A Dynamic Model of Cash Management and Payment Choices with Limited Card Acceptance

Carlos Arango
Yassine Bouhdaoui
David Bounie
Observations

• Nearly full adoption of payment cards (debit/credit) by the consumers
  – e.g. Canada 99%, US 88%, Germany 94%, Austria 86%, the Netherlands 100%.

• However, the card is not always used in transactions when the option is brought about by retailers.
  – Canada: average card acceptance rate 73%, market share of cards by volume 44%,
  – Austria: 63% vs 16%,
  – Germany: 57% vs 15%.
Observations

- Detailed market share of the card (debit/credit) conditional on the acceptance by merchants in Canada:
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- Detailed market share of the card (debit/credit) conditional on the acceptance by merchants in Canada:

A large fraction of consumers do not use card in transactions especially for low amounts.
One explanation

One explanation in the literature is the ‘Cash First’ payment policy:

- Agents use cash whenever they have enough at hand.
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  - Agents use cash whenever they have enough at hand.

- Bouhdaoui and Bounie (2012): the ‘Cash First’ rule (a cash holding criterion) fits better the payment choice of the public than the transaction size criterion.
- Arango et al. (2014): the ‘Cash First’ rule replicates well the market share of cash in three countries.
Not the only rule

- Superimposing the cumulative distribution of cash holdings gives:
Not the only rule

- Huynh et al. (2014): the share of card payments when agents have enough cash at hand:
  - 35% in Canada,
  - 21% in Austria.
Objective of the paper

Provide the micro-foundations of a ‘Card First’ payment policy.
Plan

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The model

• A model combining the cash management and payment choice decisions.
  – “Payment instrument choice is an integral part of consumers’ cash management practice” (Briglevics and Schuh, 2014).

• Includes features of the model of Alvarez and Lippi (2015):
  – Free opportunities to adjust cash balances at a probability $f$.
  – Costly cash withdrawals.
  – Variable cost for card payment $\gamma \cdot x$, with $x$ the transaction size.
• **New features:**

1. **Discrete transactions:**
   - $\pi^D(x)$ refers to the distribution of transactions.

2. **Uncertainty on the transaction size:**
   - The agent cannot anticipate the sequencing of transactions.

3. **Limited card acceptance:** $\alpha(x) < 1$, for some $x$.
   - The acceptance varies with the transaction size.
The model

• A period is divided into two subperiods:
  – 1\textsuperscript{st} subperiod: The agent decides upon adjusting cash balances.
  • Free or at a fixed cost $b$.

  – 2\textsuperscript{nd} subperiod: The agent faces a transaction with a given status for card acceptance.
  • If he pays with card: $\gamma \cdot x$.
  • If the card is not accepted and the cash holdings are not sufficient:
    – ‘unscheduled’ cash withdrawal: $\mu \cdot b$.
  • Opportunity cost for holding cash $k \cdot m$. 
The model

- Equilibrium:
  - The value functions: the expected discounted cost of payments
    - at the beginning of the first subperiod: $V_t(m)$,
    - at the beginning of the second subperiod: $v_t(m)$.

- 1\textsuperscript{st} subperiod:
  \[ V_t(m) = f \cdot v_t(\bar{m}) + (1 - f) \cdot \min \{ v_t(\bar{m}) + b , v_t(m) \} \]
  with \[ v_t(\bar{m}) = \min_{m} v_t(m). \]
The model

- $2^{nd}$ subperiod, the payment choice problem:

$$v_t(m) = \sum_x \pi^D(x) \cdot [\alpha(x) \cdot v^a_t(m, x) + (1 - \alpha(x)) \cdot v^{na}_t(m, x)]$$
The model

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• The value function when the card is accepted:

\[
v^a_t(m, x) = \begin{cases} 
  k \cdot m + \min\{\beta \cdot V_{t+1}(m - x), \gamma \cdot x + \beta \cdot V_{t+1}(m)\}; & \text{if } m \geq x \\
  k \cdot m + \gamma \cdot x + \beta \cdot V_{t+1}(m); & \text{if } m < x
\end{cases}
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- The value function when the card is not accepted:

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v_t^{na}(m, x) = \begin{cases} 
  k \cdot m + \beta \cdot V_{t+1}(m - x); & \text{if } m \geq x \\
  k \cdot m + \mu \cdot b + \beta \cdot V_{t+1}(\bar{m}); & \text{if } m < x.
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The model

- **Cash withdrawal decision** \( \Rightarrow I^w(m) \) the indicator function of costly cash withdrawals.
- **Payment choices** \( \Rightarrow I^c(m, x) \) the indicator function of cash payments when the card is accepted.
- \( p = (\bar{m}, I^w, I^c) \) fully defines a cash management and payment choice policy.
The model

