different chemical potentials in the underlying graphene sections so that these acquire FWM nonlinearities of opposite sign. Like in the previous section, the linear absorption loss \( \text{Re}(\sigma^{(1)}) \) should be kept low. As shown in Fig. 7, the chemical potentials where this requirement is met at all three involved photon energies ranges from approximately 0.6 eV to 1 eV. To establish efficient QPM conversion, we thus need to determine two chemical potentials within this range that yield strong FWM nonlinearities with opposite sign. Our earlier presented theory [22], [29] predicts that graphene’s third-order conductivity \( \text{Im}(\sigma^{(3)}(\omega_s, \omega_p, \omega_i)) \) for FWM at the considered photon energies will vary with the chemical potential as shown in Fig. 8. The qualitative trends of this curve indicate 0.6 eV and 0.77 eV as two interesting chemical potential values for QPM conversion. To our knowledge no experimental data are available for wideband FWM in graphene at such high chemical potentials, but Hendry and co-workers carried out wideband FWM experiments in graphene with \( |\mu| \approx 0 \) eV [26]. The nonlinearities reported by Hendry et al. for their wideband FWM experiment and by Gu et al. [27] for their narrow-band FWM experiment are very similar. Therefore, like we did in...
TABLE II

<table>
<thead>
<tr>
<th>Graphene conductivities (adopted from theory and experiments as explained in the text) and graphene’s contributions to the effective parameters of the converter in the QPM regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPM – gate 1</td>
</tr>
<tr>
<td>Re(σ(1)(ω)) / σ₀ (eV)</td>
</tr>
<tr>
<td>Re(σ(1)(ω)) / σ₀ (eV)</td>
</tr>
<tr>
<td>Re(σ(1)(ω)) / σ₀ (eV)</td>
</tr>
<tr>
<td>σ(3)(ω, ω, ω) / σ₀ (×10^{-16} m²/V²)</td>
</tr>
<tr>
<td>σ(3)(ω, ω, ω) / σ₀ (×10^{-16} m²/V²)</td>
</tr>
<tr>
<td>α₀,p (×10^{12} m¹)</td>
</tr>
<tr>
<td>α₀,s (×10^{12} m¹)</td>
</tr>
<tr>
<td>α₁,s (×10^{13} m¹)</td>
</tr>
<tr>
<td>γ₁,S (×10^{-3} W⁻¹)</td>
</tr>
<tr>
<td>γ₂,S (×10^{-3} W⁻¹)</td>
</tr>
<tr>
<td>γ₃,S (×10^{-2} W⁻¹)</td>
</tr>
<tr>
<td>γ₄,S (×10^{-3} W⁻¹)</td>
</tr>
</tbody>
</table>

In the calculations, room temperature is assumed, and the inter- and intra-band scattering rates are taken to be 33 meV. Only the qualitative trends of this curve should be considered; the quantitative nonlinearity values used in the numerical simulations for the converter differ from the values indicated in this graph (see text).

Fig. 7. Graphene’s linear conductivity at room temperature for the photon energies of 0.763 eV and 0.763 eV and 1.003 eV versus chemical potential [22], [29]. The inter- and intra-band scattering rates are taken to be 33 meV. The values for the real and imaginary parts are shown on the left and right axes, respectively, and for the real part only that part of the curve is shown that is below 0.2 σ₀.

Fig. 8. Graphene’s nonlinear conductivity for four-wave mixing with a pump photon energy of 0.763 eV and a signal photon energy of 0.523 eV versus chemical potential, as calculated using our earlier presented theory [22], [29].

The linear phase mismatch has been calculated using numerical mode simulations with a graphene chemical potential of 0.69 eV, which is the average of the chemical potentials of the graphene sheets under gates 1 and 2.

has changed should correspond to the sections where the FWM nonlinearity has a reversed sign as well. For a converter configuration as in Fig. 1 the sign reversal of the nonlinearity occurs every time the spiral angle varies over π. Following the same reasoning as in our earlier work [12], [14] and taking into account that the nonlinear phase mismatch is small as compared to the linear part, the condition for having FWM in the QPM regime is given by

\[ R_{avg} = \frac{1}{|\Delta k|} \]  

with \( R_{avg} \) representing the average radius of the spiral-shaped waveguide. For the pump, signal and idler wavelengths under consideration, the linear phase mismatch \( \Delta k \) in the graphene-covered waveguide equals \( 4.4 \times 10^{-4} m^{-1} \), for which Eq. (6) yields \( R_{avg} = 23 \mu m \).

Using the graphene parameter values specified above, we obtain for the effective parameters defined in Appendix the values outlined in Table II. For the remaining simulation parameters we employ the same settings as previously.

To calculate the attainable conversion efficiencies in the QPM regime for the considered graphene-covered SOI waveguide converter and for its bare counterpart, we solve Eqs. (1)-(4) for signal and idler wavelengths of 2370 nm and 1236.4 nm, and for pump input powers \( P_{p,in} \) ranging between 10 mW and 900 mW, and we plot the corresponding conversion efficiencies in Fig. 9. To quantify the QPM bandwidth of the graphene-covered converter, we also solve Eqs. (1)-(4) for varying \( \lambda_s \) and \( \lambda_i \) in the vicinity of 2370 nm and 1236.4 nm, respectively. The resulting conversion efficiencies are shown in Fig. 10. To illustrate what happens inside the spiral, Fig.
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Fig. 9. Signal-to-idler conversion efficiency in the QPM regime versus pump input power for: the graphene-covered converter (solid black line); the bare SOI waveguide converter along the best-case scenario of having a 12 dB enhancement of the conversion efficiency obtained without QPM (dashed black line). For all simulations the signal input power \( P_{s,in} \) is taken to be 250 \( \mu W \).

Fig. 10. Signal-to-idler conversion efficiency of the graphene-covered SOI waveguide converter in the QPM regime for a pump input power of 500 mW and a signal input power of 250 \( \mu W \) as a function of signal wavelength centered around 2370 nm. The latter corresponds to a pump-signal frequency spacing of 58 THz. The \(-3\) dB QPM bandwidth is found to be 3.4 THz.

Fig. 11. Spatial evolution of the idler power within the spiral converter in the QPM regime for a pump input power of 500 mW and a signal input power of 250 \( \mu W \).

11 shows how the idler power evolves within the graphene-covered spiral for a pump input power \( P_{p,in} \) of 500 mW. For all simulation results the signal input power \( P_{s,in} \) is again taken to be 250 \( \mu W \). In principle, the efficiency curve in Fig. 9 for the bare SOI waveguide converter in the QPM regime depends on which of the existing QPM schemes for SOI waveguides is being used. However, all schemes seem to have in common that an efficiency enhancement up to 12 dB can be produced as compared to a waveguide without QPM [see, e.g., [13], [15] where QPM is obtained through a Kerr–nonlinearity variation and a dispersion variation, respectively]. This 12 dB enhancement in fact represents the best-case scenario and can only be obtained at specific pump powers [13]. Nevertheless, to ensure that the performance benchmark we set for evaluating the graphene-covered converter is sufficiently high, we determine the efficiency curve for the bare SOI waveguide converter in Fig. 9 by applying for all pump powers a 12 dB enhancement of the conversion efficiency obtained without QPM.

Although we have deliberately plotted the best-case-scenario conversion efficiency for the bare SOI converter in Fig. 9, we find that the graphene-covered SOI waveguide with the new QPM scheme introduced here yields significantly higher conversion efficiencies. At low pump powers, performance improvements up to 8 dB can be achieved. The attainable conversion efficiencies of the graphene-covered converter in the QPM regime are lower than in the PPM regime, but still can exceed \(-30\) dB for sub-watt level pump powers and a propagation distance of only 350 \( \mu m \). The lower conversion efficiencies in the QPM regime as compared to the PPM regime are a direct consequence of the basic concept of QPM, where the non-zero phase mismatch, although periodically compensated, inevitably results in varying growth rates for the idler along the spiral (see Fig. 11). The main strength of the QPM concept as compared to PPM operation is that it can be applied to signal wavelengths far outside the PPM bandwidth. In this particular case, the pump-signal frequency difference equals 58 THz which significantly goes beyond the 43 THz bandwidth in the PPM regime (see Section III.A), and even beyond the 52 THz record PPM bandwidth of the extensively dispersion-engineered SOI waveguide considered earlier [11]. We remark that, when totally neglecting the linear graphene absorption in the graphene-covered converter, QPM performances close to \(-20\) dB are predicted as shown by the solid grey curve in Fig. 9.