- $V_t$ satisfies a stochastic dynamic programming problem:

$$V_t = T(V_{t+1}),$$

with

$$T(V_{t+1})(m) = \min_{p \in \mathcal{P}} \{ \tilde{r}(m, p) + \beta \cdot \sum_{m'} \tilde{\lambda}(m', m, p) \cdot V_{t+1}(m') \}.$$
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• We call an $\textbf{optimal}$ policy, a policy $p_V = (\bar{m}_V, I^w_V, I^c_V)$ associated to a $\textbf{fixed point}$ of the mapping $T$:

$$V = T(V).$$
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- We call an **optimal** policy, a policy $p_V = (\tilde{m}_V, I^w_V, I^c_V)$ associated to a **fixed point** of the mapping $T$:

$$V = T(V).$$

- **Theorem 1:** Given a model calibration $(k > 0, \gamma \geq 0, b > 0)$, $T$ has **unique fixed point** on the space of bounded functions i.e. there is a **unique** optimal policy.
The model

- Optimal policy $\Rightarrow$ the share of card payments.
The model

• Optimal policy $\Rightarrow$ the share of card payments.

• **Example**: Consider the following case:
  - **payment choice policy**: ‘Cash First’.
  - **optimal cash balance**: $\bar{m} = 100$.
  - **distribution of transactions**: only two transactions $x_1=20$ and $x_2=70$ with the same probability.
  - **card acceptance**: full.
The model

• Optimal policy ⇒ the share of card payments.

• **Example**: Consider the following case:
  – payment choice policy: ‘Cash First’.
  – optimal cash balance: $m = 100$.
  – distribution of transactions: only two transactions $x_1=20$ and $x_2=70$ with the same probability.
  – card acceptance: full.

➢ initial cash balance $m_0=100$
The model

• Optimal policy \( \Rightarrow \) the share of card payments.

• **Example**: Consider the following case:
  
  – payment choice policy: ‘Cash First’.
  
  – optimal cash balance: \( m = 100 \).
  
  – distribution of transactions: only two transactions \( x_1 = 20 \) and \( x_2 = 70 \) with the same probability.
  
  – card acceptance: full.

\[ m_{+1} = \begin{cases} 80 & \frac{1}{2} \\ 30 & \frac{1}{2} \end{cases} \]

• initial cash balance \( m_0 = 100 \)
The model

- **Optimal policy** ⇒ the share of card payments.

- **Example**: Consider the following case:
  - payment choice policy: ‘Cash First’.
  - optimal cash balance: $m = 100$.
  - distribution of transactions: only two transactions $x_1=20$ and $x_2=70$ with the same probability.
  - card acceptance: full.

- initial cash balance $m_0=100$

  - $m_1=80$ \(\frac{1}{2}\) \quad m_2=60 \(\frac{1}{4}\)
  - $m_1=30$ \(\frac{1}{2}\) \quad m_2=10 \(\frac{1}{4}\)
  - $m_2=30$ \(\frac{1}{4}\) \quad m_2=10 \(\frac{1}{4}\)    ...
Prior step: Characterize the distribution of cash holdings resulting from an optimal policy.
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The distribution of cash holdings

- The law of motion:
  - \( \pi_t^{(a)} \) and \( \pi_t^{(b)} \) refer to the distributions of cash holdings at the beginning of the first and second subperiods, respectively.
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  - \( \pi_t^{(a)} \) and \( \pi_t^{(b)} \) refer to the distributions of cash holdings at the beginning of the first and second subperiods, respectively.
  - Law of motion of \( \pi_t^{(a)} \) and \( \pi_t^{(b)} \) for \( m \neq \bar{m} \):

\[
\pi_t^{(b)}(m) = (1 - f) \cdot (1 - \Gamma_w(m)) \cdot \pi_t^{(a)}(m)
\]
The distribution of cash holdings

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  - $\pi_t^{(a)}$ and $\pi_t^{(b)}$ refer to the distributions of cash holdings at the beginning of the first and second subperiods, respectively.
  - Law of motion of $\pi_t^{(a)}$ and $\pi_t^{(b)}$ for $m \neq \bar{m}$:
    \[
    \pi_t^{(b)}(m) = (1 - f) \cdot (1 - \Gamma^w(m)) \cdot \pi_t^{(a)}(m)
    \]
    
    and
    \[
    \pi_{t+1}^{(a)}(m) = \sum_x \alpha(x) \cdot \pi^D(x) \cdot \left( \Gamma^c(m + x, x) \cdot \pi_t^{(b)}(m + x) + (1 - \Gamma^c(m, x)) \cdot \pi_t^{(b)}(m) \right)
    + \sum_x (1 - \alpha(x)) \cdot \pi^D(x) \cdot \pi_t^{(b)}(m + x).
    \]
  - The normalization condition gives the value for $m = \bar{m}$. 
The distribution of cash holdings

- **Lemma:** Agents following an optimal policy give up cash holdings above the optimal cash balance $\bar{m}$ after a finite number of periods.