Fig. 10 illustrates that thanks to the chirped nature of the QPM periodicity in the spiral-shaped graphene-covered SOI waveguide, efficient QPM operation is not just obtained at one discrete signal wavelength, but over a continuous band around the design signal wavelength \( \lambda_s = 2370 \) nm. Its \(-3 \Delta \) -bandwidth equals 3.4 THz. As such, we find that the considered graphene-covered foundry-compatible spiral-shaped SOI waveguide converter not only allows for broadband PPM conversion but also for QPM conversion over considerable bandwidths centered at signal wavelengths very far away from the pump wavelength.

IV. CONCLUSION AND OUTLOOK

The use of graphene as a waveguide cover layer opens up different routes towards wideband FWM conversion in foundry-compatible SOI structures. For the spiral-shaped graphene-covered foundry-compatible SOI waveguide studied here with a length of a few hundreds of microns, we found PPM conversion efficiencies that approach \(-20\) dB and that are up to 8 dB higher than those of the bare SOI waveguide. At the same time, the PPM signal bandwidth is less than 20% smaller than the record PPM bandwidth achievable in...
extensively dispersion-engineered SOI waveguides that are not foundry-compatible. The latter can also yield \(-20\) dB conversion efficiencies with pump powers that are up to 10 times lower than those considered here, but in that case waveguide lengths more than 40 times longer and footprints more than 4 times larger than those used here are required. We also found that the new graphene-enabled QPM scheme where the ‘ideal’ scenario of periodically reversing the nonlinearity sign is implemented yields conversion efficiencies that are up to \(8\) dB higher than those attainable with the existing QPM schemes for bare SOI waveguides in the best-case scenario. QPM conversion efficiencies exceeding \(-30\) dB have been shown for a 3.4 THz-wide band centered at a signal wavelength that is even outside the record PPM bandwidth of SOI waveguides incompatible with foundry fabrication constraints. The possibility of switching between the PPM and QPM conversion modes is an additional asset that further leverages the practical usability of graphene-covered foundry-compatible SOI waveguide converters.

As pointed out in the paper, the broadband character of graphene’s nonlinear optical behavior has already been experimentally investigated above the one-photon absorption onset (i.e. \(hv > 2|\mu|\) or \(|\mu| < hv/2\)) through, amongst others, wideband FWM experiments in graphene with \(|\mu| \approx 0\) eV [26]. Figs. 3 and 8 plotted for given photon energies show that, as long as \(|\mu| < hv/2\), graphene’s nonlinear characteristics are practically constant as a function of chemical potential. Equivalently, when fixing the chemical potential, graphene’s nonlinearities only weakly depend on the photon energy as long as \(hv \gg 2|\mu|\). As a result, graphene samples with \(|\mu| \approx 0\) eV indeed exhibit very broadband nonlinear optical properties from the microwave domain to the visible region [26], [47]. In this paper, however, we have considered graphene’s optical nonlinearities at a working point below the one-photon absorption threshold. We have found that also below the one-photon absorption threshold strong nonlinearities can be obtained over a broad range of photon energies, provided that the graphene chemical potential is tuned to a properly chosen value. Taking into account that with electrolyte top gates the chemical potential can be varied (in absolute value) from 0 eV to beyond 1 eV, the operation bandwidth of the graphene-covered waveguide converter presented here could hypothetically also range from the microwave to the visible domain. In practice, however, it will only cover the spectral domains where the waveguide material is transparent, which in the case of a silicon waveguide comprises the near- and mid-infrared.

Besides employing graphene, it is also interesting to consider using more recently introduced two-dimensional materials such as MoS\(_2\) [48], [49] and black phosphorus [50], [51] for wavelength conversion purposes. Graphene turns out to have one of the highest Kerr nonlinear indices over the widest bandwidth as compared to other two-dimensional materials (see, e.g., [49] and [51]) because of the resonant enhancement over a broad range of photon energies in graphene’s linear, gapless band structure. Another special feature of graphene is its electrically or chemically tunable chemical potential which allows controlling both its nonlinear optical properties and its linear absorption losses. These characteristics could also be controlled to some extent in the above mentioned two-dimensional materials with a bandgap, since it is possible to modify their bandgap size [50]. Whereas the gapped two-dimensional materials are likely to be outperformed by graphene when targeting wideband wavelength conversion based on third-order nonlinear processes, they do offer the advantage that they naturally exhibit a second-order optical nonlinearity [52] whereas graphene does not. From a practical point of view, materials like MoS\(_2\) and black phosphorus are still quite challenging to prepare and process [50], but this will most likely improve over time as has been the case with graphene. Therefore, for future nonlinear optical devices one could think of using multilayer hetero-structures involving both graphene and the emerging gapped two-dimensional materials, as this could allow combining the best of both worlds.

### APPENDIX A

We assume a graphene-covered silicon waveguide running in the \(\zeta\) direction. The electric field associated with a waveguide mode is written as proportional to

\[
e_\mu(x, y)e^{ik_\mu\zeta},
\]

where \(\mu = p, s, i\) indicates the mode profile appropriate for the pump, signal or idler; we take the normal to the top of the waveguide to be \(\hat{y}\). When determining these modal fields, we include in the Maxwell equations the linear optical properties of silicon (assumed lossless at the frequencies of interest), and treat the graphene as a thin layer with thickness \(d_\|\) on top of the waveguide. For the modal field simulations, it is allowed to take \(d_\|\) in a range around 0.3 nm for which the mode simulation results converge. We include only the imaginary part of the linear graphene conductivity \(\sigma^{(1)}(\omega)\) in determining the waveguide mode, converting it to an effective dielectric tensor affecting the optical response of the thin layer only in the \(x\) and \(\zeta\) directions. We neglect any optical response of the graphene in the \(y\) direction. The real part of the linear graphene conductivity, which describes linear loss, and the nonlinear effects are included by allowing the mode amplitudes to be slowly varying, in the usual “coupled mode equation” approach. Our approach is similar in spirit to that of Lin et al. [9], but we use a strategy [53] that allows for both modal and material dispersion. The group velocities, including modal and material dispersion, of the modes are defined as

\[
v_\mu = \frac{d\omega_\mu(k)}{dk},
\]