- **Theorem 2:** An optimal cash management and payment choice policy gives rise to a **unique** stationary distribution of cash holdings.
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‘Card First’ payment policy

(work in progress)

- **Theorem**: Agents facing only one transaction size with a limited card acceptance make an **optimal ‘Card First’ payment** for at least one cash balance if the cost of the card is sufficiently low.

Formally:

Let \( x > 0 \) with \( \alpha(x) < 1 \) and \( \pi^D(x) = 1 \), we have:

\[ \exists \gamma_c > 0 \text{ such that } \forall \gamma \leq \gamma_c, \exists m \geq x \text{ satisfying } I^c(m, x) = 0. \]
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Simulations for Canada

- Survey commissioned by the BoC in 2009:
  - Study individual payment patterns
    - Focus on transactions at the point of sale.

**Distribution of transactions**

**Acceptance rate of cards**
Model calibration

• **Reference scenario:**
  – Cost of a cash withdrawal: \( b = 1.5\$ \).
  – Penalty factor on unscheduled cash withdrawals: \( \mu = 5 \).
  – Probability of free withdrawal opportunities: \( f = 4\% \).

• We test a **non restrictive** set of values for \( \gamma \):
  – from 0% to 2%.

• Perform robustness checks for \( b \) from 0.5$ to 3.5$. 
Model calibration

• Methodology:

1. Determine the optimal policy by computing iterations of $V_t$ starting from an initialization $V_0$.
   Convergence criteria: $\|V_{t+1} - V_t\| < 1 \times 10^{-4}$.

2. Compute the distribution of cash holdings resulting from the optimal policy.
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Results

Reference scenario (‘Card First’ payments in blue)

$\gamma = 0\%$

$\gamma = 0.4\%$

- x-axis: Cash holdings
- y-axis: Transaction size
Results

Reference scenario (‘Card First’ payments in blue)

\[ \gamma = 1\% \]

\[ \gamma = 2\% \]

x-axis: Cash holdings
y-axis: Transaction size
Results

- Share of ‘Card First’ payments as a function of cash holdings:
Results

- Share of ‘Card First’ payments as a function of cash holdings:

Upper bound on cash holdings in the stationary equilibrium ($\bar{m}$)
Results

- Share of ‘Card First’ payments as a function of cash holdings:
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- Share of ‘Card First’ payments as a function of cash holdings

![Graph showing the share of 'Card First' payments as a function of cash holdings. The graph has a horizontal axis labeled with cash holdings ranging from 0 to 220, and a vertical axis labeled with percentage from 0% to 100%. Three curves are shown for different values of $\gamma$: $\gamma=0\%$, $\gamma=0.4\%$, and $\gamma=1\%$. The curves demonstrate how the share of 'Card First' payments decreases as cash holdings increase.]
Results

• Share of ‘Card First’ payments as a function of cash holdings:
Results

- Share of ‘Card First’ payments as a function of cash holdings:
Results

- Impact of the cost of withdrawal, $b$:
  
  Average share of ‘Card First’ payments

![Graph showing the impact of cost of withdrawal on 'Card First' payments]
Results

• Simulations fit well statistics on the average share of cash holdings (84$) and the global share of card payments (50%) for:
  - $\gamma = 0.4\%$ and $b=1.5$ (3% of daily expenditures), close to the reference scenario of Alvarez and Lippi (2015).
Summary

- Introducing a model of cash management and payment choice with discrete transactions and limited acceptance of cards.

- Uncertainty on the transaction size and the acceptance of cards give rise to an optimal ‘Card First’ payment policy.
  - Cash burning effect: the share of ‘Card First’ payments tends to decrease with the cash balances.
  - An optimal policy can incorporate both the ‘Cash First’ and the ‘Card First’ rules for different levels of cash balances.

- Our reference scenario fits well the observed average cash holdings and the global market share of card payments.
Future research

- The impact of the card acceptance on the payment choices.
- Assess precautionary cash holdings: withdrawals before cash balances are depleted.
Thank you for your attention.

Questions?