and thus include both contributions to the full dispersion relation \(\omega_\mu(k)\) of mode \(\mu\). This slightly modifies the normalization condition of the modes [54]; the strategy has been used earlier for the description of quantum nonlinear optical effects [55], and the inclusion of nonlinear effects mirrors the treatment here. We use the function \(s_{\mu}(x, y)\) (\(s_{\mu}(x, y)\) = 1) to indicate the location of the silicon (graphene), taking \(s_{\mu}(x, y) = 1\) (\(s_{\mu}(x, y) = 1\)) where the silicon (the thin layer modeling the graphene) is present and \(s_{\mu}(x, y) = 0\) (\(s_{\mu}(x, y) = 0\)) where the silicon (the thin layer modeling
the graphene is not present. We find the following formulas for the effective parameters of the graphene-covered silicon waveguide:
\[ \begin{align*}
\bar{\alpha}_p &= \alpha_{p, S_i} + \alpha_{p, g} \\
&= \alpha_{p, S_i} + (2v_p d_g I_p)^{-1} \text{Re}(\sigma^{(1)}(\omega_p)) \int e_p^*(x', y') \cdot e_p^\parallel(x', y') s_g(x', y') \, dx' \, dy', \\
\bar{\alpha}_s &= \alpha_{s, S_i} + \alpha_{s, g} \\
&= \alpha_{s, S_i} + (2v_s d_g I_s)^{-1} \text{Re}(\sigma^{(1)}(\omega_s)) \int e_s^*(x', y') \cdot e_s^\parallel(x', y') s_g(x', y') \, dx' \, dy', \\
\bar{\alpha}_i &= \alpha_{i, S_i} + \alpha_{i, g} \\
&= \alpha_{i, S_i} + (2v_i d_g I_i)^{-1} \text{Re}(\sigma^{(1)}(\omega_i)) \int e_i^*(x', y') \cdot e_i^\parallel(x', y') s_g(x', y') \, dx' \, dy', \\
\bar{\gamma}_S &= \gamma_{S, S_i} + \gamma_{S, g} \\
&= (4v_p^2 I_p^2)^{-1} 3\epsilon_0 \omega_p \chi_S^{(3),ijkl} \int (e_p^i(x', y'))^* e_p^l(x', y') (e_p^k(x', y'))^* e_p^j(x', y') s_S(x', y') \, dx' \, dy' \\
&\quad + (4v_p^2 d_g I_p^2)^{-1} 3\epsilon_0 \omega_S \chi_S^{(3),ijkl} \int (e_p^i(x', y'))^* e_p^l(x', y') (e_p^k(x', y'))^* e_p^j(x', y') s_S(x', y') \, dx' \, dy', \\
\bar{\gamma}_C &= \gamma_{C, S_i} + \gamma_{C, g} \\
&= (2v_p d_g I_p) \int (e_p^i(x', y'))^* e_p^l(x', y') (e_p^k(x', y'))^* e_p^j(x', y') s_S(x', y') \, dx' \, dy' \\
&\quad + (2v_p d_g I_p) \int (e_p^i(x', y'))^* e_p^l(x', y') (e_p^k(x', y'))^* e_p^j(x', y') s_S(x', y') \, dx' \, dy', \\
\bar{\gamma}_M &= \gamma_{M, S_i} + \gamma_{M, g} \\
&= (4v_p I_p \sqrt{v_s v_i I_s I_i})^{-1} 3\epsilon_0 \omega_s \chi_M^{(3),ijkl} \int (e_s^i(x', y'))^* (e_p^l(x', y'))^* e_p^k(x', y') e_p^j(x', y') s_M(x', y') \, dx' \, dy' \\
&\quad + (4v_p I_p \sqrt{v_s v_i I_s I_i})^{-1} 3\epsilon_0 \omega_M \chi_M^{(3),ijkl} \int (e_s^i(x', y'))^* (e_p^l(x', y'))^* e_p^k(x', y') e_p^j(x', y') s_M(x', y') \, dx' \, dy', \\
\bar{\gamma}_2 &= \gamma_{2, S_i} + \gamma_{2, g} \\
&= (4v_p I_p \sqrt{v_s v_i I_s I_i})^{-1} 3\epsilon_0 \omega_s \chi_2^{(3),ijkl} \int (e_s^i(x', y'))^* (e_p^l(x', y'))^* e_p^k(x', y') e_p^j(x', y') s_M(x', y') \, dx' \, dy' \\
&\quad + (4v_p I_p \sqrt{v_s v_i I_s I_i})^{-1} 3\epsilon_0 \omega_M \chi_2^{(3),ijkl} \int (e_s^i(x', y'))^* (e_p^l(x', y'))^* e_p^k(x', y') e_p^j(x', y') s_M(x', y') \, dx' \, dy'.
\end{align*} \]
Here, the fields $\epsilon_p^{\mu}(x,y)$ indicate only the $x$ and $\zeta$ components of $\mathbf{e}_\mu(x,y)$. Furthermore,
\[
I_\mu = \int dx dy \epsilon_0 x \left( \frac{ v_{\text{phase},x}(x,y;\omega_\mu)}{v_{\text{group},x}(x,y;\omega_\mu)} \right)^2 \epsilon_p^{\mu}(x,y) + \frac{ v_{\text{phase},y}(x,y;\omega_\mu)}{v_{\text{group},y}(x,y;\omega_\mu)} \epsilon_p^{\mu}(x,y) + \frac{ v_{\text{phase},\zeta}(x,y;\omega_\mu)}{v_{\text{group},\zeta}(x,y;\omega_\mu)} \epsilon_p^{\mu}(x,y) \right)^2
\]

where
\[
v_{\text{phase},(x,y,\zeta)}(x,y,\omega) = \frac{c}{n_{x,y,\zeta}(x,y;\omega)}, \quad (8)
\]

and
\[
v_{\text{group},(x,y,\zeta)}(x,y,\omega) = \left(1 + \frac{ \omega}{n_{x,y,\zeta}(x,y;\omega)} \right) \frac{ n_{x,y,\zeta}(x,y;\omega)}{\omega}, \quad (9)
\]

are respectively the local material phase and group velocities. For $(x,y)$ at a point in the silicon we have $n_{x,y}(x,y;\omega) = n_{x,y}(x,y;\omega) = \sqrt{\epsilon_{Si}(\omega)}$, where $\epsilon_{Si}$ is the (assumed real) relative dielectric constant of silicon. For $(x,y)$ at a point in the graphene and the PEO layers we have $n_{x,y}(x,y;\omega) = n_{x,y}(x,y;\omega) = \sqrt{\epsilon_{GE}(\omega)}$, where we only take the lossless part of the graphene response into account in determining the waveguide modes; $n_{x,y}(x,y;\omega) = 1$ since we neglect any optical response of graphene perpendicular to the plane in which it lies. Finally, $n_{x,y}(x,y;\omega) = n_{x,y}(x,y;\omega) = \sqrt{\epsilon_{SiO_2} = \sqrt{\epsilon_{PEO}}} in the SiO_2 (PEO) surrounding the waveguide, and $n_{x,y}(x,y;\omega) = n_{x,y}(x,y;\omega) = 1$ in air.

We were able to verify the expressions above for the effective linear loss parameters $\alpha_{p/s,i}$ by comparing the values of $\alpha_{p/s,i}$ with the linear loss values that the commercial Numerical MODE software readily calculates for graphene-covered waveguides. An excellent correspondence was obtained with relative differences of only a few percent.

Turning now to the expressions for the nonlinear parameters $\tilde{\kappa}^{S/C1/C2/M1/M2}$ we can assume in first order that all nonlinear susceptibilities $\chi^{(3)}$ of silicon used in these formulas are equal. The nonlinear conductivities of graphene are given by
\[
\sigma^{(3)}_{S} = \sigma^{(3),ijkl}(-\omega_p;\omega_p, -\omega_p, \omega_p),
\]
\[
\sigma^{(3)}_{C1} = \sigma^{(3),ijkl}(-\omega_s;\omega_p, -\omega_p, \omega_s),
\]
\[
\sigma^{(3)}_{C2} = \sigma^{(3),ijkl}(-\omega_s;\omega_s, -\omega_s, \omega_p),
\]
\[
\sigma^{(3)}_{M1} = \sigma^{(3),ijkl}(-\omega_s;\omega_s, -\omega_s, \omega_s),
\]
\[
\sigma^{(3)}_{M2} = \sigma^{(3),ijkl}(-\omega_s;\omega_p, -\omega_p, \omega_s).
\]

For the numerical simulations in the paper we approximate $\sigma^{(3)}_{C1/C2}$ by $\sigma^{(3)}_{S}$ and $\sigma^{(3)}_{M1}$ by $\sigma^{(3)}_{M2}$. This approximation, which is generally used in the literature for silicon, allows for relating all nonlinear conductivities of graphene in Eq. (7) to experimentally determined values. We point out that, although to our knowledge the PEO electrolyte we consider is not known to exhibit particularly strong nonlinear optical effects, there exist other polymer electrolytes with relatively high nonlinear optical coefficients of the order of $\chi^{(3)} = 10^{-18}$ esu [46]. But even when using such a highly nonlinear electrolyte cover layer, its contribution to the waveguide’s effective nonlinear parameters would be more than 30 times smaller than that from the graphene sheet. Therefore, we can disregard the nonlinearity contribution from the electrolyte cover layer in the formulas for the effective nonlinear parameters in Eq. (7). We do take the silicon waveguide into account in these formulas because, even though silicon’s $\chi^{(3)}$ is smaller than $1.4 \times 10^{-18}$ esu, it contributes significantly more to the effective nonlinear parameters than the electrolyte. This is due to the much higher field strength in the silicon waveguide as compared to that in the electrolyte cover layer. Finally, we remark that all effective coefficients in Eq. (7) depend on the position along the waveguide because of the position dependence of the graphene chemical potential realized with the electrolyte gates on top.

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